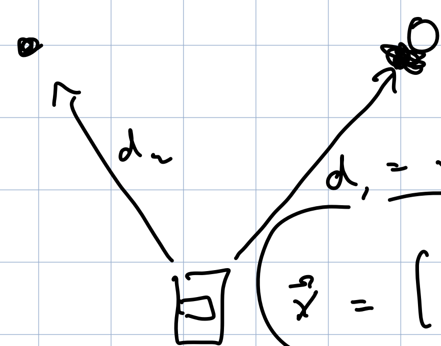
$$\vec{y} = \vec{g}(\vec{x}) + \vec{r} \quad , \quad \vec{g}(\vec{x}) = G\vec{x} + \vec{b}$$

$$J(\vec{x}) = (\vec{y} - \vec{g}(\vec{x}))^T R^{-1} (\vec{y} - g(\vec{x}))$$

$$= (\hat{y} - G\vec{x} - \vec{b})^T R^{-1} (\vec{y} - G\vec{x} - \hat{b})$$

$\tilde{y} = \hat{y} - \vec{b}$

$$= (\tilde{y} - G\vec{x}) R^{-1} (\tilde{y} - G\vec{x})$$



$$\begin{pmatrix} s_1^x \\ s_1^y \end{pmatrix}$$

d_2

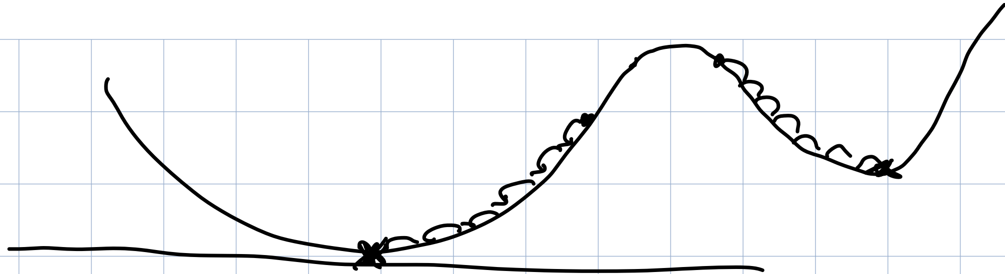
$$d_1 = \sqrt{(s_1^x - p^x)^2 + (s_1^y - p^y)^2}$$

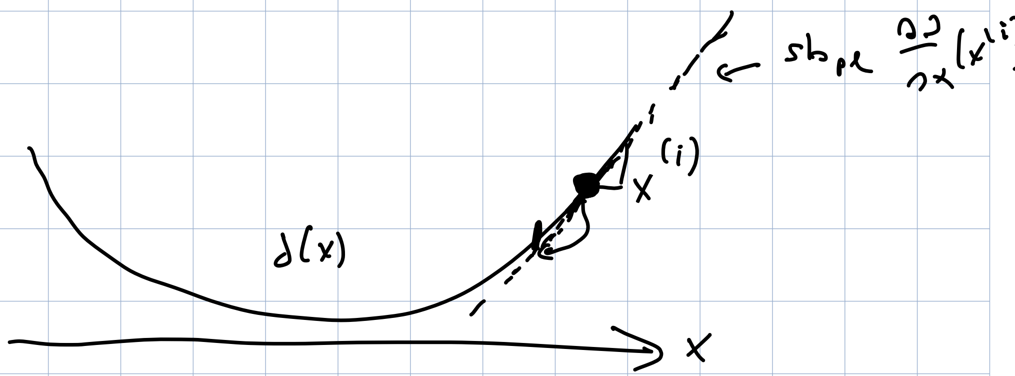
$$\vec{x} = \begin{pmatrix} p^x \\ p^y \end{pmatrix}$$

$$y_1^R = d_1 + r_1^R =$$
$$= \sqrt{(s_1^x - p^x)^2 + (s_1^y - p^y)^2} + r_1^R$$
$$y_2^R = \sqrt{(s_2^x - p^x)^2 + (s_2^y - p^y)^2}$$

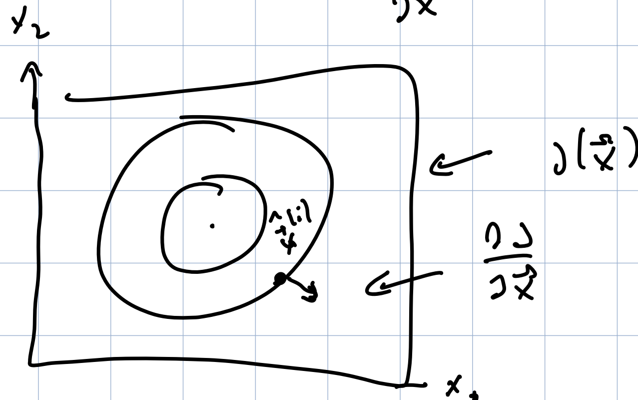
$$\underbrace{\begin{pmatrix} y_1^R \\ \vdots \\ y_n^R \end{pmatrix}}_{\vec{y}} = \underbrace{\begin{pmatrix} \sqrt{(s_1^x - p^x)^2 + (s_1^y - p^y)^2} \\ \sqrt{(s_2^x - p^x)^2 + (s_2^y - p^y)^2} \\ \vdots \\ \sqrt{(s_n^x - p^x)^2 + (s_n^y - p^y)^2} \end{pmatrix}}_{\vec{g}(\vec{x})} + \underbrace{\begin{pmatrix} r_1^R \\ r_2^R \\ \vdots \\ r_n^R \end{pmatrix}}_{\vec{r}}$$

$$\vec{x} = \begin{pmatrix} p^x \\ p^y \end{pmatrix}$$





$$J(x^{(i)}) + \frac{\partial J}{\partial x}(x^{(i)})\,(x - x^{(i)})$$



$$\hat{\vec{x}}^{(i+1)} = \hat{\vec{x}}^{(i)} - \gamma \cdot \frac{\partial J}{\partial \vec{x}}(\hat{\vec{x}}^{(i)})$$

$$J(\vec{x}) = \sum_{n=1}^{N} (y_n - g_n(\vec{x}))^2$$

$$\frac{\partial J}{\partial \vec{x}} = \sum_{n=1}^{N} \frac{\partial}{\partial \vec{x}} \left(y_n - g_n(\vec{x})\right)^2$$

$$= \sum_{n=1}^{N} -2\left(y_n - g_n(x)\right) \cdot \frac{\partial g_n(\vec{x})}{\partial \vec{x}}$$

$$= -2 \begin{bmatrix} \frac{\partial g_1}{\partial \vec{x}} & \cdots & \frac{\partial g_N}{\partial \vec{x}} \end{bmatrix} \begin{pmatrix} y_1 - g_1(\vec{x}) \\ \vdots \\ y_N - g_N(\vec{x}) \end{pmatrix}$$

Jacobian $$G_x(\vec{x}) = \begin{bmatrix} \partial g_1 / \partial x_1 & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_N} \\ \vdots & & \ddots & \\ \partial g_N / \partial x_1 & & & \frac{\partial g_N}{\partial x_N} \end{bmatrix}$$

$$= -2\, G_x^T(\vec{x}) \left(\hat{y} - g(\vec{x})\right)$$

$$\hat{x}^{(i+1)} = \hat{x}^{(i)} + \gamma \cdot G_x^T(\hat{x}^{(i)}) \left(\hat{y} - g(\hat{x}^{(i)})\right)$$

$\hat{x}^{(i)}$

$$\vec{y} = \vec{g}(\vec{x}) + \vec{r}$$

$$J(\vec{x}) = \left(\vec{y} - \vec{g}(\vec{x})\right)^T R^{-1} \left(\vec{y} - \vec{g}(\vec{x})\right)$$

$$\vec{g}(\vec{x}) = \vec{g}(\hat{x}^{(i)}) + G_x(\hat{x}^{(i)})(\vec{x} - \hat{x}^{(i)})$$

$$\Rightarrow \vec{y} = \vec{g}(\hat{\vec{x}}^{(i)}) + G_x(\hat{\vec{x}}^{(i)})(\vec{x} - \hat{\vec{x}}^{(i)}) + \vec{r}$$

$$= \underbrace{G_x(\hat{\vec{x}}^{(i)})}_{G}\vec{x} + \underbrace{\left(\vec{g}(\hat{\vec{x}}^{(i)}) - G_x(\hat{\vec{x}}^{(i)})\hat{\vec{x}}^{(i)}\right)}_{\vec{b}} + \vec{r}$$

$$= G\vec{x} + \vec{b} + \vec{r}$$

$$\tilde{\vec{y}} = \vec{y} - \vec{b} = \vec{y} - \vec{g}(\hat{\vec{x}}^{(i)}) + G_x(\hat{\vec{x}}^{(i)})\hat{\vec{x}}^{(i)}$$

$$\hat{\vec{x}}^{(i+1)} = (G^T R^{-1} G)^{-1} G^T R^{-1} \tilde{\vec{y}}$$

$$= \left(G_x^T(\hat{\vec{x}}^{(i)}) R^{-1} G_x(\hat{\vec{x}}^{(i)})\right)^{-1} G_x^T(\hat{\vec{x}}^{(i)}) \cdot R^{-1} \cdot \left(\vec{y} - \vec{g}(\hat{\vec{x}}^{(i)}) + G_x(\hat{\vec{x}}^{(i)})\hat{\vec{x}}^{(i)}\right)$$

$$= \left(G_x^T(\hat{\vec{x}}^{(i)}) R^{-1} G_x(\hat{\vec{x}}^{(i)})\right)^{-1} G_x^T(\hat{\vec{x}}^{(i)}) \cdot R^{-1} \cdot \left(y - g(x^{(i)})\right)$$

$$+ \underbrace{\left(G_x^T(\hat{\vec{x}}^{(i)}) R^{-1} G_x(\hat{\vec{x}}^{(i)})\right)^{-1} G_x^T(\hat{\vec{x}}^{(i)}) \cdot R^{-1} \cdot G_x(\hat{\vec{x}}^{(i)})\hat{\vec{x}}^{(i)}}_{\hat{\vec{x}}^{(i)}}$$