Vihdoinkin: TÄYDET DYNAAMISET KENTÄT

$\bar{E}$	V/m	$\bar{H}$	A/m
$\bar{D}$	As/m^2	$\bar{B}$	Vs/m^2
ε	As/Vm	μ	Vs/Am
ρ	As/m^3	$\bar{J}$	A/m^2

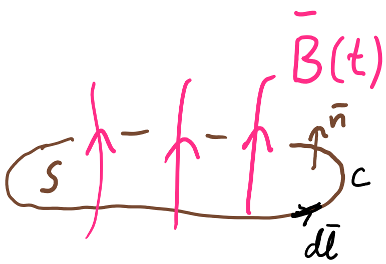
MAXWELLIN YHTÄLÖT:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho \qquad \nabla \cdot \bar{B} = 0$$

$\bar{f}(\bar{r}, t)$

Faradayn laki $\quad \nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$



Gaussin lause $\quad \int_V \nabla \cdot \bar{f}\, dV = \oint_S \bar{f} \cdot \overline{dS}$

Stokesin lause $\quad \int_S \nabla \times \bar{f} \cdot \overline{dS} = \oint_C \bar{f} \cdot \overline{d\ell}$

$$\int_S \nabla \times \bar{E} \cdot d\bar{S} = \underbrace{\oint \bar{E} \cdot d\bar{l}}_{\text{smv}}$$

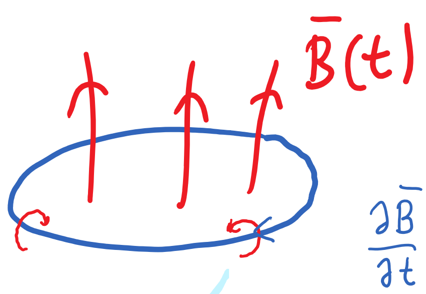
$$= -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} = -\frac{\partial}{\partial t} \overbrace{\int_S \bar{B} \cdot d\bar{S}}^{\Phi \text{ (MAGN.VUO)}}$$

$$[\Phi] = \frac{Vs}{m^2} \cdot m^2 = Vs = Wb$$

(weber)

$$[\text{smv}] = V$$

$$\text{smv} = -\frac{\partial \Phi}{\partial t}$$



$\dfrac{\partial \bar{B}}{\partial t} > 0$

smv = sähkömotorinen voima
emf = electromotive force

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emf = electromotive force

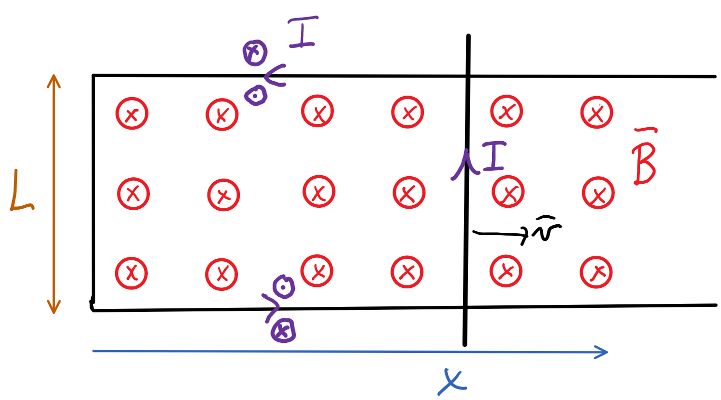
$\frac{\partial B}{\partial t} > 0$

★

Lenzin laki:

Indusoitunut sähkövirta aiheuttaa sellaisen oman magneettikentän, joka vastustaa alkuperäistä muutosta

Muuttuva piiri vakiomagneettikentässä



LORENTZIN VOIMA

$$\bar{F} = Q\, \underbrace{\bar{v} \times \bar{B}}_{\text{ekvivalenttinen sähkökenttä}} \; (= Q\bar{E}_{ekv})$$

$$|smv| = E_{ekv} L = v B L, \quad v = \frac{dx}{dt}$$

$$\Phi = \int \bar{B} \cdot d\bar{S} = BA, \quad A = xL$$

$$|smv| = B \frac{d(\overbrace{xL}^{A})}{dt} = \frac{d}{dt}(BA) = \frac{d}{dt}\Phi$$

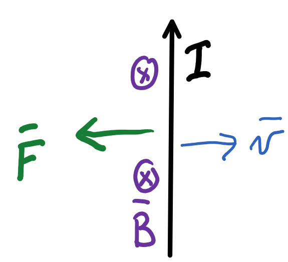
SÄHKÖTEHO

$$P_s = \underbrace{vBL}_{smv} \cdot \underbrace{\frac{vBL}{R}}_{I} = \frac{(vBL)^2}{R}$$

MEKAANINEN TEHO

$$F = BIL$$

$$\frac{vBL}{R}$$

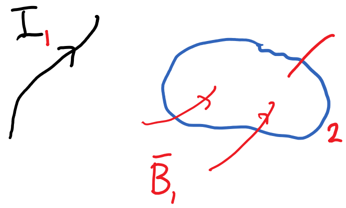


$$P_m = \frac{F\,dx}{dt} = Fv = BILv = \frac{(vBL)^2}{R}$$

Induktanssi L

Magneettivuo Φ

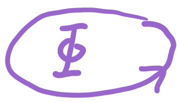
Magneettivuon tiheys $\bar{B}$

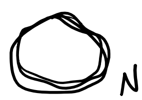


$\Phi_{12} = L_{12} I_1$

KESKINÄIS-INDUKTANSSI

$[L_{12}] = \frac{Vs}{A} = H$ (henry)

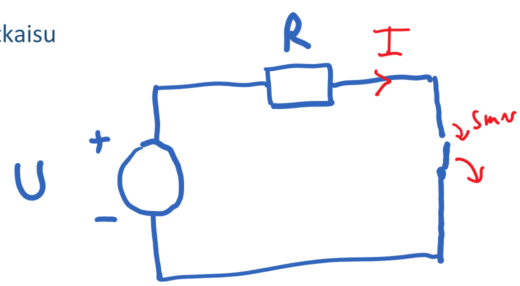


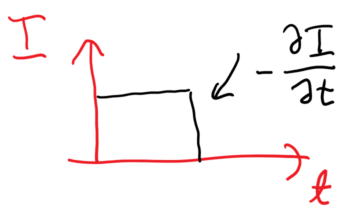


$L \sim N^2$

Virtapiirin katkaisu

Kipinä!





$$\Phi = LI \Rightarrow \text{smv} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$ (SUURI, POSITIIVINEN)

Pyörrevirta (virvelström, eddy current)

https://aalto.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=2b331402-074f-4b81-8d32-aeba00dafe6c

Siirrosvirta $\frac{\partial \bar{D}}{\partial t}$ yksikkö $\frac{As}{m^2} \cdot \frac{1}{s} = \frac{A}{m^2}$

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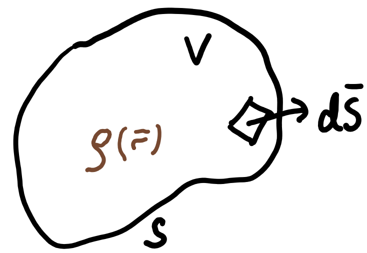
$$\nabla\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \underbrace{\nabla\cdot\left(\nabla\times\bar{H}\right)}_{0} = \nabla\cdot\bar{J} + \nabla\cdot\frac{\partial \bar{D}}{\partial t}$$

$$\nabla\cdot\bar{J} = -\frac{\partial}{\partial t}\underbrace{\nabla\cdot\bar{D}}_{\rho}$$



$$\underbrace{\int_V \nabla\cdot\bar{J}\,dV = \oint_S \bar{J}\cdot d\bar{S}}_{I_{ulos}} = \int_V -\frac{\partial\rho}{\partial t}\,dV$$

$$= -\frac{\partial}{\partial t}\int_V \rho\,dV = -\frac{\partial}{\partial t}\overbrace{Q_{kok}}^{\text{VARAUS V:N SISÄLLÄ}}$$

Aaltoyhtälö

$$\boxed{\bar{J} = 0 \quad \rho = 0 \quad \bar{E} = ? \quad \varepsilon_0, \mu_0}$$

$$\nabla\times\bar{E} = -\frac{\partial}{\partial t}\bar{B} = -\mu_0\frac{\partial}{\partial t}\bar{H}$$

$$\nabla\times\bar{H} = +\frac{\partial}{\partial t}\bar{D} = \varepsilon_0\frac{\partial}{\partial t}\bar{E}$$

$$\nabla\cdot\bar{D} = 0$$

$$\varepsilon_0\nabla\cdot\bar{E} = 0$$

$$\underbrace{\nabla\times\left(\nabla\times\bar{E}\right)}_{\nabla\underbrace{(\nabla\cdot\bar{E})}_{=0} - \nabla^2\bar{E}} = -\mu_0\frac{\partial}{\partial t}\nabla\times\bar{H} = -\mu_0\frac{\partial}{\partial t}\left(\varepsilon_0\frac{\partial}{\partial t}\bar{E}\right)$$

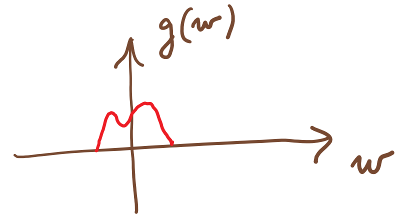
$$\Rightarrow \quad \nabla^2\bar{E}(\bar{r},t) - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\bar{E}(\bar{r},t) = 0$$

$$\bar{E}(\bar{r},t) = \bar{u}\,f(z,t) \qquad \bar{u}\left(\underbrace{\nabla^2 f(z,t)}_{\frac{\partial^2}{\partial z^2}f(z,t)} - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}f(z,t)\right) = 0$$

$$\frac{\partial^2}{\partial z^2}f(z,t) - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}f(z,t) = 0$$

$$\frac{\partial^2}{\partial z^2} f(z,t) - \underbrace{\mu_0 \varepsilon_0}\, \frac{\partial^2}{\partial t^2} f(z,t) = 0$$

$$g(z \mp vt)$$



$$\frac{\partial}{\partial z} g(z \mp vt) = g'(z \mp vt)$$

$$\frac{\partial^2}{\partial z^2} g(z \mp vt) = g''(z \mp vt)$$

$$\frac{\partial}{\partial t} g(z \mp vt) = \mp v\, g'(z \mp vt)$$

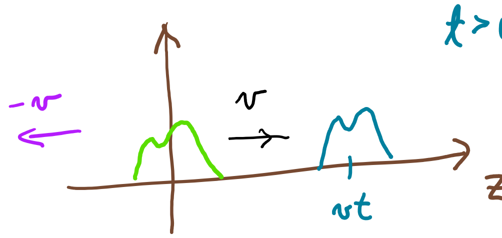
$$\frac{\partial^2}{\partial t^2} g(z \mp vt) = (\mp v)^2 g''(z \mp vt)$$

$$\Rightarrow \quad \frac{\partial^2}{\partial z^2} g(z \mp vt) = \frac{1}{\underbrace{v^2}} \frac{\partial^2}{\partial t^2} g(z \mp vt)$$

$t = 0 \Rightarrow g(z)$

$t > 0 \Rightarrow g(\underbrace{z - vt}_{=0})$

$z = vt$



$$v^2 = \frac{1}{\mu_0 \varepsilon_0} = c^2$$

VALON NOPEUS $\quad c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299\,792\,458 \,\dfrac{m}{s}$

Viivästyneet potentiaalit

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \nabla \times \Big(\underbrace{\bar{E} + \frac{\partial \bar{A}}{\partial t}}_{-\nabla \phi}\Big) = 0$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho$$

$$\nabla \cdot \bar{B} = 0 \Rightarrow \bar{B} = \nabla \times \bar{A}$$

$$\rho,\ \bar{J} \to \phi,\ \bar{A} \to \bar{E},\ \bar{H}$$

$$-\nabla \phi - \frac{\partial \bar{A}}{\partial t}$$

$$\nabla \times \bar{B} = \nabla \times (\nabla \times \bar{A}) = \mu_0 \bar{J} + \mu_0 \frac{\partial}{\partial t} \varepsilon_0 \bar{E}$$

$$\nabla \underbrace{\nabla \cdot \bar{A}} - \nabla^2 \bar{A} = \mu_0 \bar{J} - \mu_0 \varepsilon_0 \nabla \frac{\partial \phi}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \bar{A}$$

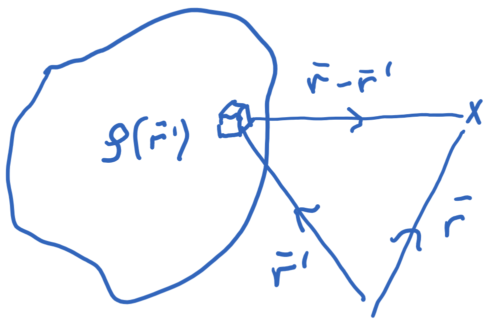
$\nabla \cdot \bar{A} = -\mu_0 \varepsilon_0 \dfrac{\partial \phi}{\partial t}$ (LORENZIN MITTA)

$$\nabla^2 \bar{A} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \bar{A} = -\mu_0 \bar{J}$$

$$\nabla \cdot \bar{D} = \rho \quad \Rightarrow \quad \varepsilon_0 \nabla \cdot \bar{E} = \rho \quad \Rightarrow \quad \nabla \cdot \left(-\nabla \phi - \frac{\partial \bar{A}}{\partial t} \right) = \frac{\rho}{\varepsilon_0}$$

$$\nabla^2 \phi + \frac{\partial}{\partial t} \nabla \cdot \bar{A} = -\rho/\varepsilon_0 \quad \Rightarrow \quad \nabla^2 \phi \underline{- \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}} \phi = -\rho/\varepsilon_0$$

STATIIKKA



$$\phi(\bar{r}) = \int \frac{\rho(\bar{r}')\, dV'}{4\pi \varepsilon_0 |\bar{r} - \bar{r}'|}$$

VIIVÄSTYS

DYNAMIIKKA

$$\phi(\bar{r}, t) = \int \frac{\rho\left(\bar{r}', t - \frac{|\bar{r} - \bar{r}'|}{v}\right) dV'}{4\pi \varepsilon_0 |\bar{r} - \bar{r}'|}$$

$$\left(v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \right)$$

$$\bar{A}(\bar{r}, t) = \int \frac{\mu_0 \bar{J}\left(\bar{r}', t - \frac{|\bar{r} - \bar{r}'|}{v}\right)}{4\pi |\bar{r} - \bar{r}'|} dV'$$