PHYS-E055102, Low Temperature Physics, Exercise 5 DL 12.10.2022

1. Derive an expression for the conductance of a cylindrical pipe in the case of viscous laminar flow.

   Hint: Use the Navier-Stokes equation in a steady state to solve the velocity field inside the cylinder.

2. The ^{3}He circulation of a dilution refrigerator is maintained by a booster pump (with a pumping speed $S = 4\,\mathrm{m^3/s}$ below 10 Pa pressure). The pumping line consists of cylindrical pipe sections, which have the following lengths and average temperatures:

part #	length l (m)	T (K)
1	0.1	1
2	0.3	4.2
3	0.6	77
4	5	293

   The temperature of the evaporation chamber (still) is kept at 0.6 K, so that only ^{3}He is evaporated. At this temperature, the vapor pressure of helium mixture is 4.6 Pa. The desired flow rate is 2 mmol/s. Determine the diameters of the pipe sections so that the pressure difference over each section is the same. Use the experimental temperature dependence $\eta_3 = 4.22 \cdot 10^{-7}\,(T/\mathrm{K})^{0.647}\,\mathrm{s\,Pa}$ for the viscosity of ^{3}He gas (compare to ideal gas: $\eta = 9.2 \cdot 10^{-7}\,(T/\mathrm{K})^{0.5}\,\mathrm{s\,Pa}$).

3. A dewar is filled with liquid ^{4}He at normal pressure and 4.2 K temperature. The bath is cooled by pumping. What fraction of the liquid is evaporated before it becomes superfluid ($T = 2.17\,\mathrm{K}$)? The latent heat is $L \approx 22\,\mathrm{J/g}$. The ^{4}He enthalpy is $H(4.2\,\mathrm{K}) = 9.25\,\mathrm{J/g}$ and $H(2.17\,\mathrm{K}) = 3.5\,\mathrm{J/g}$.

4. A ^{3}He evaporation cryostat, which has separate ^{3}He and ^{4}He evaporators, is pumped by mechanical pumps, which have a pumping speed $S = 35\,\mathrm{m^3/h}$. See Fig. 1 for a schematic. The conductance of the pumping lines is $C = 25\,\mathrm{m^3/h}$. The heat load of the ^{4}He evaporator consists of the ^{4}He enthalpy arriving into the evaporator, the thermally anchored ^{3}He return capillary, and a 0.1 mW external load. The ^{3}He evaporator is loaded by the ^{3}He arriving through the ^{4}He evaporator and an external load of 0.1 mW. What are the

equilibrium temperatures of the two evaporators? Use $L_4 = 85\,\mathrm{J/mol}$ and $L_3 = 25\,\mathrm{J/mol}$ for the heats of evaporation, $C_3 = (2.1 + 2.2\,T/\mathrm{K})\,\mathrm{J/(mol\,K)}$ for the ^{3}He specific heat, and assume that 40% of the cooling power of the ^{4}He evaporator goes into cooling the ^{4}He liquid itself ($4.2\,\mathrm{K} \rightarrow T_4$). The ^{4}He and ^{3}He vapor pressures behave in the temperature range in question approximately as $P_4 = 640 e^{-10.6\,\mathrm{K}/T}\,\mathrm{kPa}$ and $P3 = 46 e^{-3.7\,\mathrm{K}/T}\,\mathrm{kPa}$, respectively. The parts of the piping marked with flow impedances Z_{43} and Z_{04} are considered thin in order to uphold some pressure differences across the system.

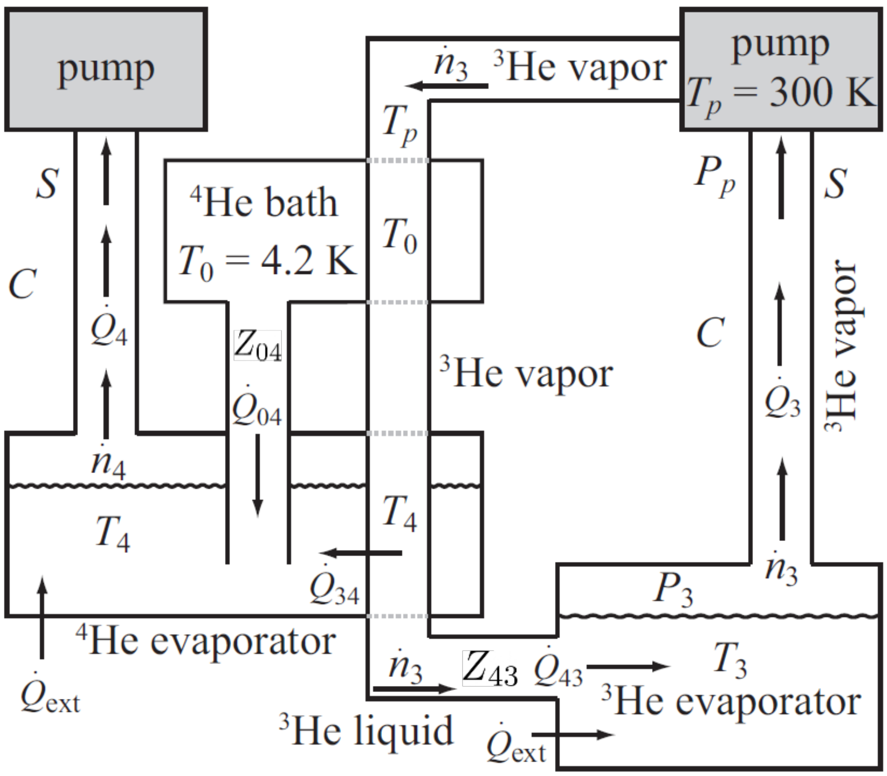

Figure 1: Schematic of a ^{3}He evaporation cryostat.