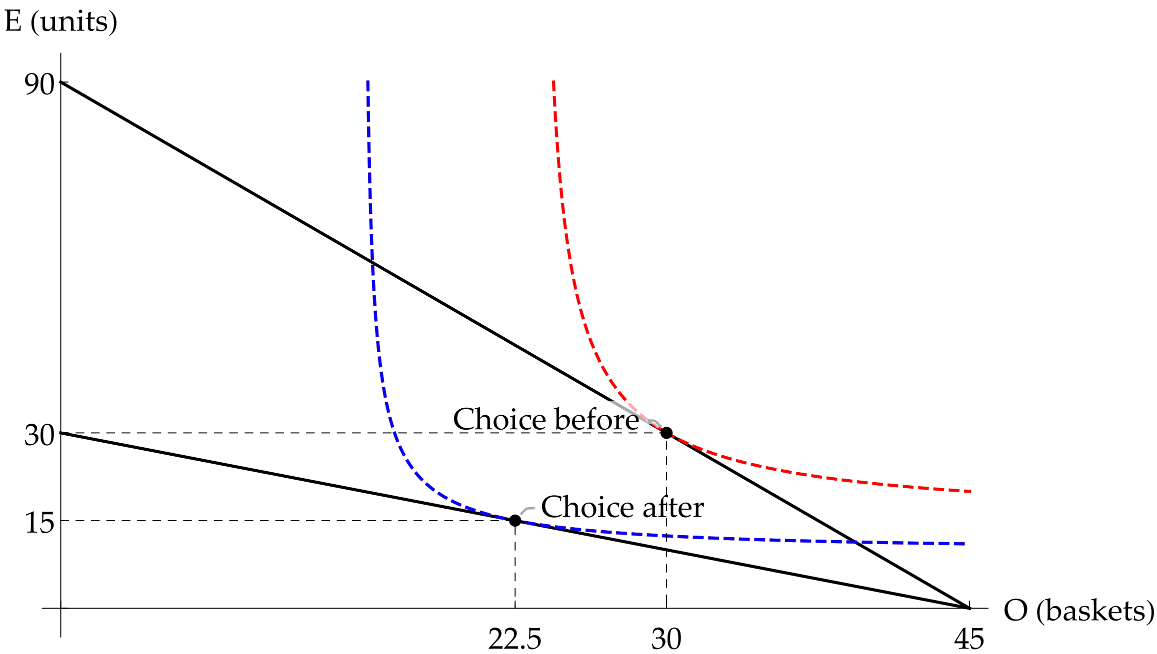
Model Solutions 5

1. (a) Anna's disposable income is $1200 \times (1 - 0.25) = 900$. Let q_E and be the quantity she purchases electricity and q_B that of other goods. Denote the price of electricity by p_E. The following baskets use up her budget in full and therefore constitute the budget line:

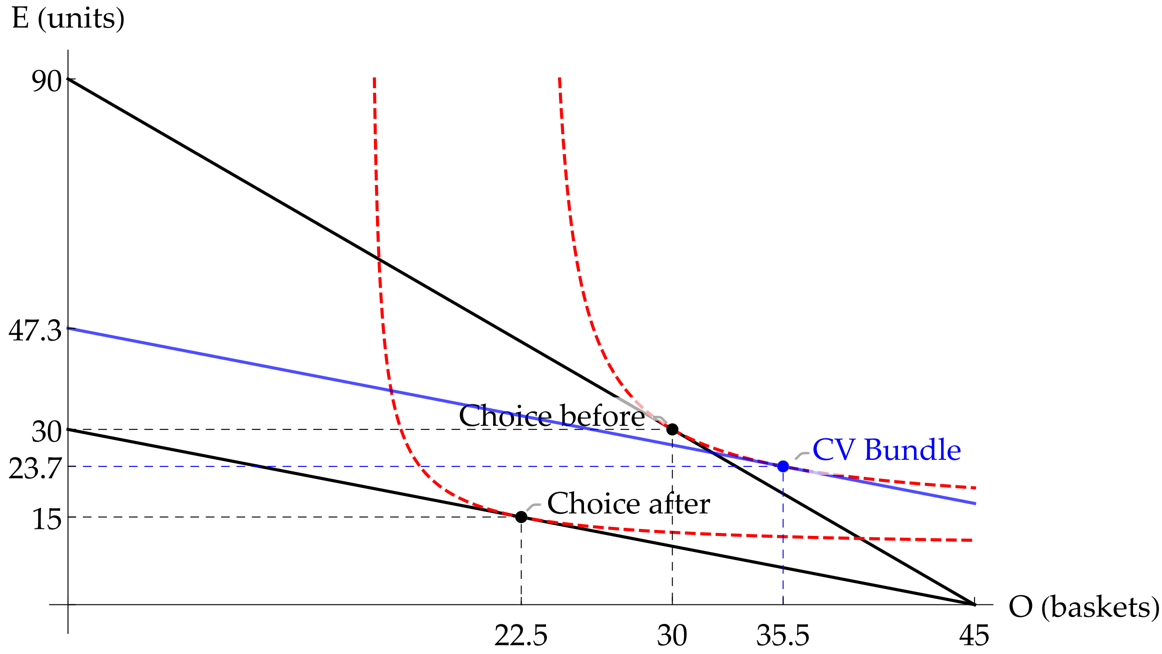
$$10q_E + 20q_B = 900 \Leftrightarrow$$
$$q_E = \frac{-20q_B + 900}{p_E}.$$

Plugging in prices $p_E = 10$ and $p_E = 30$ yield $q_E = -2q_B + 90$ and $q_E = -(2/3)q_B + 30$, respectively.

(b) Anna's spending of electricity last period was third of her budget, i.e. $10q_E = 900/3 \Leftrightarrow q_E = 30$ and after the price change we have $30q_E = 900/2 \Leftrightarrow q_E = 15$.

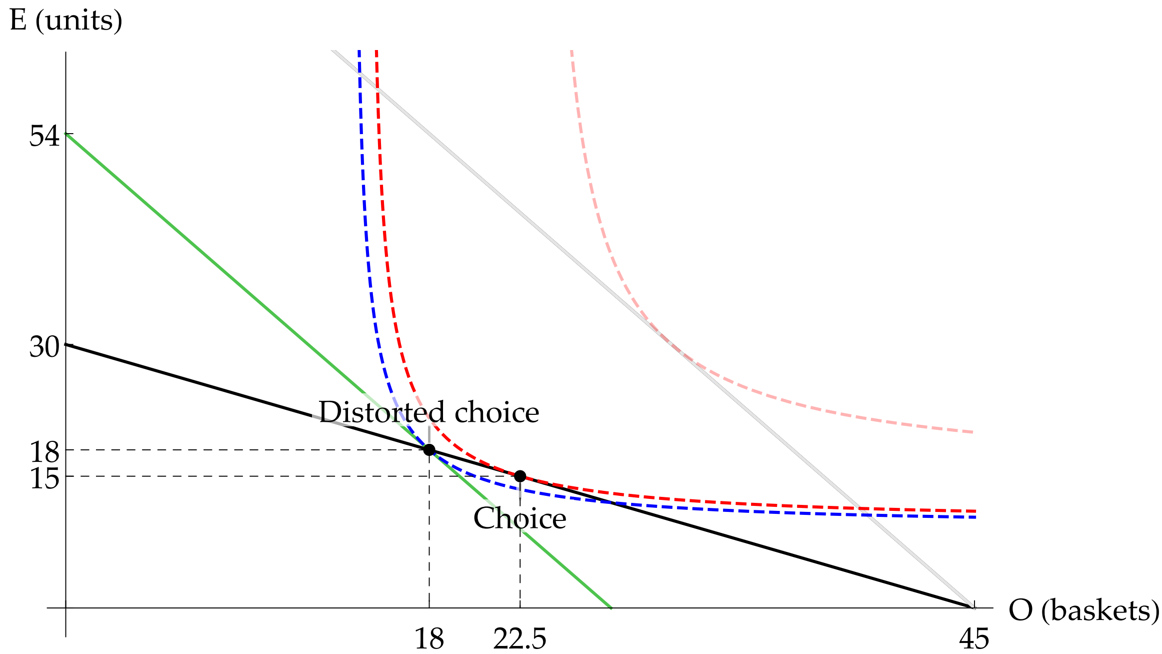


(c) Anna spends $900/3 - 900/2 = 150$ euros more on electricity. A change in relative prices affects the slope of the budget line. If Anna gets more money, the budget line shifts upwards but stays parallel to the original as the relative prices remain intact. We add to her budget the amount of money that makes her new, tilted budget line just touch the indifference curve she was before the price change so that the original level of utility is restored. This amount is called the compensating variation (CV).



(d) The government policy doesn't really give Ann more money but distorts the relative prices, preventing Anna from reaching the utility level she'd have in absence of the policy.

The amount money Anna spends remains unchanged: she gets the same amount back as a subsidy that she got taxed, and the price of the basket of other good remains unchanged. Therefore the new consumption bundle must lie on the budget line she'd have without the policy. Since she was choosing the optimal bundle then and now chooses a different bundle, she's worse off.



2. (a) At the cost-minimizing input choice the technical rate of substitution (the slope of the isoquant) is equal to the ratio of prices (the slope of the isocost line).

$$\frac{\partial q(x,y)/\partial x}{\partial q(x,y)/\partial y} = \frac{p_x}{p_y} \tag{1}$$
$$\frac{10\sqrt{y}}{5x/\sqrt{y}} = \frac{300}{100}$$
$$\frac{2y}{x} = 3.$$

Solving x we see that the cost-minimizing input choice must satisfy $x^*(y) = 2y/3$.

The cost-minimizing input combination that yields 1000 robots must therefore satisfy

$$q(x^*(y), y) = 1000$$
$$10\frac{2y}{3}\sqrt{y} = 1000$$
$$y^* = 5 \times 5^{\frac{1}{3}} 6^{\frac{2}{3}} \approx 28.2$$
$$x^* = x^*(y^*) = (2/3)y^* \approx 18.8.$$

(b) The logic is unchanged from 2a except that we treat p_y now as an unknown:

$$\frac{10\sqrt{y}}{5x/\sqrt{y}} = \frac{300}{p_y}$$
$$\frac{2y}{x} = \frac{300}{p_y} \implies$$
$$x^* = 2y\frac{p_y}{300}.$$

As before,

$$q(x^*(y), y) = 1000$$
$$10 \times 2y\frac{p_y}{300}\sqrt{y} = 1000$$
$$y^*(p_y) = \frac{100 \times 15^{2/3}}{p^{2/3}}.$$

(c) Notice that $x^*(y) = 2y\frac{p_y}{p_x}$. Energy's share of costs is $\frac{p_y y}{p_y y + p_x x} = \frac{1}{1+2p_y}$ and therefore doesn't depend on the price of tungsten. It remains unchanged.

3. (a) The equilibrium after the tax cut is given by

$$P^S(q) = P^D(q) \Leftrightarrow$$
$$2q = 200 - 0.5q \implies$$
$$q_a^* = 80 \implies$$
$$p_a^* = 160.$$

Before, with the 40 TD/TWh tax, there's equally big wedge between the supply and the demand in the equilibrium:

$$\begin{aligned} P^S(q) + 40 &= P^D(q) \Leftrightarrow \\ 2q + 40 &= 200 - 0.5q \implies \\ q_b^* &= 64 \implies \\ p_b^* &= 168. \end{aligned}$$

Welfare is given by calculating consumer surplus, producer surplus and the change in tax revenue together.

After the cut $W_a = CS_a + PS_a + T_a = ((200 - 160)80/2) + (160 \times 80/2) + 0 = 8000$.

Before, $W_b = CS_b + PS_b + T_b = ((200 - 168)64/2) + ((168 - 40) \times 64/2) + 40 \times 64 = 7680$.

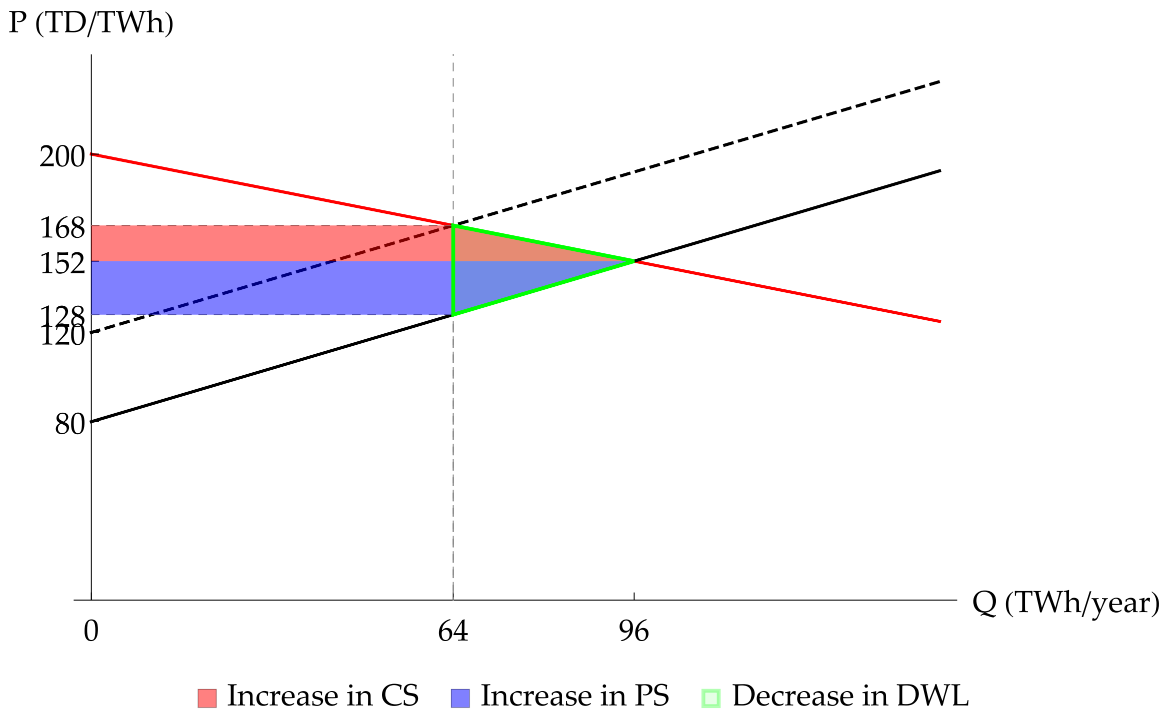
Change in welfare is $W_a - W_b = 320$.

(b) The welfare only changes through changes in the equilibrium quantities as long as the tax revenue isn't used for something useful. The traded quantity remains unchanged, so does the welfare generated in the market. As the tax is removed, the government revenue goes to zero and this is transferred to the producers.

(c) In the long-run equilibrium,

$$\begin{aligned} P^S_{LR}(q) &= P^D(q) \Leftrightarrow \\ 80 + 0.75q &= 200 - 0.5q \implies \\ q_{LR}^* &= 96 \implies \\ p_{LR}^* &= 152. \end{aligned}$$

The price decreases from 168 to 152 TD/TWh. The government still doesn't earn a penny so the decrease in government revenue is $40 \times 64 = 2560$.



(d)

4. (a) With a binding price ceiling, there are more consumers who are willing to purchase at the market price than suppliers that are willing to supply. The situation described here corresponds the best-case scenario where those who value the electricity the most get to purchase.

   In the absence of the ceiling, the equilibrium is given by

$$P^S(q) = P^D(q) \Leftrightarrow$$
$$40 + 2q = 200 - 0.5q \implies$$
$$q^* = 64 \implies$$
$$p^* = 168$$

   Producer surplus is $PS = 64(168 - 40)/2 = 4096$ and consumer surplus $CS = 64(200 - 168)/2 = 1024$ and welfare the sum of these two, 5120.

   The equilibrium quantity under the ceiling is

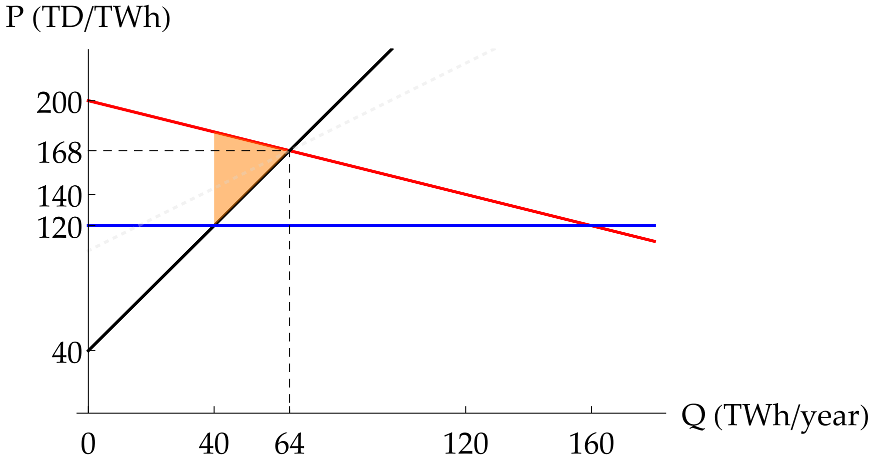
$$P^S(q) = 120 \Leftrightarrow$$
$$40 + 2q = 120 \implies$$
$$\hat{q}_b = 40.$$

   Producer surplus is $PS = 40(120 - 40)/2 = 1600$ and consumer surplus (a shape of trapezoid) $CS = 40((200 - 120) + ((200 - 40 \times 0.5) - 120))/2 = 2800$ and welfare the sum of these two, 4400. Welfare is decreased by 720.

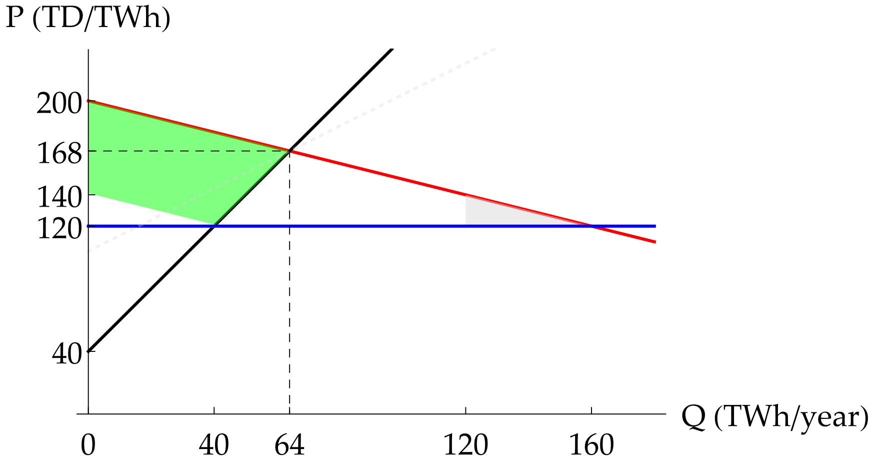
(b) The situation described here corresponds the worst-case scenario where those who value the electricity the least get to purchase. Quantity traded and producer surplus remain the same.

Consumer surplus is $CS = 40(140-120)/2 = 400$ and welfare 2000. Welfare is decreased by 3120.

(c) The best-case scenario. Deadweight loss is highlighted.



The worst-case scenario, with deadweight highlighted in green. None of the consumers that would've bought without the ceiling get to purchase now. Consumer surplus solely consists of the grey area. Those consumers wouldn't have purchased in the absence of the ceiling since their valuation is less than 168, the competitive market price.



(d) The long-run equilibrium quantity under the ceiling is

$$P^S_{LR}(q) = 120 \Leftrightarrow$$
$$104 + 2 = 120 \implies$$
$$\hat{q} = 16.$$

Consumer surplus (a similar trapezoid as in 4c, only truncated at the new equilibrium quantity $q = 16$ rather than $q = 40$) is $CS = 40((200-120)+((200-16\times0.5)-120))/2 = 1216$. Consumer surplus increases as $1216 - 1024 = 192 > 0$.