ELEC-E8101 Digital and Optimal Control

Exercise 6

1. Design a deadbeat type state feedback controller for the following system.

$$\begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 0,5 \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x}(k) \end{cases}$$

2. Design a deadbeat type state observer for the system in the previous problem.

3. Consider the two tank system, which has the state-space representation

$$\frac{dx(t)}{dt} = \begin{bmatrix} -0.0197 & 0 \\ 0.0178 & -0.0129 \end{bmatrix} x(t) + \begin{bmatrix} 0.0263 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

and with sampling time 12 the corresponding pulse transfer operator is

$$H(q) = \frac{0.030q + 0.026}{q^2 - 1.65q + 0.68}.$$

a. Determine a state feedback controller such that the closed loop characteristic equation is $z^2 - 1.55z + 0.64 = 0$. In continuous-time systems this corresponds to values $\zeta = 0.7$ and $\omega_n = 0.027$ rad/s in the second order dynamics.

b. Find a suitable controller to eliminate the stationary state error (*i.e.* add an integrator).

c. Simulate the systems of a- and b-parts.

4. (*) Show that if the state given by a state observer is used in the state feedback, the whole system has the poles of the state feedback and the state observer.