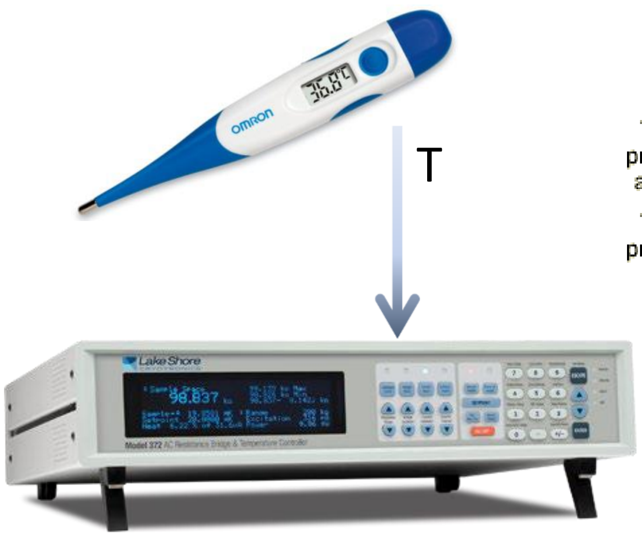
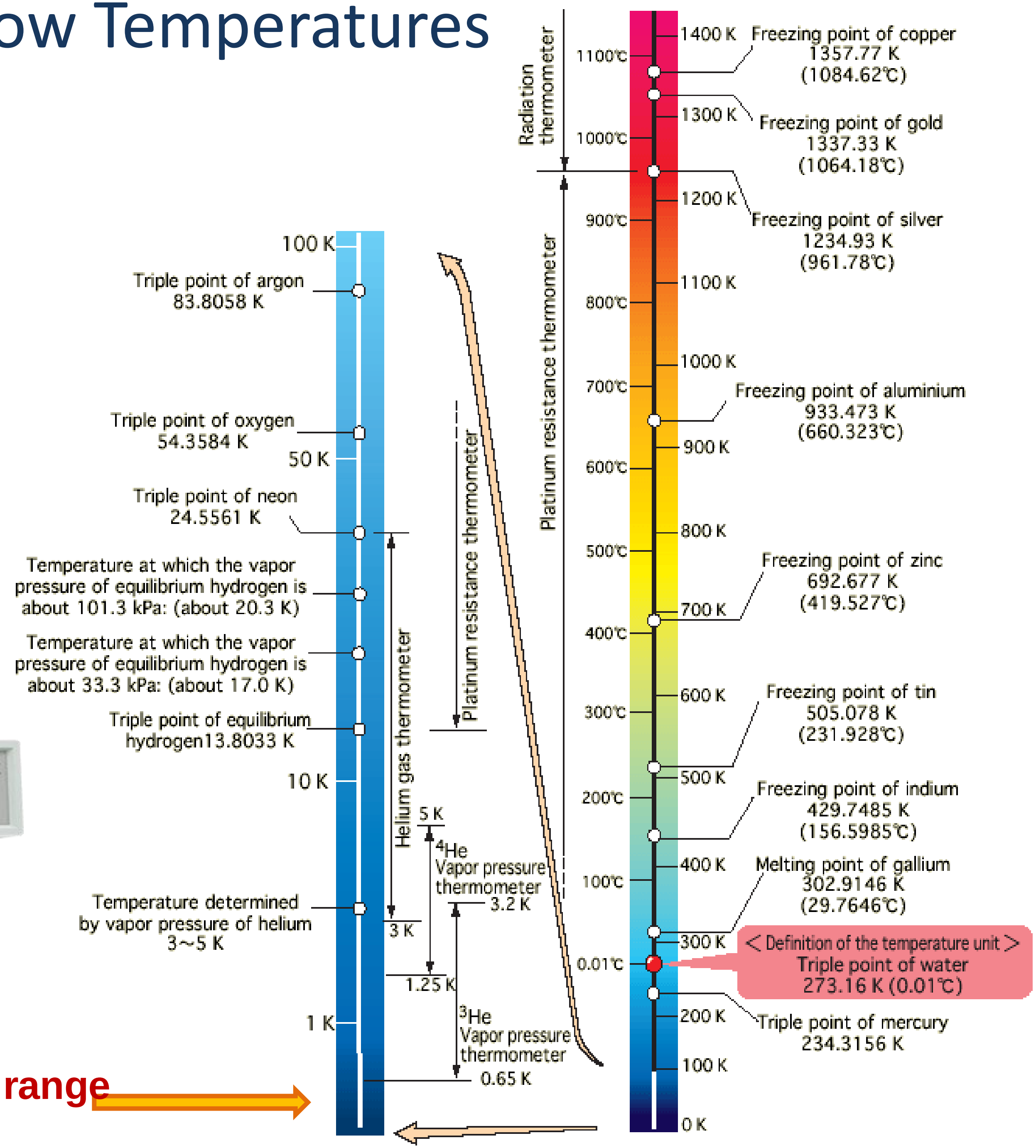
Thermometry at Low Temperatures





Our temperature range

International Temperature Scale of 1990 (ITS-90)

The table below lists the defining fixed points of ITS-90.

Substance and its state	Defining point (range)			
	K	°C	°R	°F
Triple point of hydrogen	13.8033	−259.3467	24.8459	−434.8241
Triple point of neon	24.5561	−248.5939	44.2010	−415.4690
Triple point of oxygen	54.3584	−218.7916	97.8451	−361.8249
Triple point of argon	83.8058	−189.3442	150.8504	−308.8196
Triple point of mercury	234.3156	−38.8344	421.7681	−37.9019
Triple point of water[note 1]	273.16	0.01	491.69	32.02
Melting point[note 2] of gallium	302.9146	29.7646	545.2463	85.5763
Freezing point[note 2] of indium	429.7485	156.5985	773.5473	313.8773
Freezing point[note 2] of tin	505.078	231.928	909.140	449.470
Freezing point[note 2] of zinc	692.677	419.527	1,246.819	787.149
Freezing point[note 2] of aluminium	933.473	660.323	1,680.251	1,220.581
Freezing point[note 2] of silver	1,234.93	961.78	2,222.87	1,763.20
Freezing point[note 2] of gold	1,337.33	1,064.18	2,407.19	1,947.52
Freezing point[note 2] of copper	1,357.77	1,084.62	2,443.99	1,984.32

1. ^ *The triple point of water is frequently approximated by using the melting point of water at standard conditions for temperature and pressure.*
2. ^ a b c d e f g h *Melting and freezing points are distinguished by whether heat is entering or leaving the sample when its temperature is measured. See melting point for more information.*

Various phase transitions are defined as fixed points over the scale

– These are agreed upon in an international scale ITS-90

– The scale extends only down to 0.65 K !

– No official scale exists at mK-range

Provisional PLTS-2000 gives temperatures from 1 mK to 1 K in terms of the melting pressure of 3He

The Provisional Low-temperature Scale of 2000 PLTS-2000

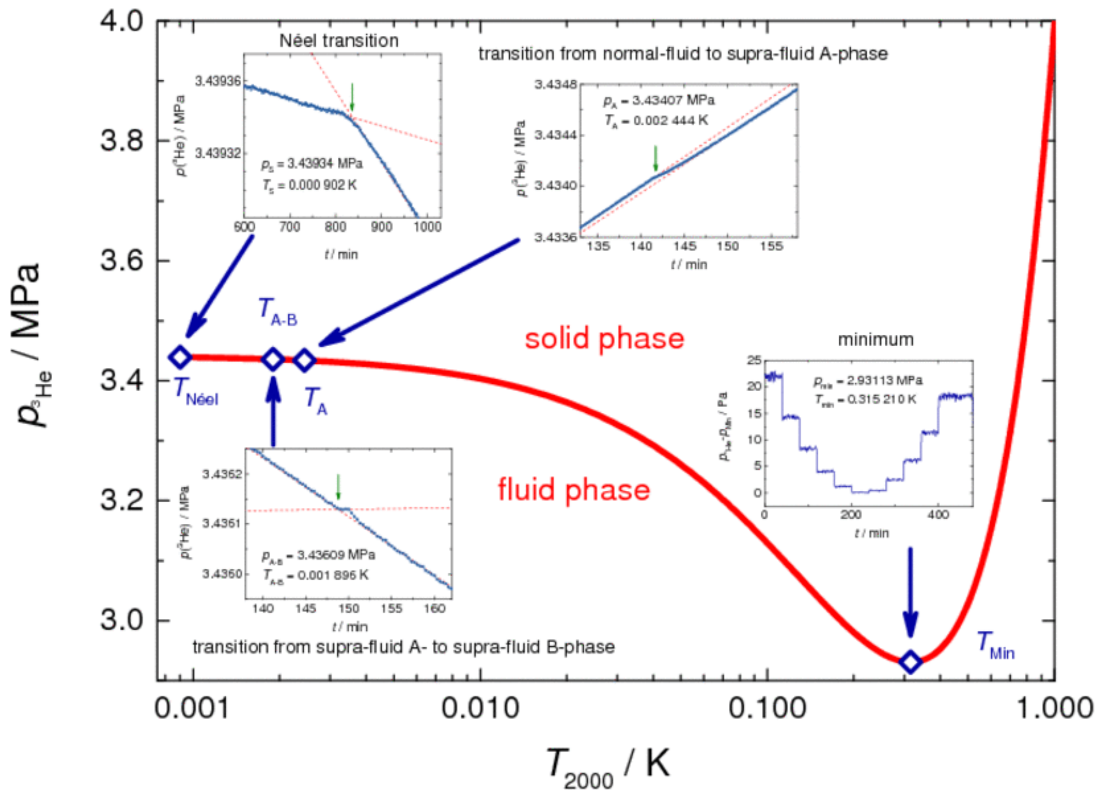


Table 1. Pressure and temperature values of the fixed points of the ^3He melting pressure according to the PLTS-2000

Fixed point	$p_{^3He}$ / MPa	T_{2000} / K
Minimum	2.93113	0.315 240
A transition	3.43407	0.002 444
A-B transition	3.43609	0.001 896
Néel transition	3.43934	0.000 902

The scale is defined by the following equation relating the melting pressure p of ^3He to temperature T_{2000}:

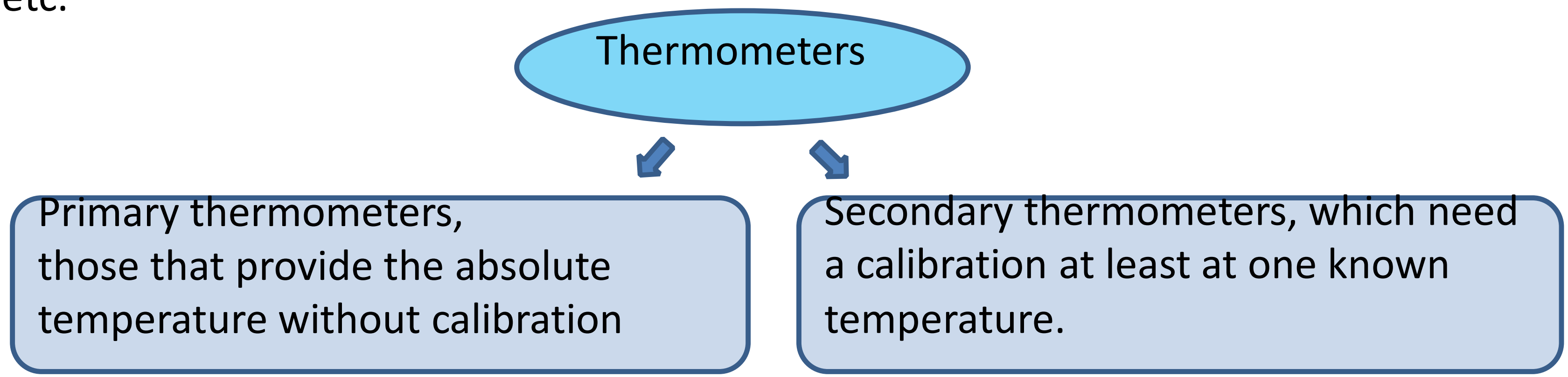
$$p\,/\,\mathrm{MPa} = \sum_{i=-3}^{+9} a_i \left(T_{2000}\,/\,\mathrm{K}\right)^i$$

$a_{-3} = -1.385\,544\,2 \cdot 10^{-12}$
$a_{-2} = 4.555\,702\,6 \cdot 10^{-9}$
$a_{-1} = -6.443\,086\,9 \cdot 10^{-6}$
$a_0 = 3.446\,743\,4 \cdot 10^{0}$
$a_1 = -4.417\,643\,8 \cdot 10^{0}$
$a_2 = 1.541\,743\,7 \cdot 10^{1}$
$a_3 = -3.578\,985\,3 \cdot 10^{1}$
$a_4 = 7.149\,912\,5 \cdot 10^{1}$
$a_5 = -1.041\,437\,9 \cdot 10^{2}$
$a_6 = 1.051\,853\,8 \cdot 10^{2}$
$a_7 = -6.944\,376\,7 \cdot 10^{1}$
$a_8 = 2.683\,308\,7 \cdot 10^{1}$
$a_9 = -4.587\,570\,9 \cdot 10^{0}$

Thermometry at Low Temperatures

Any quantity that changes with temperature can, in principle, be used as a thermometer. Usefulness of a particular thermometric quantity in each application is determined by how well it satisfies a number of other criteria:

- wide operation range with simple and monotonic dependence on temperature,
- low self-heating,
- fast response and measurement time,
- ease of operation,
- immunity to external parameters, in particular to magnetic field,
- small size,
- small thermal mass,
- etc.



Primary thermometers are rare and they are typically difficult to operate, but nevertheless they are very valuable, e.g., in calibrating secondary thermometers.

Thermometry at Low Temperatures

Some problems with measuring temperature of mesoscopic systems:

- Size of the system (smallest size of the system with well defined temperature)?
- Which temperature is measured (electrons, phonons, nuclei spins, etc.)?
- How good thermal contact between thermometer and measured sample?
- Equilibrium, quasiequilibrium and nonequilibrium temperatures.

Books:
O.V.Lounasmaa “Experimental principals and methods below 1K”
Frank Pobell ”Matter and Methods at Low Temperatures”
Christian Enss, Siegfried Hunklinger ”Low-Temperature Physics”

Thermal Properties at Low Temperature

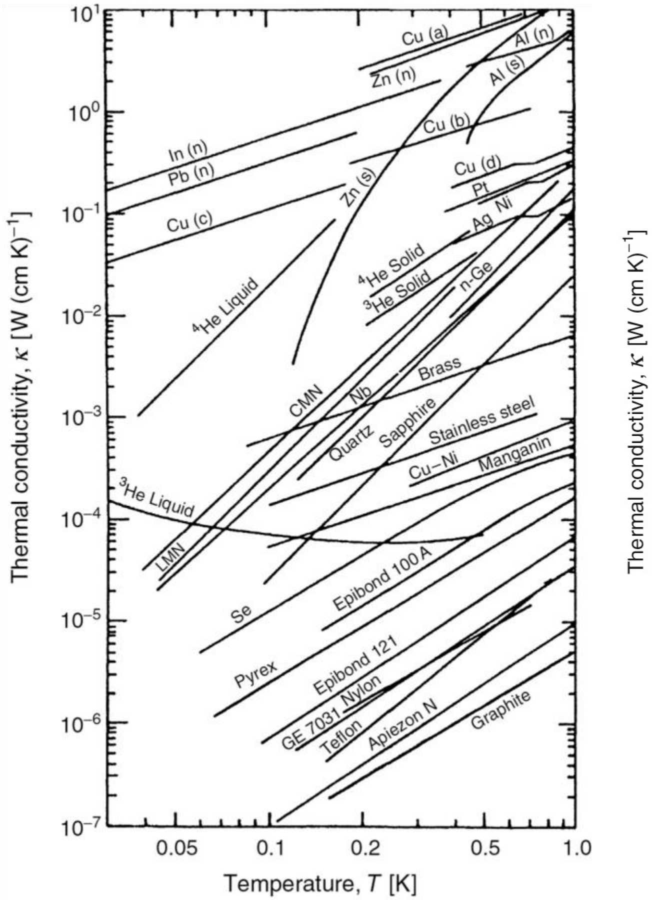
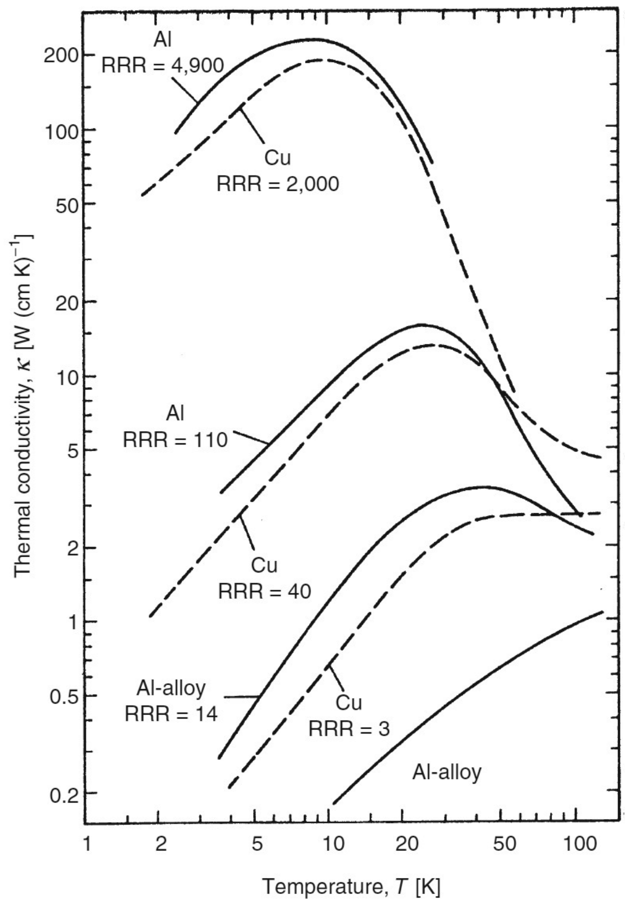
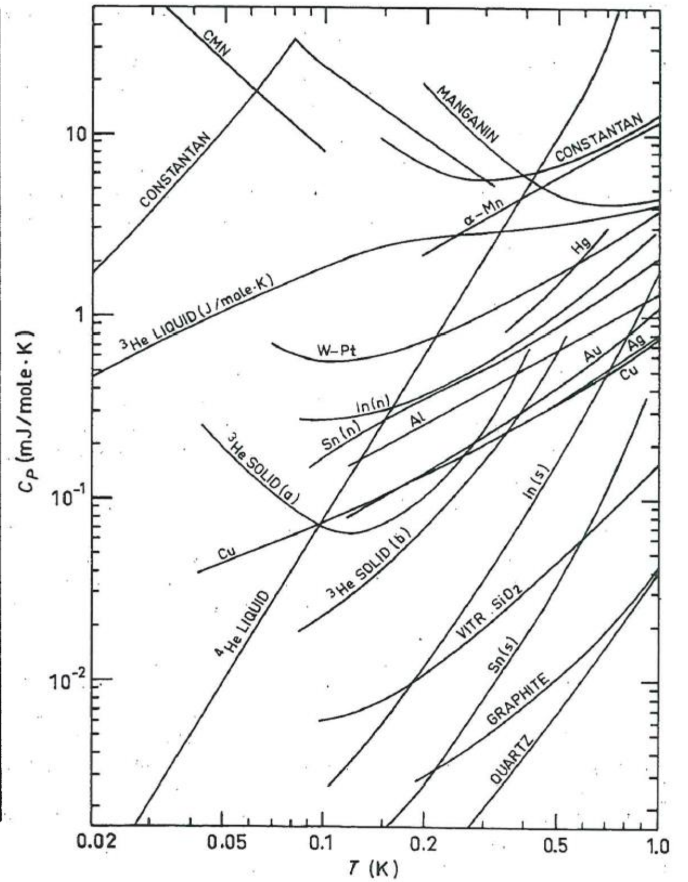


Fig. 3.21. Typical thermal conductivities κ of various materials at $T < 1\,K$.





Wiedemann-Franz law: $\kappa/\sigma = L_0 T$

- Thermal conductivity and heat capacity decreasing with temperature
- Strongly depend on the material, material quality and impurities

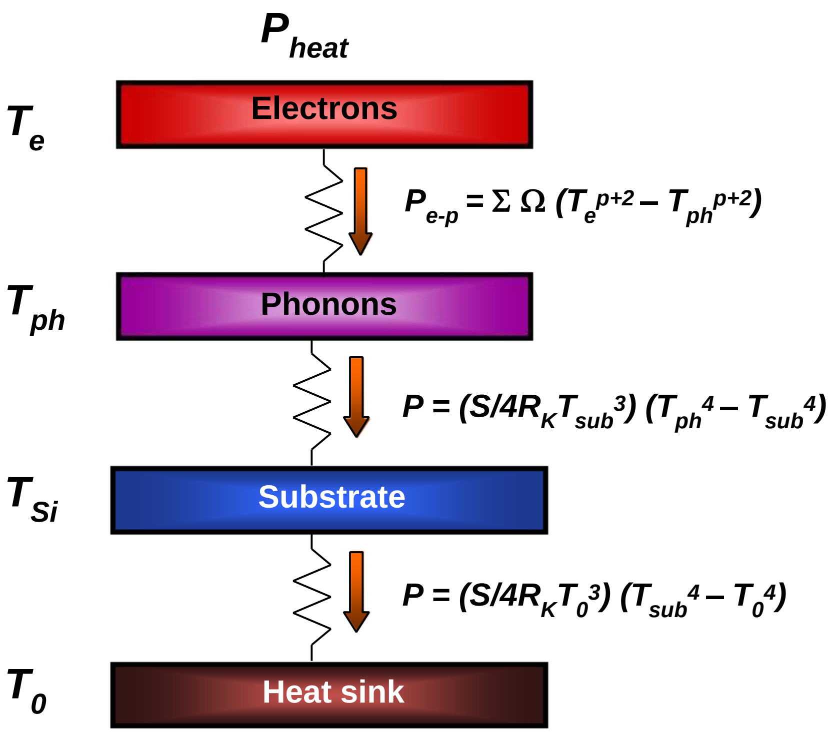
Limitation of cooling of the circuits
Thermal resistance of system components

Thermal resistances (Cu film on Si substrate)

	50 mK	500 mK
$\Re_{e\text{-}ph}$ ($\propto T^{-4}$) (d = 0.1μm)	**160 mK/nW** (1 mm^2)	**16 mK/nW** (10^{-3} mm^2) **16 mK/μW** (1 mm^2)

Kapitza resistance

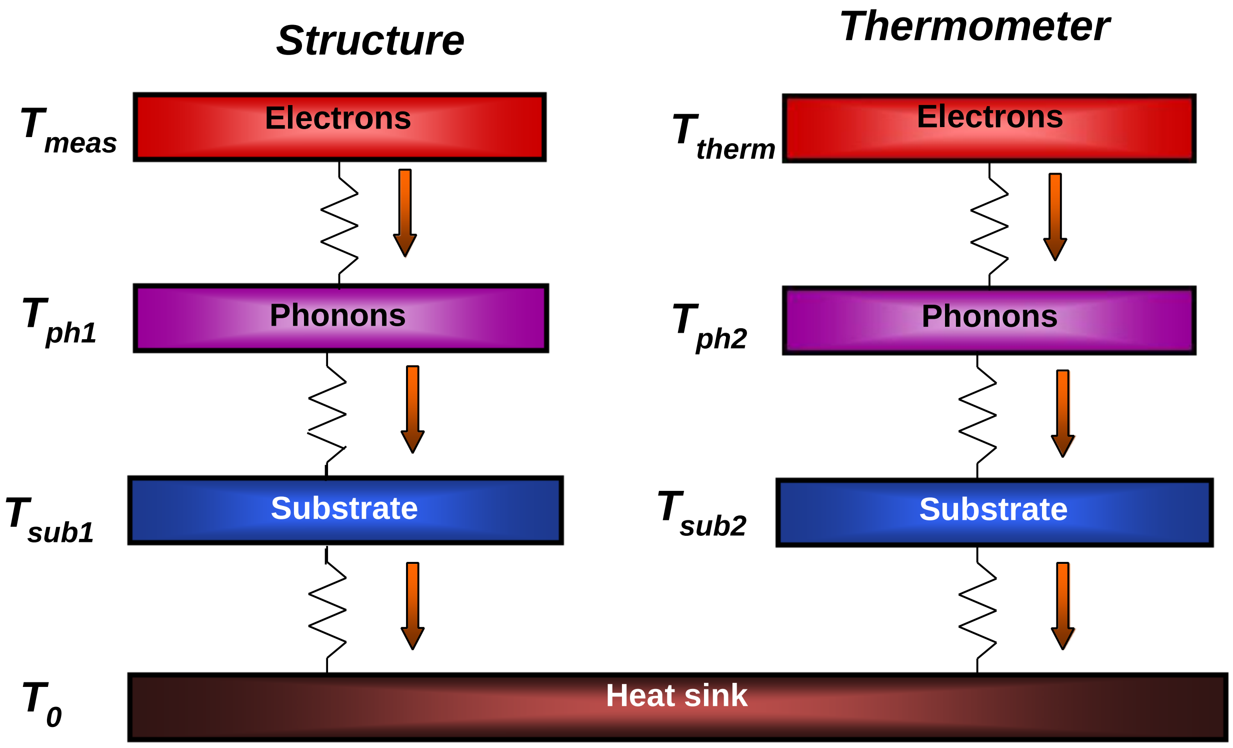
$\Re_{K\,(Si\text{-}Cu)}$ ($\propto T^{-3}$) B=14 K^4/(Wcm^2)	**12 mK/(nW mm^2)**	**12 mK/(μW mm^2)**



- Good thermal contact between thermometer and sample
- Self overheating of thermometer
- Possible difference between phonon and electron temperatures

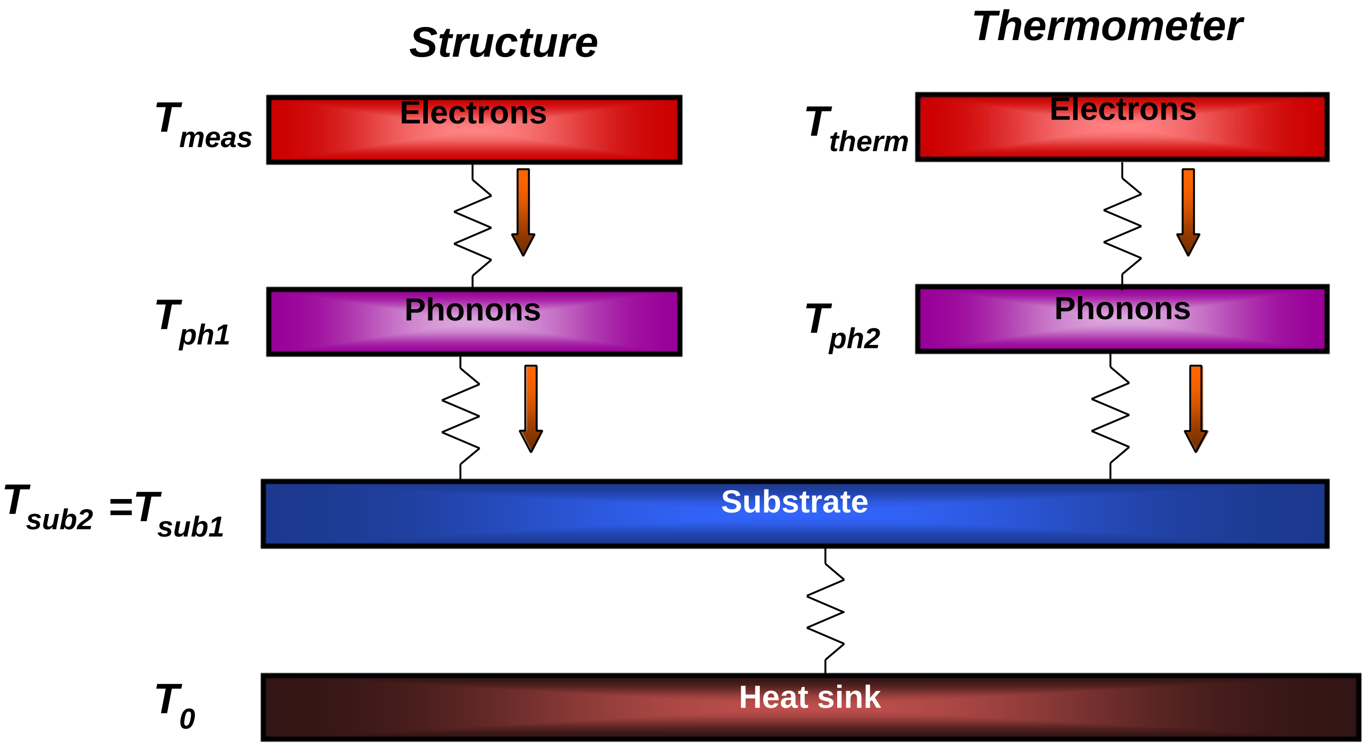
Thermal contact between thermometer and measured structure

Typical configuration



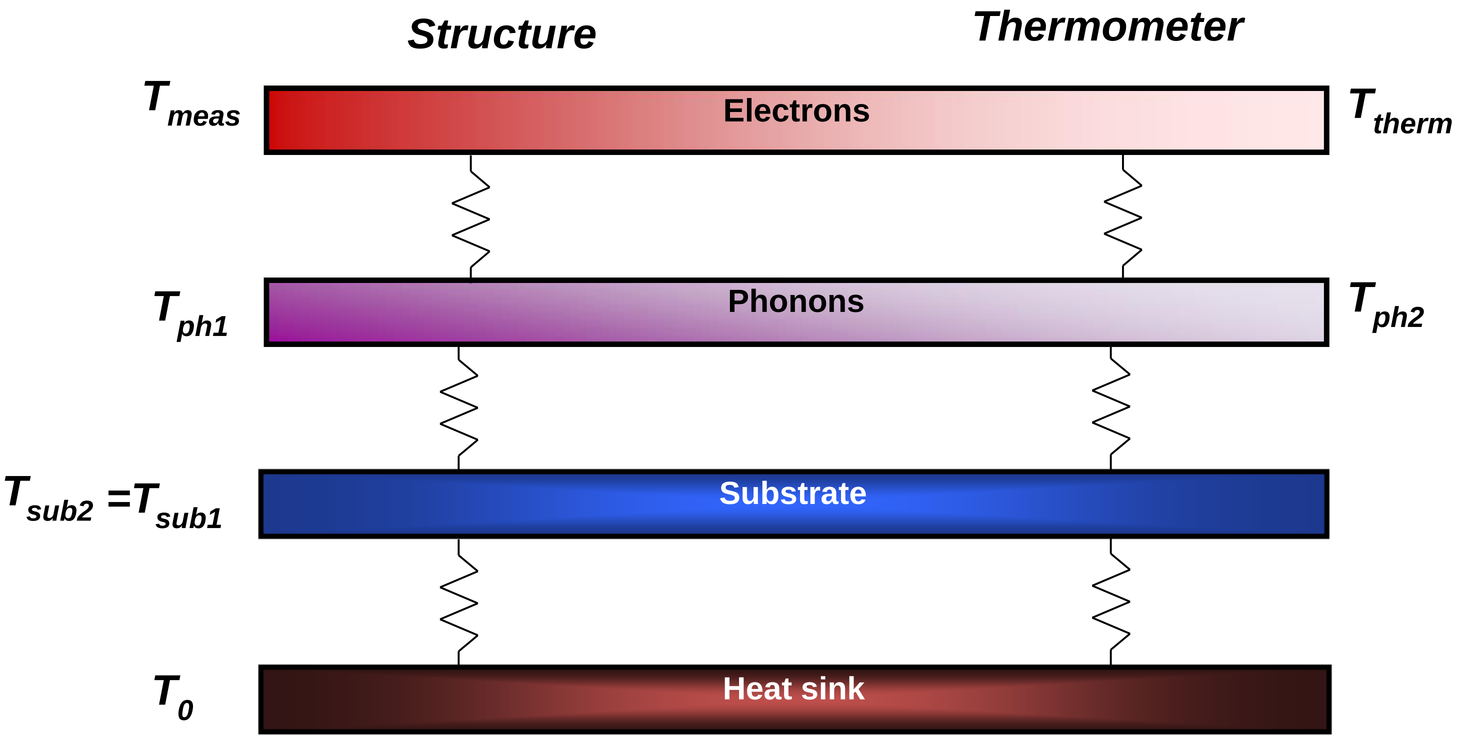
Thermal contact between thermometer and measured structure

On-chip thermometer

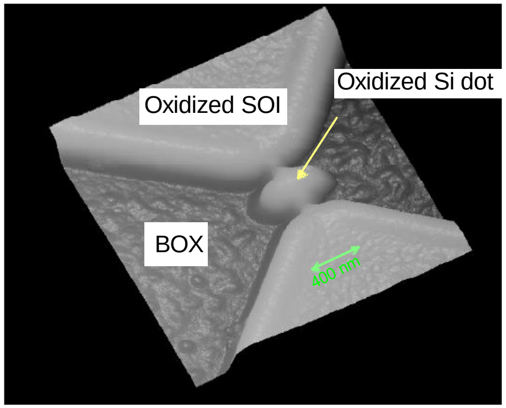


Thermal contact between thermometer and measured structure

Direct temperature measurement: utilization of measured structure as thermometer

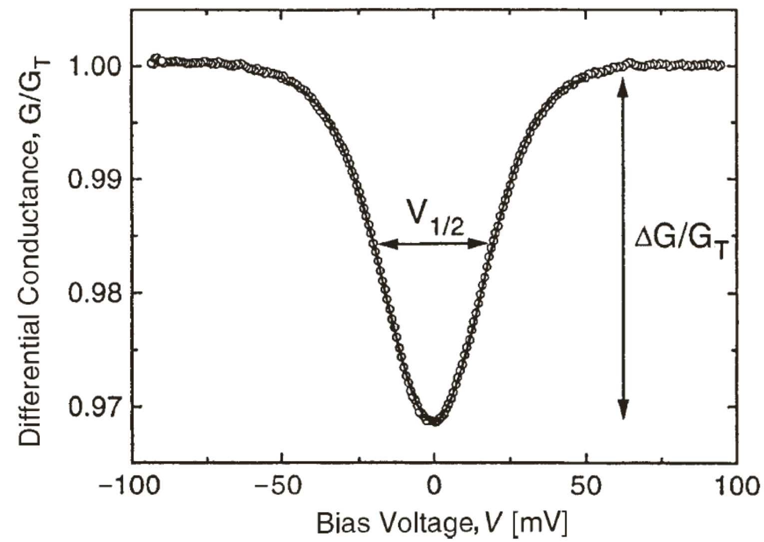


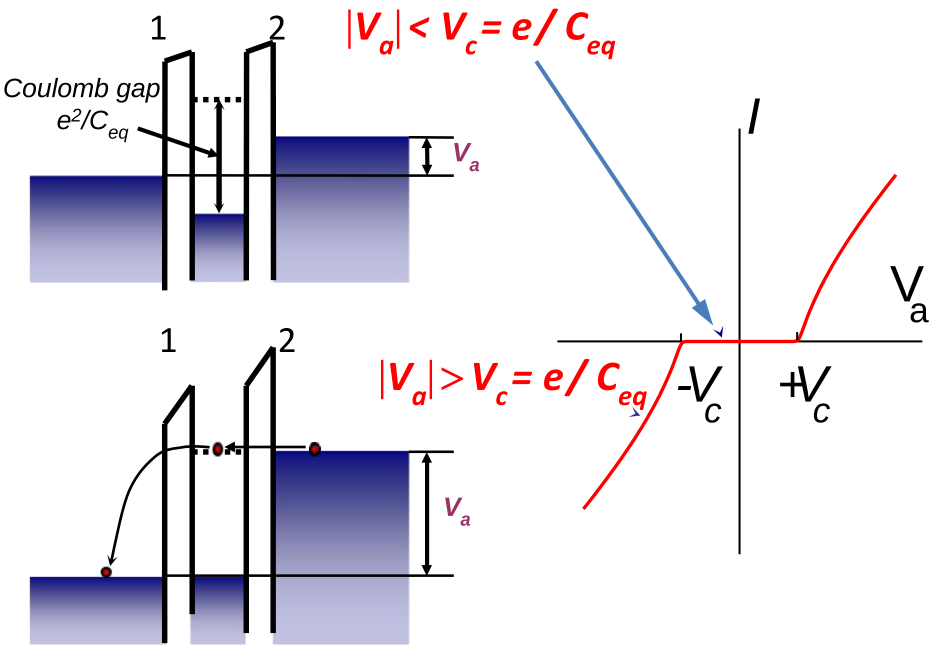
Primary Thermometers
Coulomb blockade thermometer (CBT)



Weak Coulomb blockade regime: $k_B T \approx E_C$

J. Pekola et al, PRL 73 (1994) 2903





$$V_{1/2} = 5.439 N k_B T / e$$

Secondary mode

$$\Delta G / G_T = E_C / 6 k_B T$$

$$G(V)/G_T = 1 - (E_C / k_B T) g(eV / N k_B T)$$

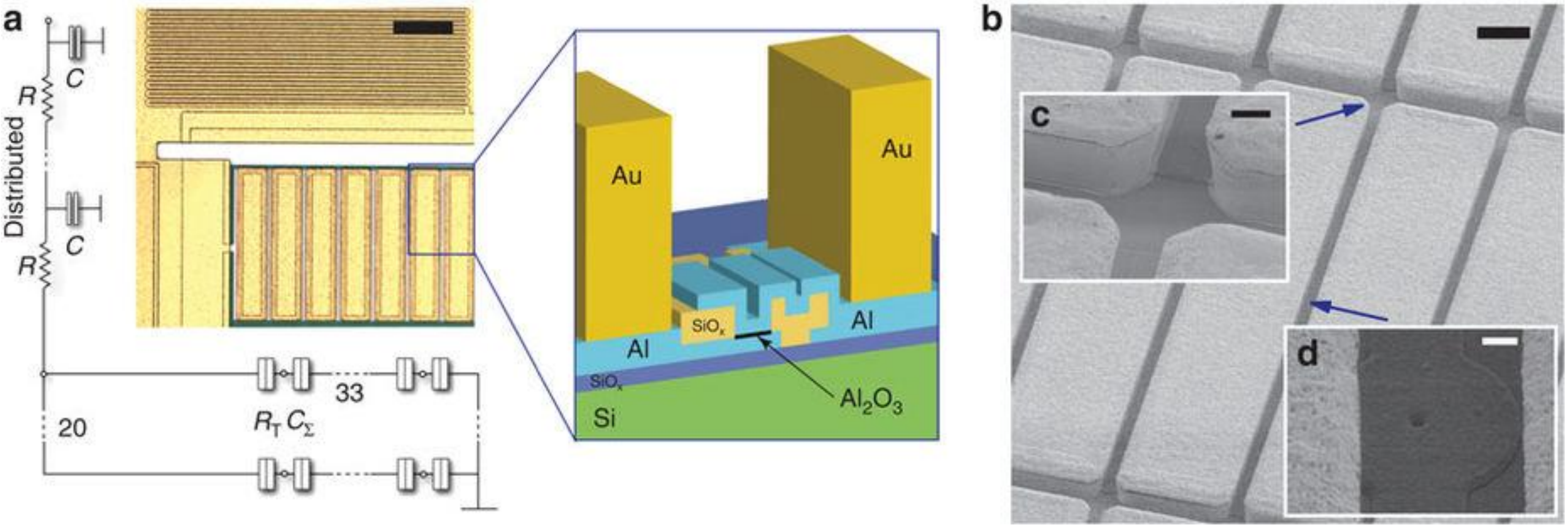
$$g(x) = (x \sinh x - 4 \sinh^2 x/2)/(8 \sinh^4 x/2)$$

Coulomb blockade thermometer

Improved thermalization of the CBT islands:

$$G_{ep} = \Sigma\Omega(T_e^5 - T_{ph}^5)$$

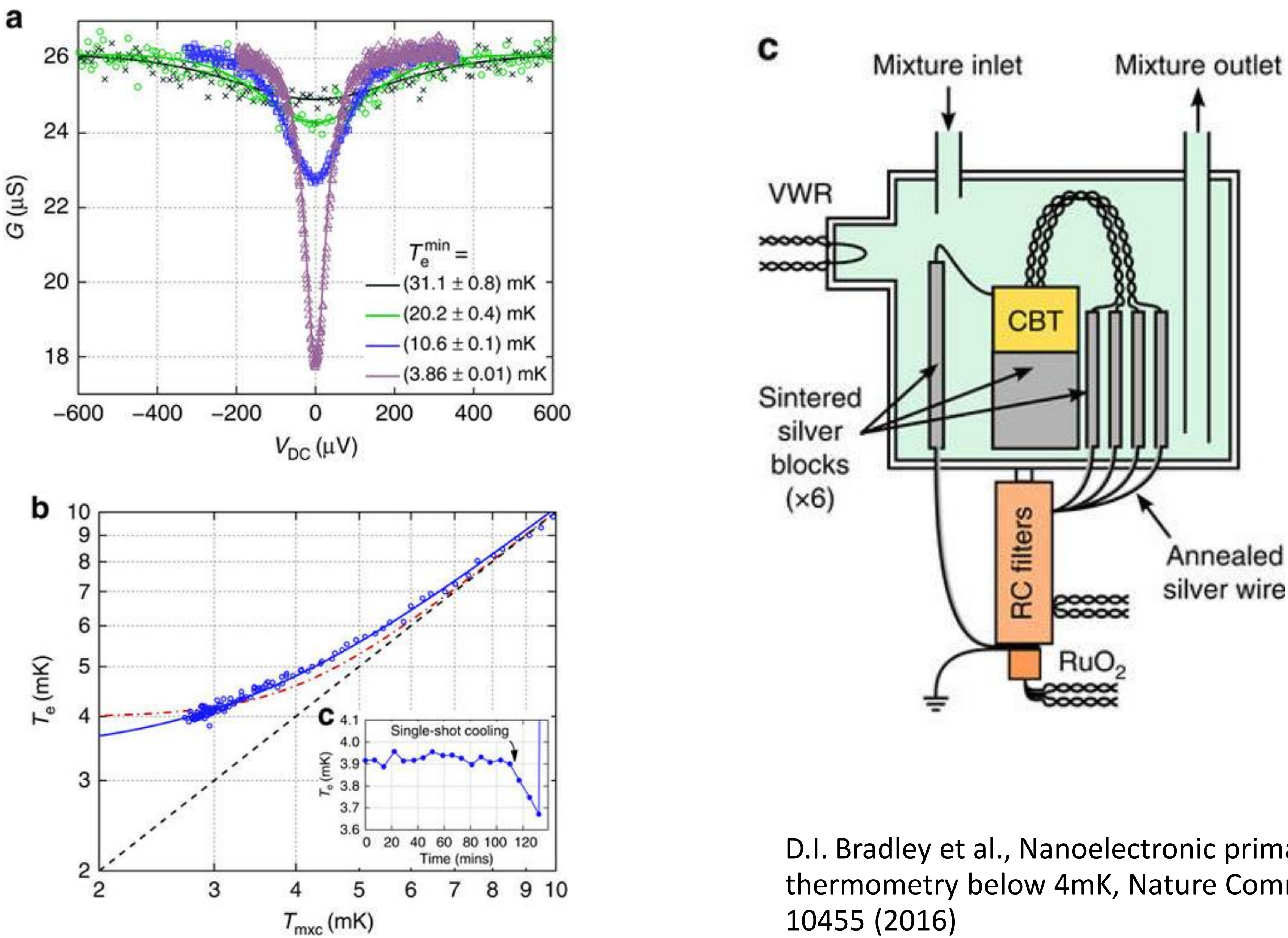
- S nearly independent of material
  => increase volume



- **Masked electroplating** of Au on top of the CBT islands
- Electroplating can produce **10 μm-thick**, low-stress films
- a nominal thickness of 5 μm for the thermalization blocks.

D.I. Bradley et al., Nanoelectronic primary thermometry below 4mK, Nature Comm. **7**,10455 (2016)

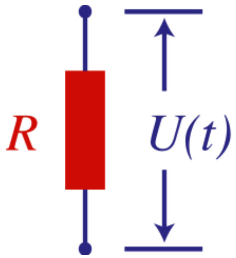
Coulomb blockade thermometer



D.I. Bradley et al., Nanoelectronic primary thermometry below 4mK, Nature Comm. **7** , 10455 (2016)

Noise Thermometry

Johnson & Nyquist (1928)

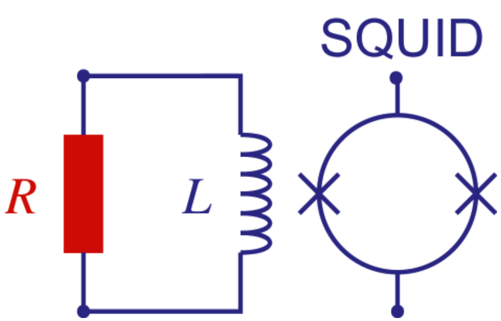


thermal voltage fluctuations across a conductor

$$\langle V^2 \rangle = 4k_B T R \Delta f$$

Power at low temperatures is rather small
For 10 kΩ and bandwidth 100 kHz at T = 1 K :

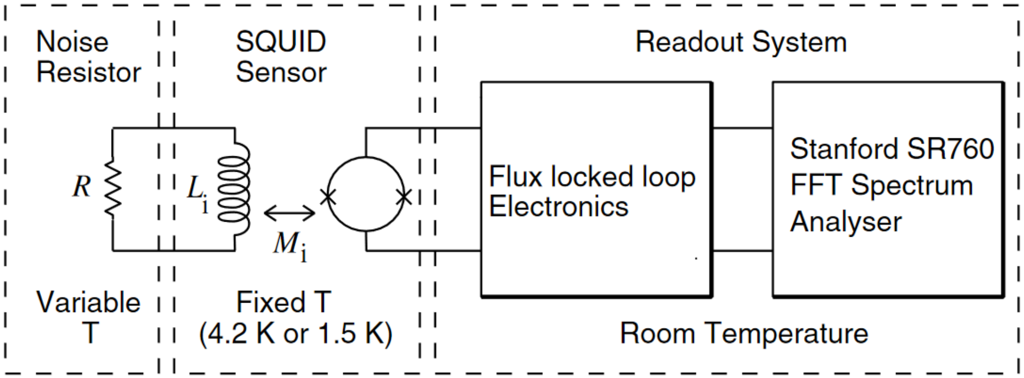
- voltage fluctuations 2×10^{-7} V
- noise power: < 10^{-18} W

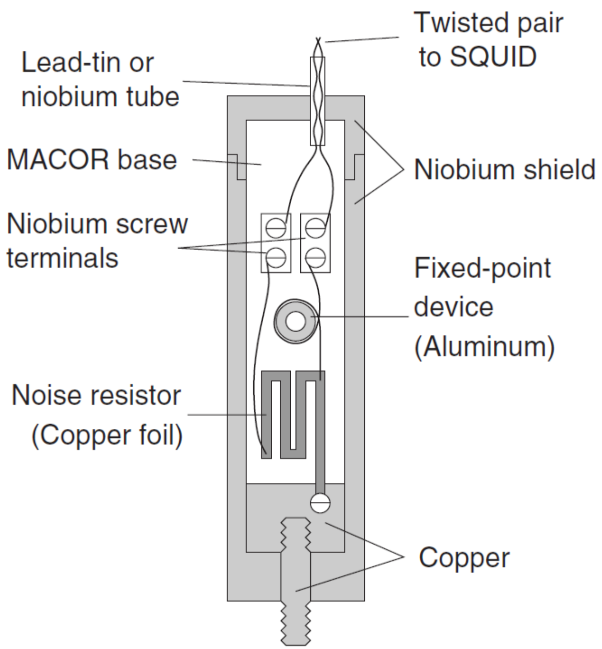


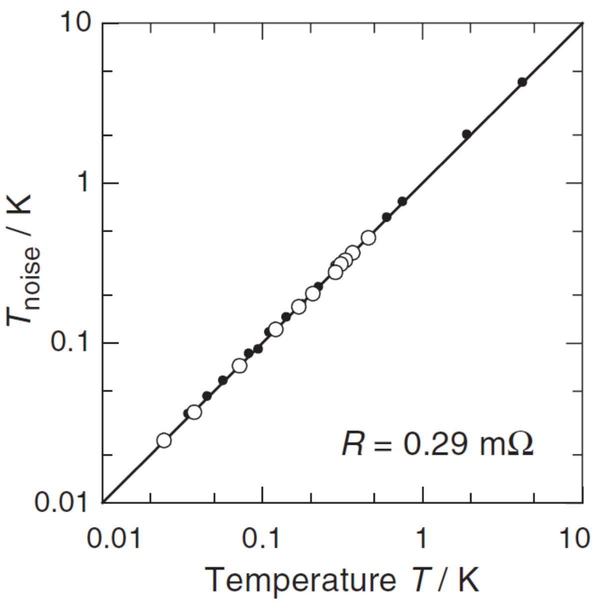
SQUID readout

$$\langle I^2 \rangle = \frac{4k_B T}{R} \Delta f \qquad \omega_c = R / L$$

Noise Thermometry







C.P. Lusher et al. Meas. Sci. Technol. **12**, 1 (2001)

Probing flux noise with SQUID

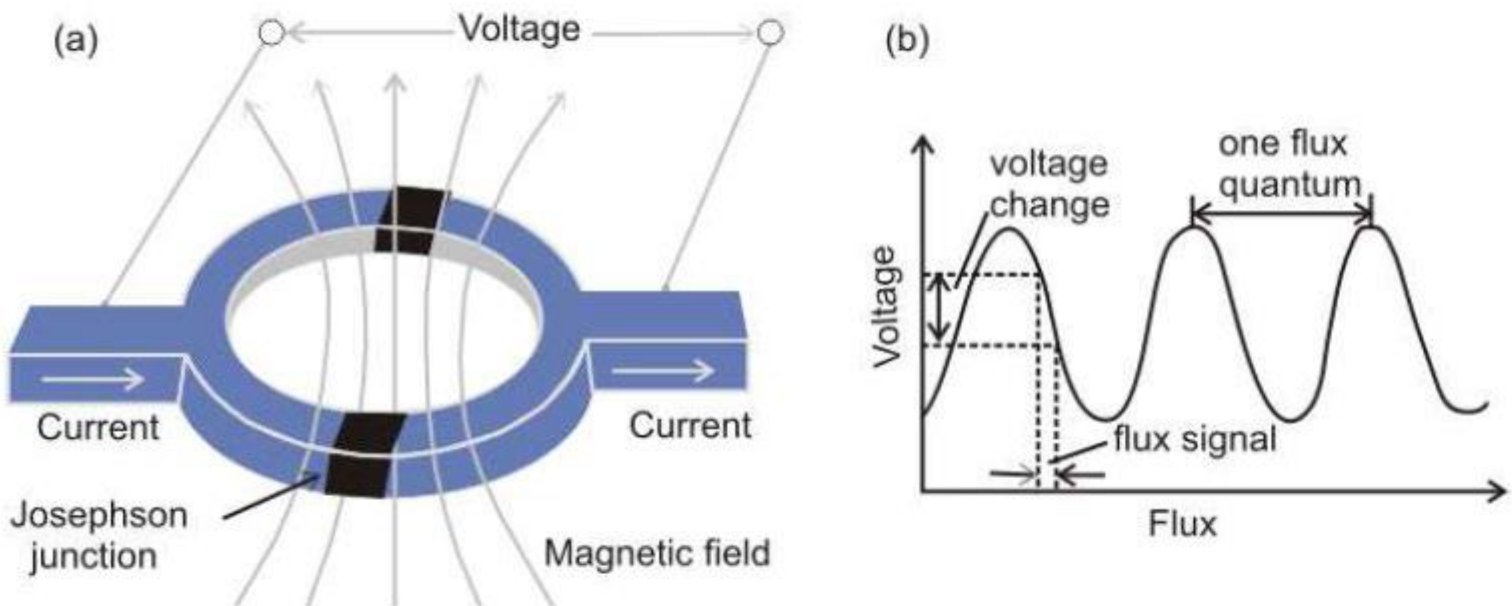
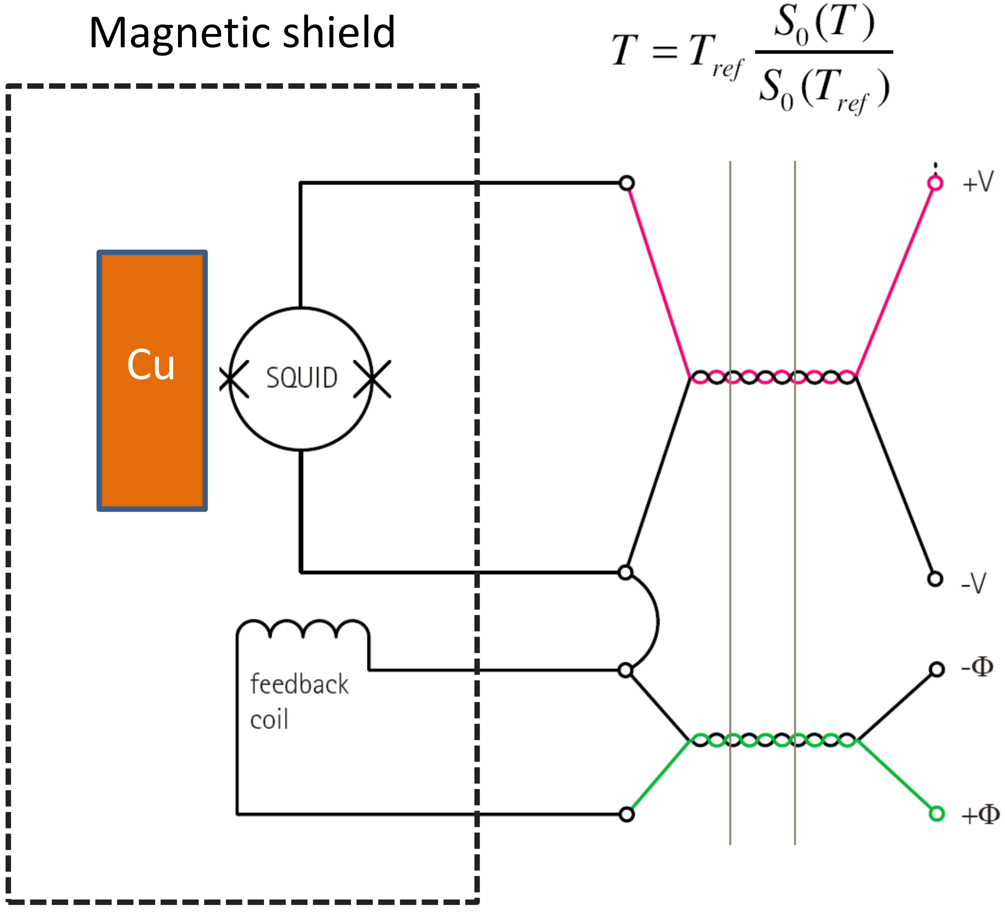


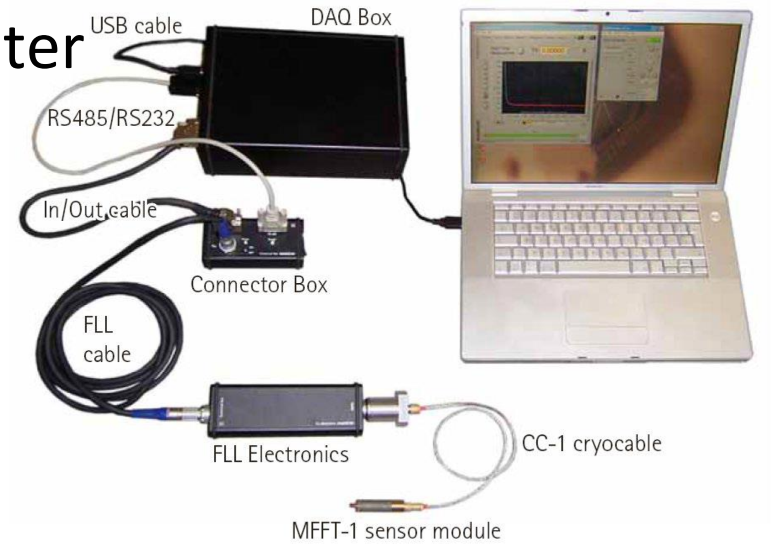
Figure 2: *a)* Symmetrical DC-SQUID device. *b)* Voltage across the device as a function of external flux.

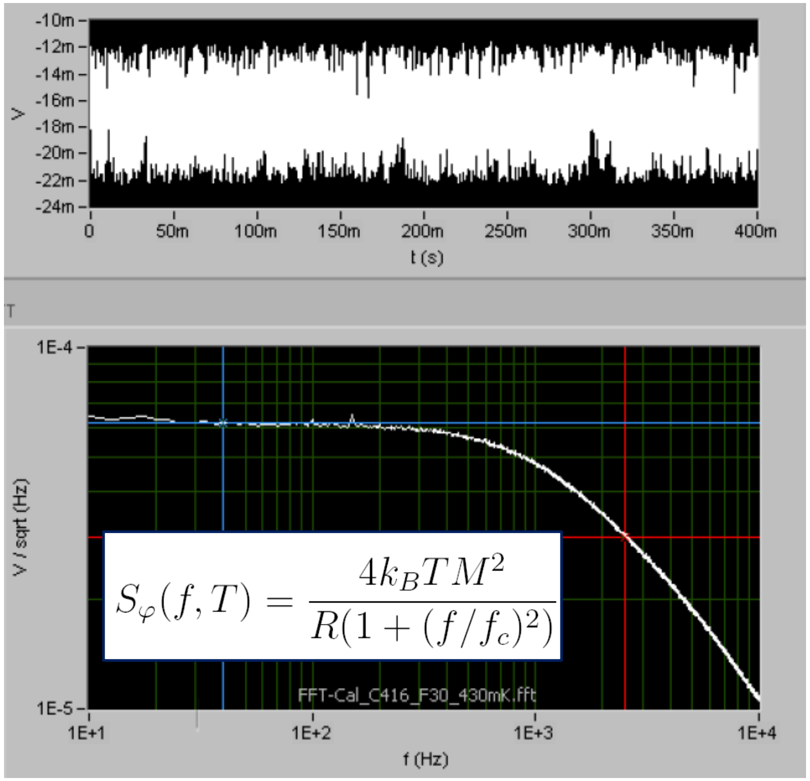
Magnetic-Field-Fluctuation Thermometer

http://www.magnicon.com/

$$T = T_{ref} \frac{S_0(T)}{S_0(T_{ref})}$$

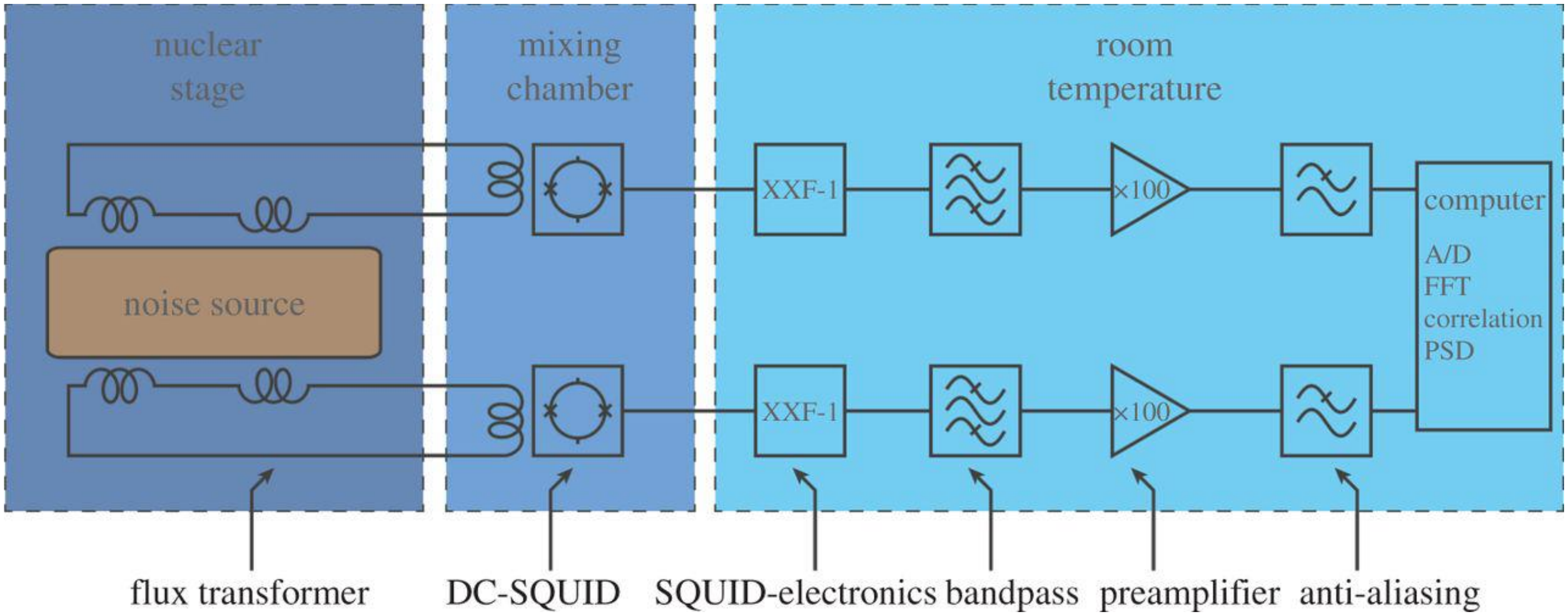






- Linear temperature range from 1mK to 1K
- Better than 1% resolution in temperature with only 30s measurement time

Cross Correlation configuration



Signal processing with Cross Correlation for Noise Reduction

Channel 1: $A_1(t) = U(t) + N_1(t)$

Channel 2: $A_2(t) = U(t) + N_2(t)$

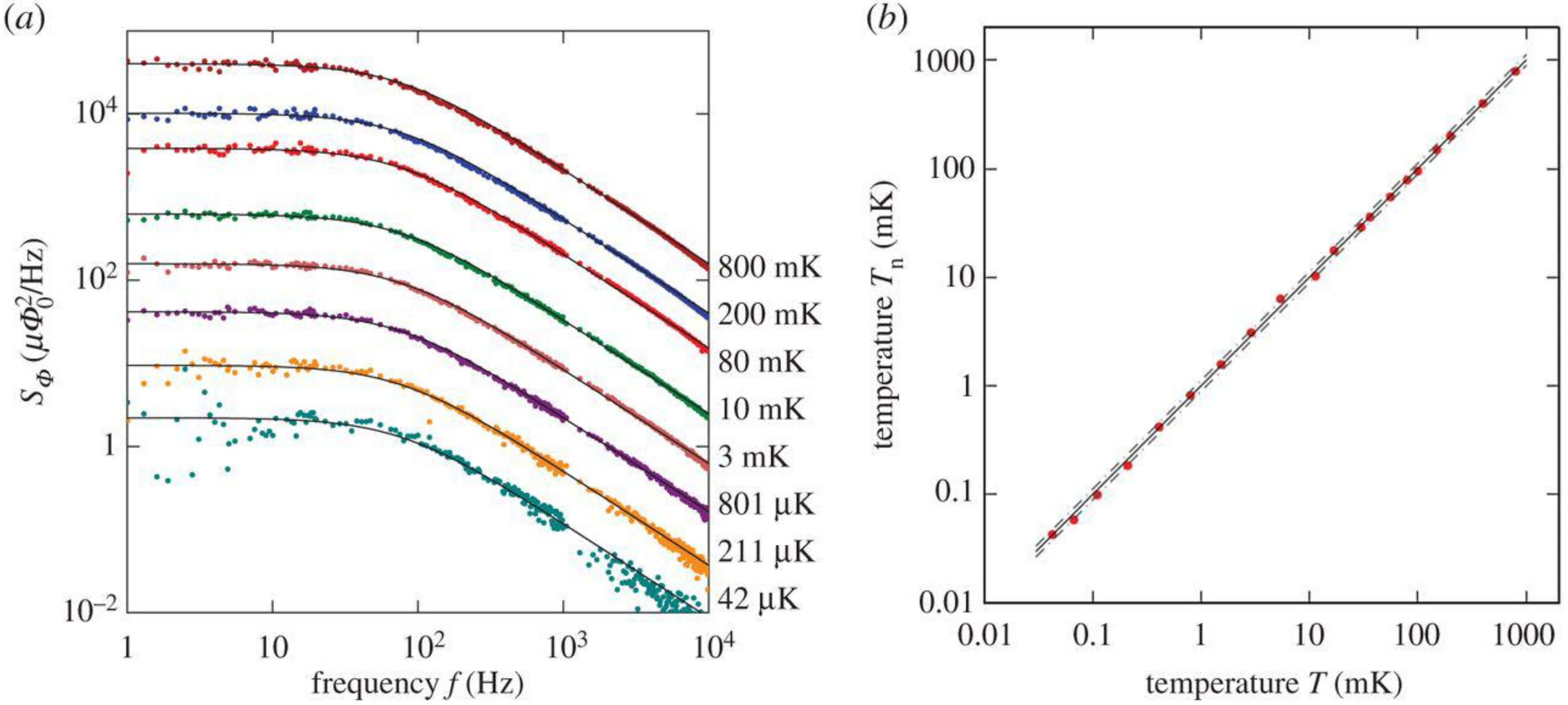
Cross Correlation: $$C(t') = \lim_{T_W \to \infty} \frac{1}{T_W} \int_0^{T_W} A_1(t) A_2(t + t') \mathrm{d}t = R(t)$$

$$\Rightarrow \quad S_\Phi(f) = 2 \int_{-\infty}^{\infty} R(t)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t$$

D. Rothfuss et al. Phil. Trans. R. Soc. A **374**, 20150051 (2016)

Cross Correlation configuration

- Linear temperature range extended to 50 μK
- Measurement time 400 s at lowest temperatures
- Signal down with lower T ⟺ NMR signal up with lower T



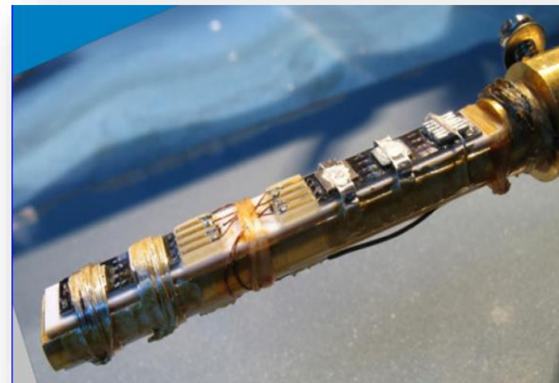
D. Rothfuss et al. Phil. Trans. R. Soc. A **374**, 20150051 (2016)

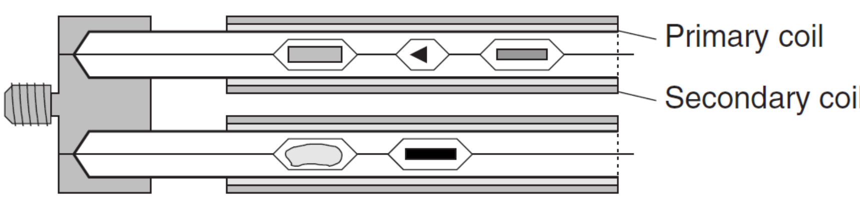
Secondary Thermometers

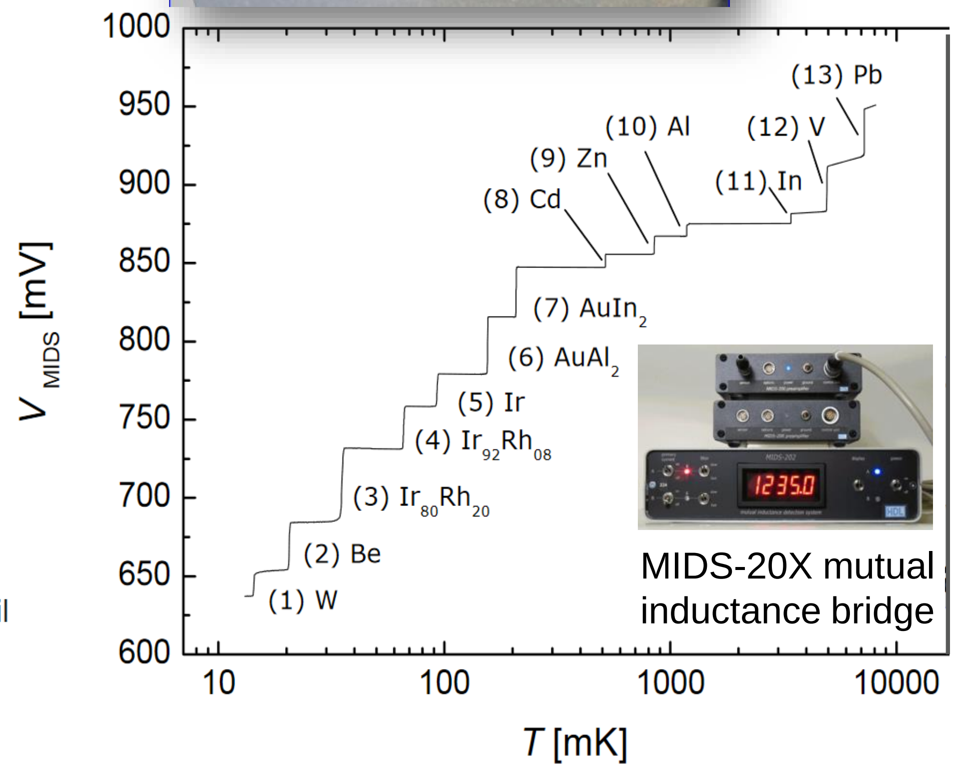
Superconducting Fixed Point Thermometer

hdleiden.home.xs4all.nl/srd1000

#	material	T_C [mK]	W_C [mK]	U_C [%]
1	W	15	< 0.2	< 0.26
2	Be	21	< 0.3	< 0.28
3	$Ir_{80}Rh_{20}$	30	< 0.5	< 0.34
4	$Ir_{92}Rh_{08}$	65	< 0.5	< 0.16
5	Ir	98	< 0.5	< 0.10
6	$AuAl_2$	145	< 0.5	< 0.06
7	$AuIn_2$	208	< 1	< 0.10
8	Cd	520	< 1	< 0.04
9	Zn	850	< 2	< 0.05
10	Al	1180	< 4	< 0.06
11	In	3400	< 4	< 0.02
12	V	4900	< 20	< 0.08
13	Pb	7200	< 6	< 0.02

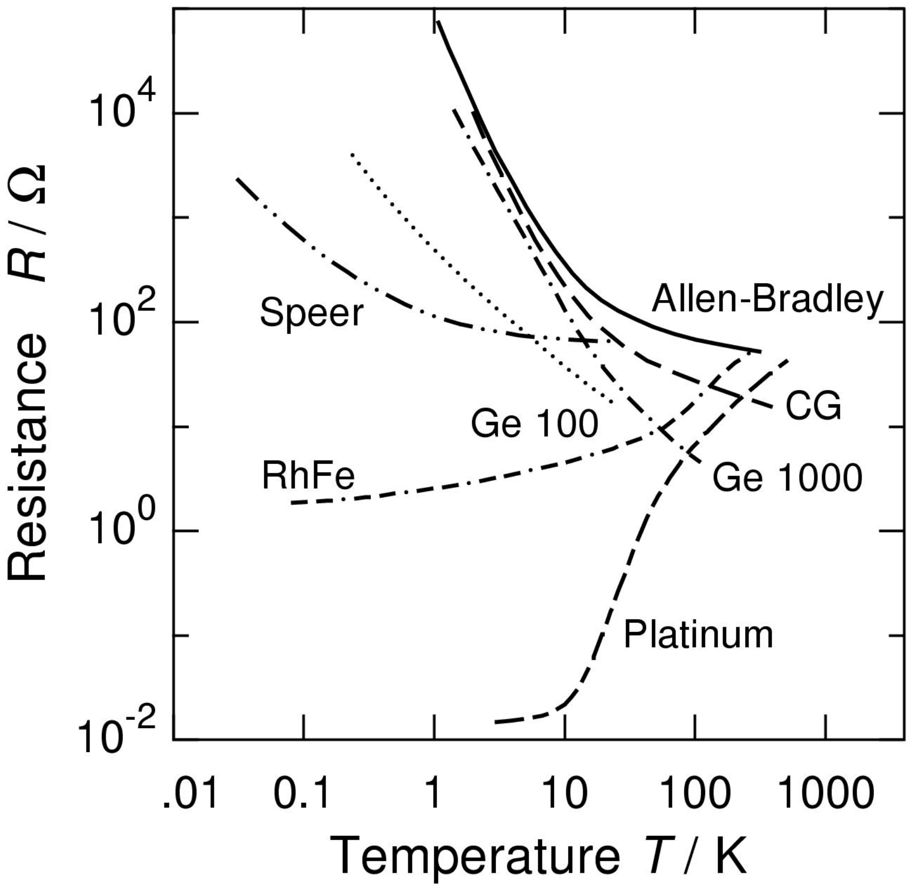






Resistance Thermometers

- Most common thermometers
- Inexpensive, easy to use



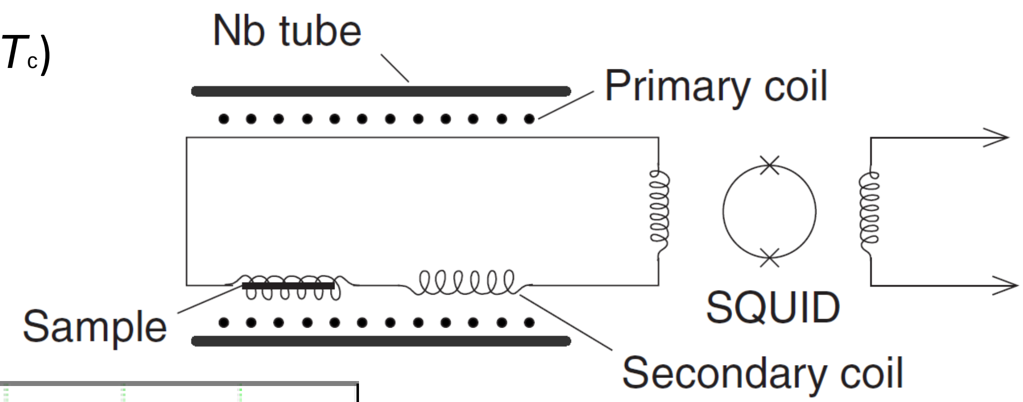
Possible problems:

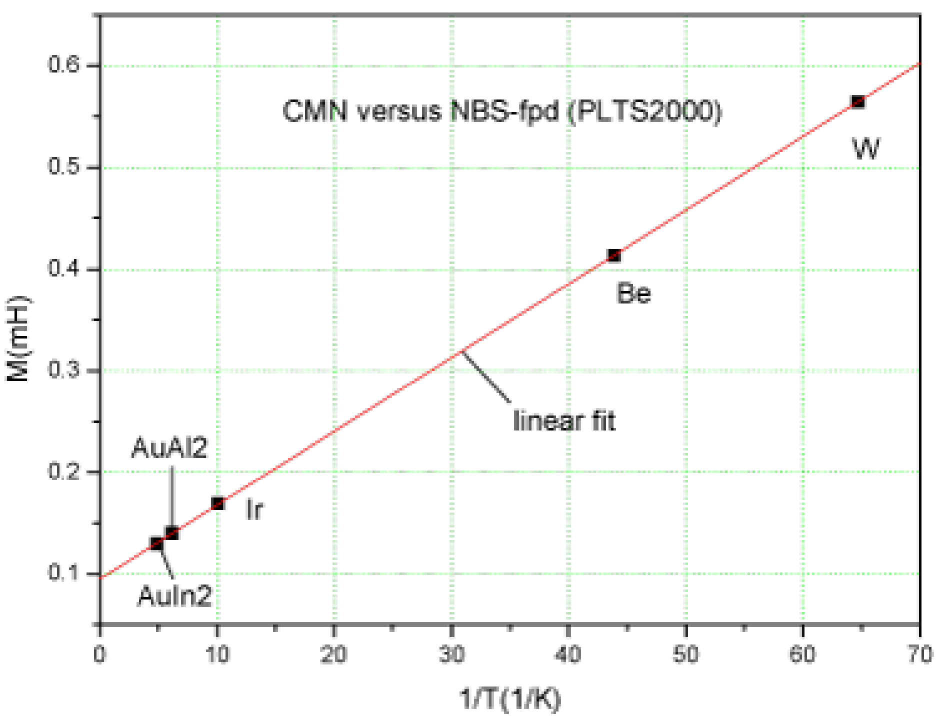
- Thermal contact between thermometer and sample;
- Power dissipation in thermometer (self heating);
- Thermoelectric power;
- ***Overheating by noise***;
- Very weak electron-phonon coupling below 100 mK;
- ***Not fully reproducible*** in different cooling cycles;
- Not trivial temperature dependence;
- ***Sensitivity to external magnetic field***;
- Calibration may depend on excitation current/voltage;
- …

Magnetic susceptibility thermometers

Paramagnetic Materials: CMN (cerium magnesium nitrate), Au:Er, Pd:Fe

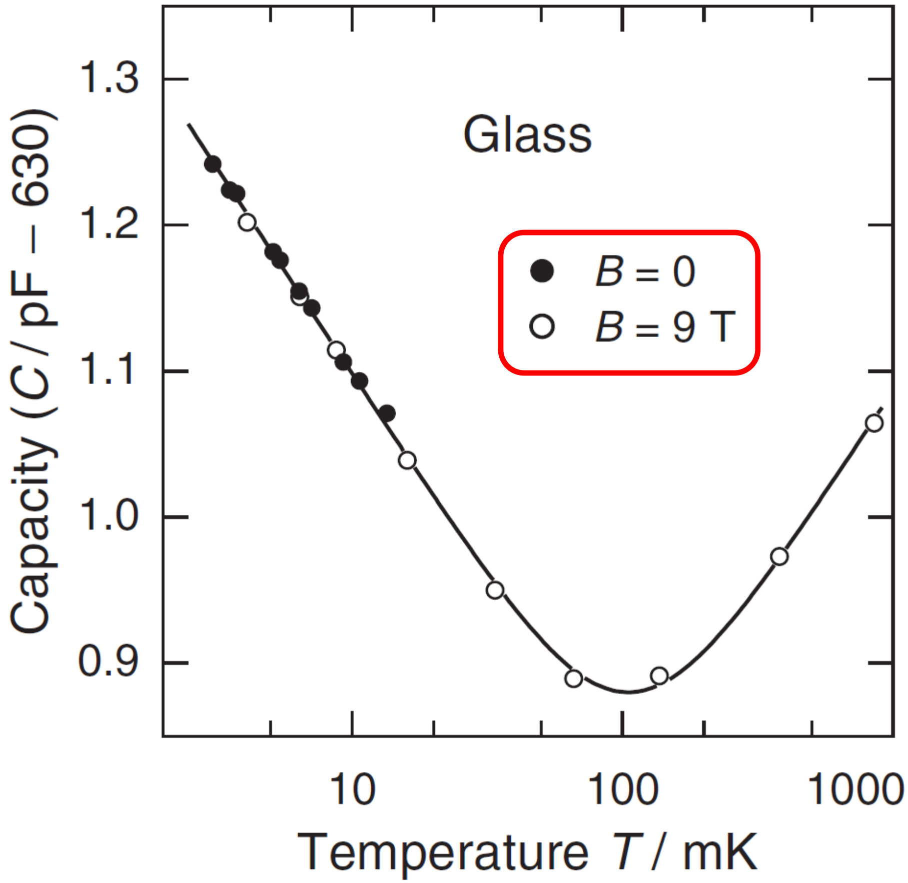
Curie–Weiss law: $\chi = C/(T - T_c)$

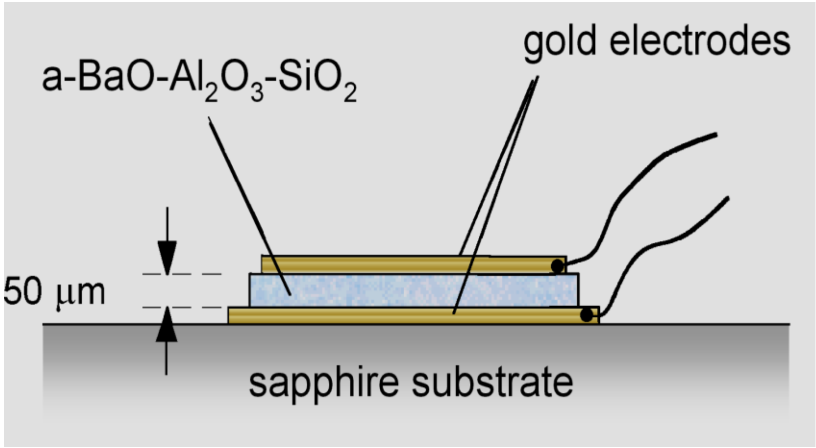




- Coil to generate magnetization $M = \chi B$
- Gradient coils to subtract background

Capacitive Thermometer





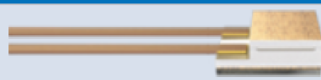
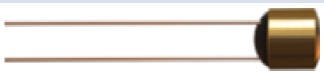
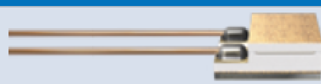
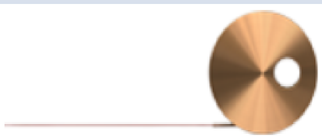
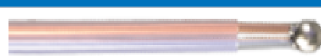
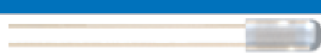
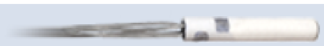
Capacitance C of a glass capacitance thermometer as a function of temperature

S. A. J. Wiegers, R. Jochemsen, C. C. Kranenburg, and G. Frossati, Rev. Sci. Inst. **58**, 2274 (1987)

Basic Commercial Cryogenic Thermometers

Cryogenic Temperature Sensors

www.lakeshore.com

		Temperature range	Standard curve	Below 1 K	Can be used in radiation	Performance in magnetic field
Diodes						
Silicon		1.4 K to 500 K	■			Fair above 60 K
GaAlAs		1.4 K to 500 K				Fair
Negative Temperature Coefficent RTDs						
Cernox®		0.10 K to 420 K		■	■	Excellent above 1 K
Germanium		0.05 K to 100 K		■	■	Not Recommended
Ruthenium Oxide (Rox™)		0.01 K to 40 K	■	■	■	Good below 1 K*
Other						
Thermocouples		1.2 K to 1543 K	■			Fair
Capacitance		1.4 K to 290 K				Excellent
Positive Temperature Coefficient RTDs						
Platinum		14 K to 873 K	■		■	Fair above 30 K
Rhodium-Iron		1.4 K to 500 K			■	Not reccommended below 77 K

CERNOX: Zirconium reactively sputtered in a nitrogen-oxygen atmosphere. The resulting thin film is comprised of conducting zirconium nitride embedded within a zirconium oxide nonconducting matrix.

Application of Hybrid Junctions for Thermometry NIN, NIS, SIS, SsS, $S_1S_2S_1$, SGS, SNS, SSmS

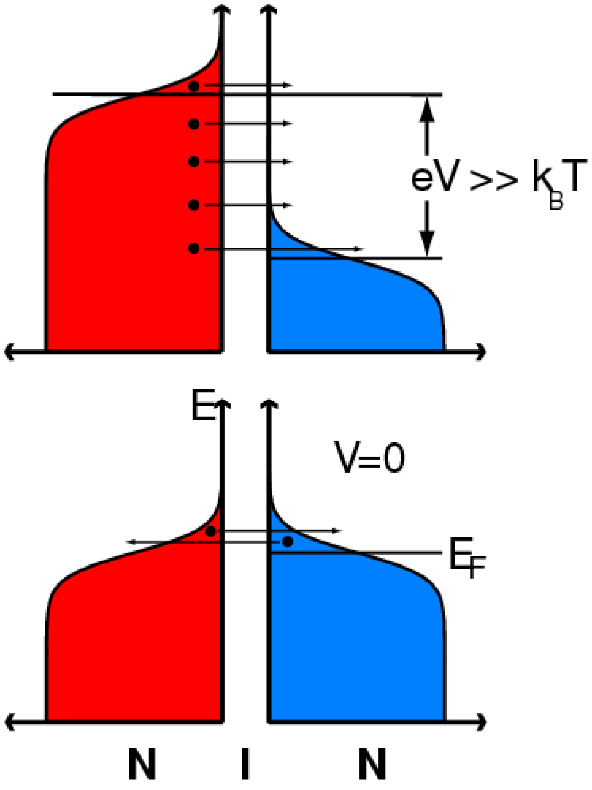
- ✓ Tunneling characteristics through a barrier separating two conductors with nonequal densities of states are usually temperature dependent.
- ✓ The barrier may be a solid insulating layer, a Schottky barrier, a vacuum gap, or a weak link.
- ✓ Two tunneling mechanisms: Cooper pair tunneling (Josephson effect) and quasiparticle tunneling.

Specific aspects of Junction thermometry:

- Small submicrometer size
- Large surface to volume ratio
- Small thermal constant
- Easy to implement in different microcircuits and devices
- Compatible with different fabrication techniques
- Direct measurement of electron temperature in normal and superconducting materials
- Require advanced electronics and filtering from external noise

NIN: Shot-Noise Thermometer

L. Spietz et al., Science 300, 1929 (2003); APL 89, 183123 (2006)



High bias regime => Shot noise:

$$S_I = 2eI$$

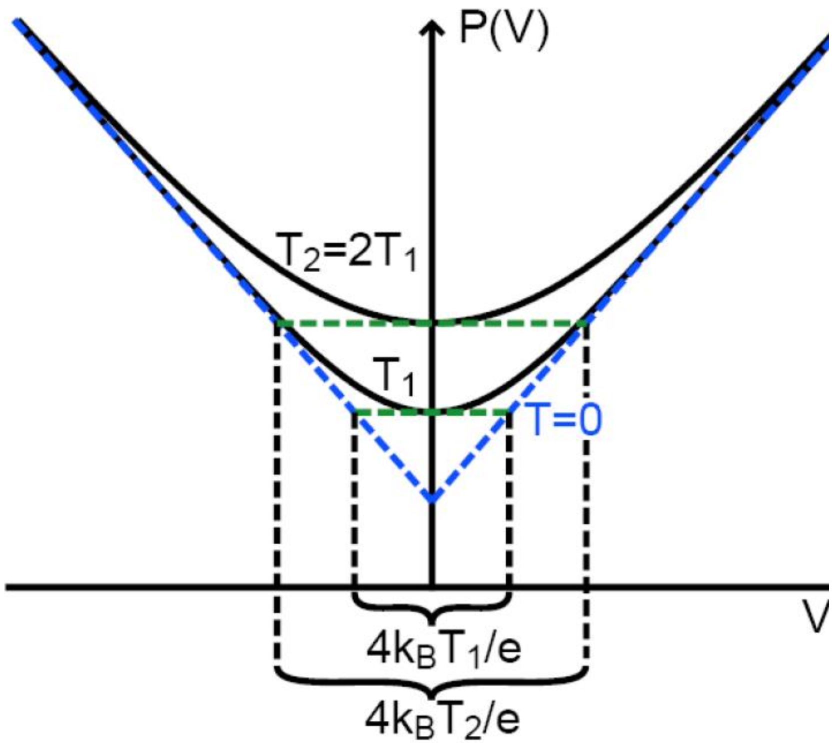
Zero bias regime => Johnson noise:

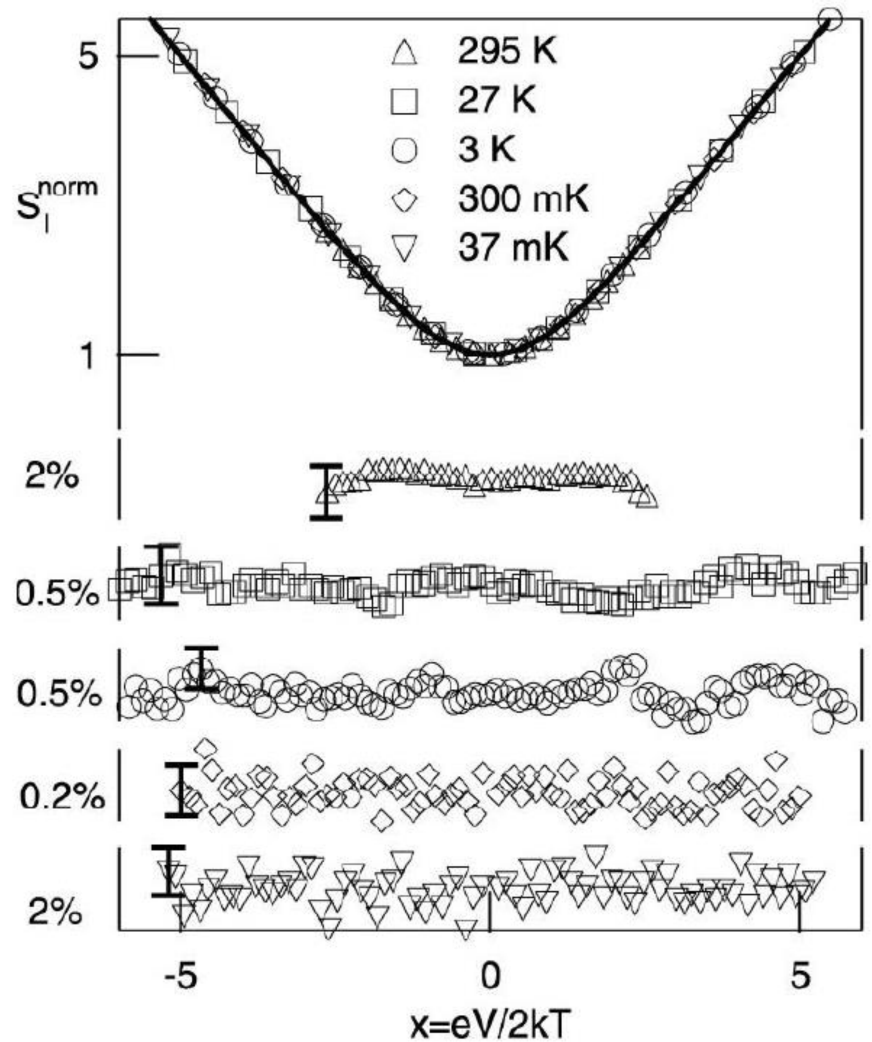
$$S_I = 4k_BT/R_T$$

NIN: Shot-Noise Thermometer

L. Spietz et al., Science 300, 1929 (2003); APL 89, 183123 (2006).

$$S_I(V) \simeq \frac{2eV}{R}\coth\left(\frac{eV}{2k_BT}\right)$$





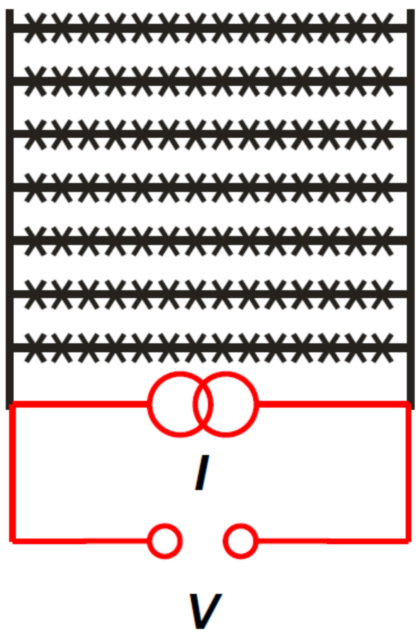
NININ: CBT Thermometer

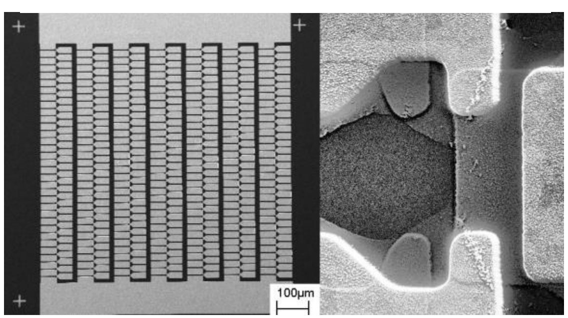
Multijunction configuration **CBT**

J. Pekola et al, PRL 73 (1994) 2903

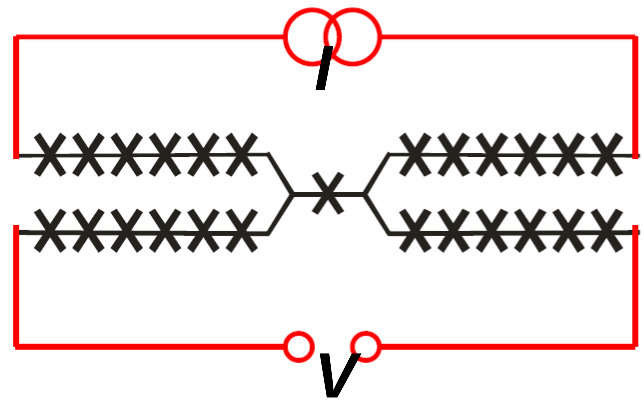
Single junction configuration **SJT**

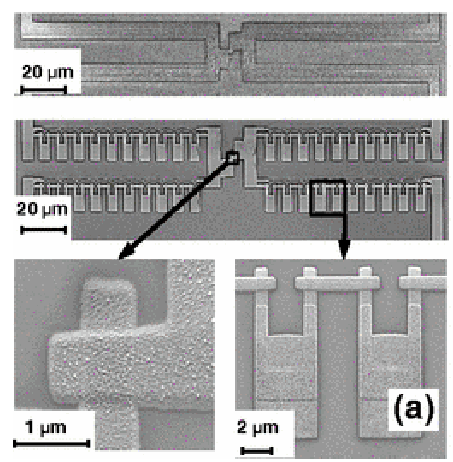
J. Pekola et al, Phys. Rev. Lett. 101, 206801

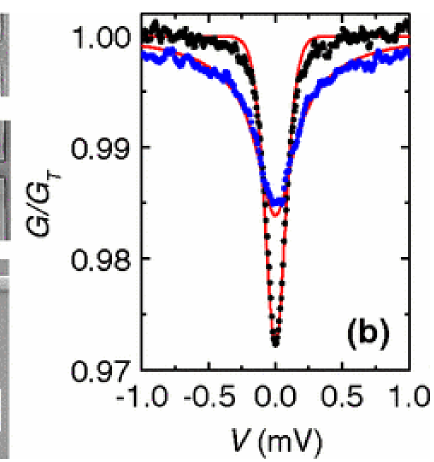




$$V_{1/2} = 5.439 \frac{N k_B T}{e}$$







Probing of superconductor temperature by SIS junction

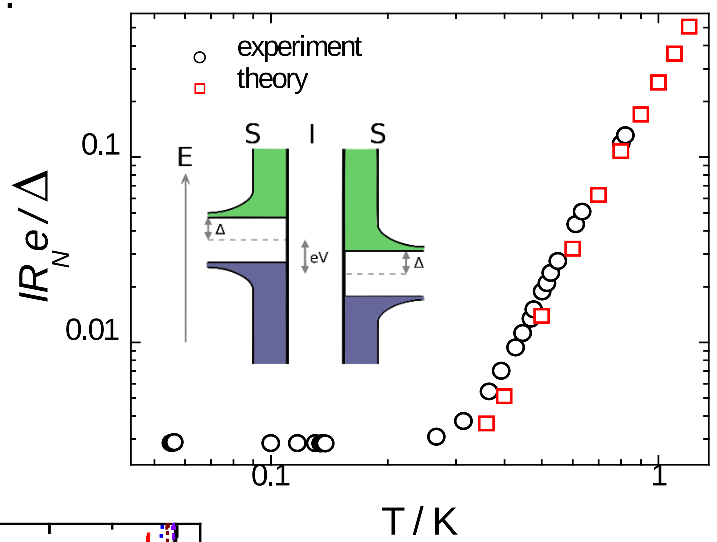
Quasiparticle current in SIS junction at finite V:

$$I = \frac{1}{eR_T}\int dE N_L(\tilde{E}) N_R(E)[f_L(E) - f_R(E)]$$

Superconducting density of states:

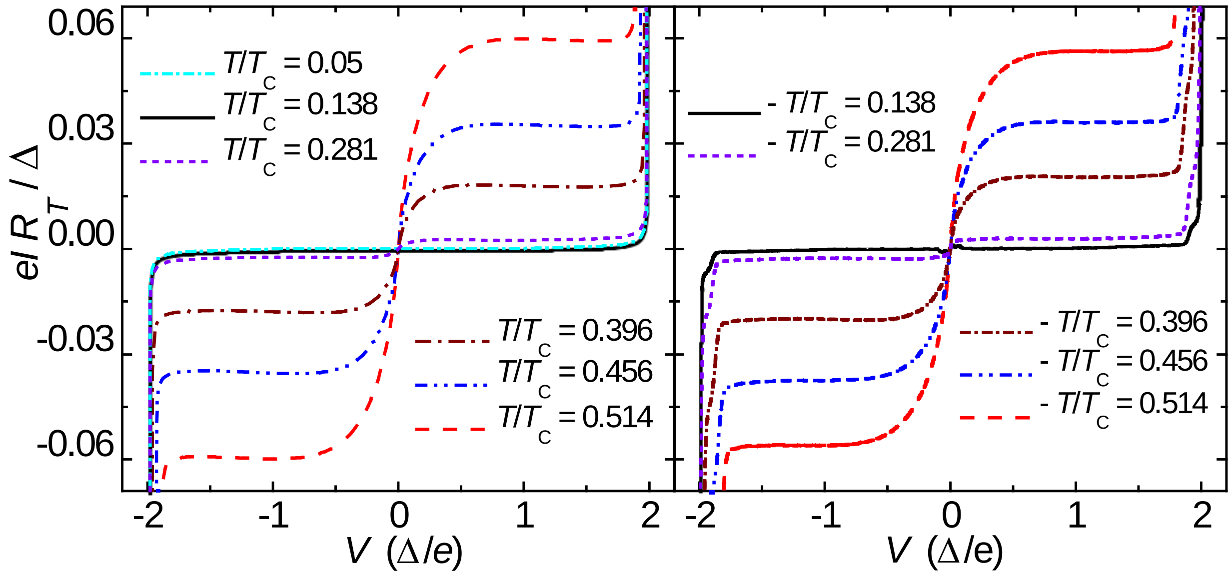
$$N_S(E) = \left| \mathrm{Re}\left[\frac{E + i\Gamma}{\sqrt{(E+i\Gamma)^2 - \Delta^2}} \right] \right|$$

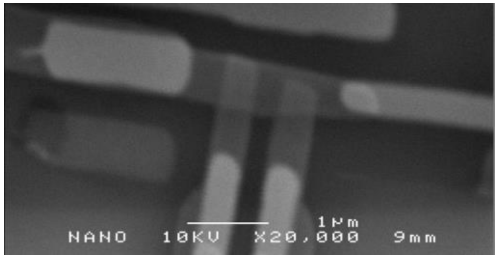
Γ: Dynes parameter



$T_{S1}=T_{S2}$:

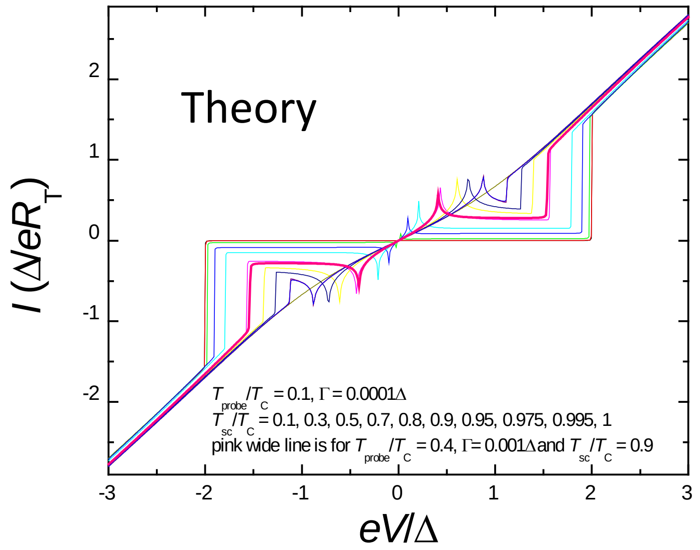
Theory Experiment

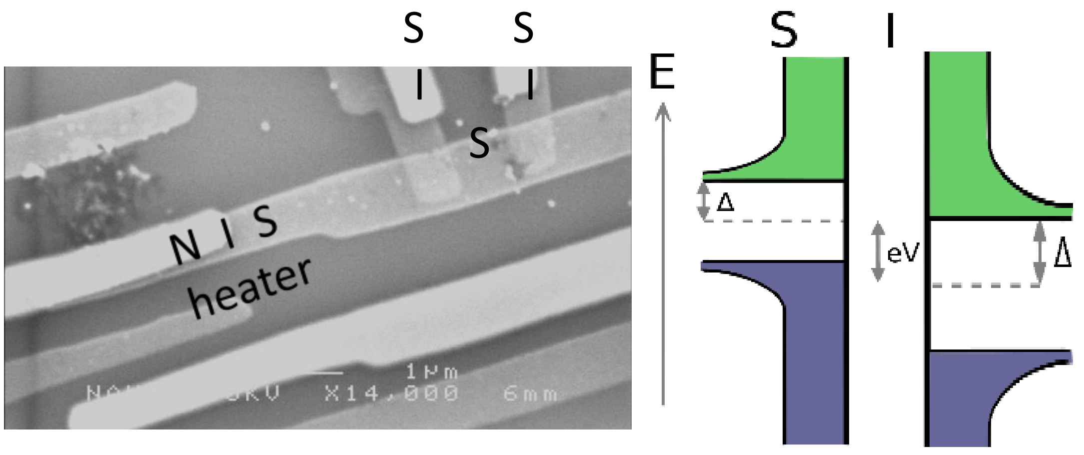


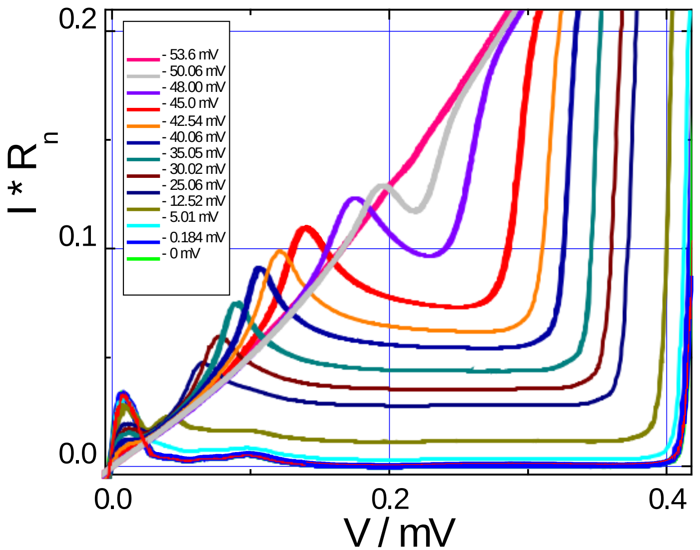


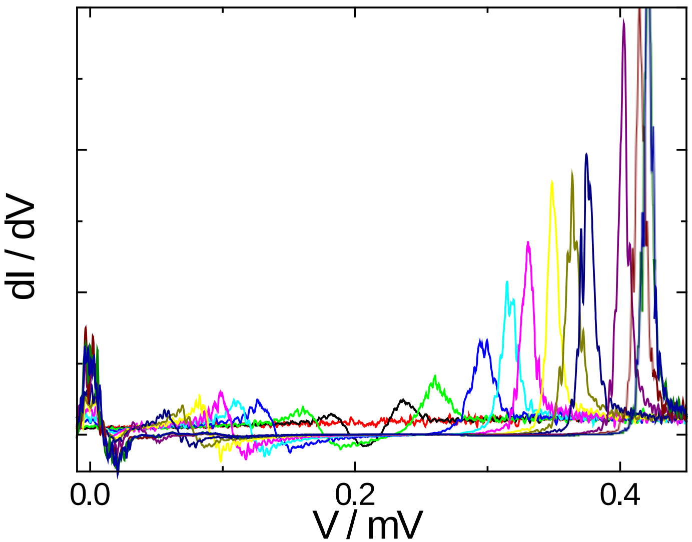
Rev. Mod. Phys. **78** (2006) 2017

Probing of superconductor temperature by SIS junction ($T_{S1}>T_{S2}$)

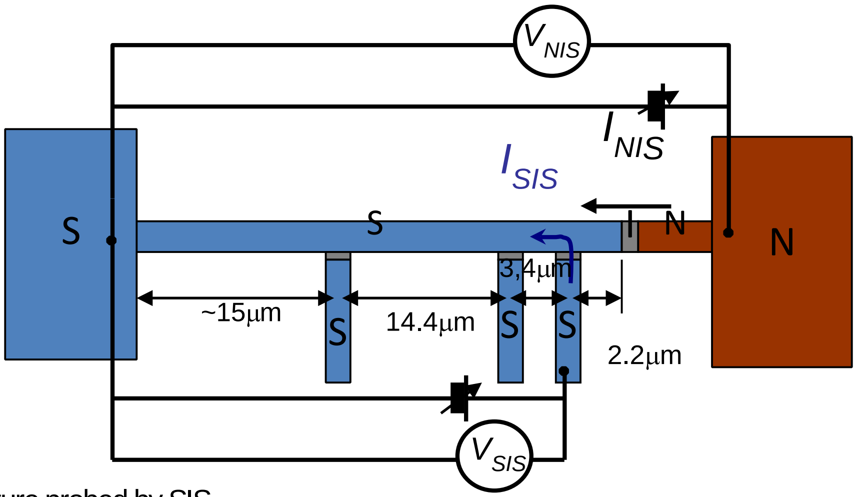




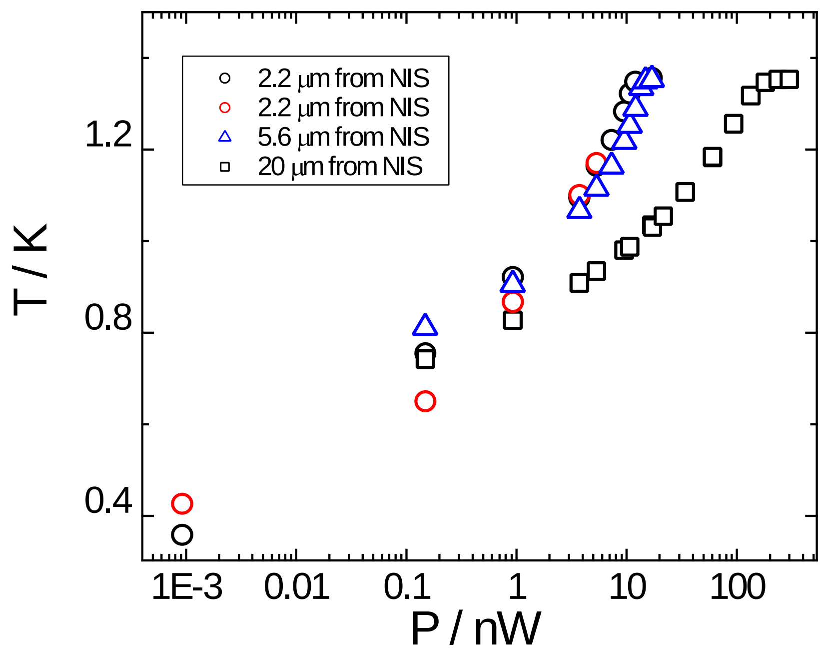


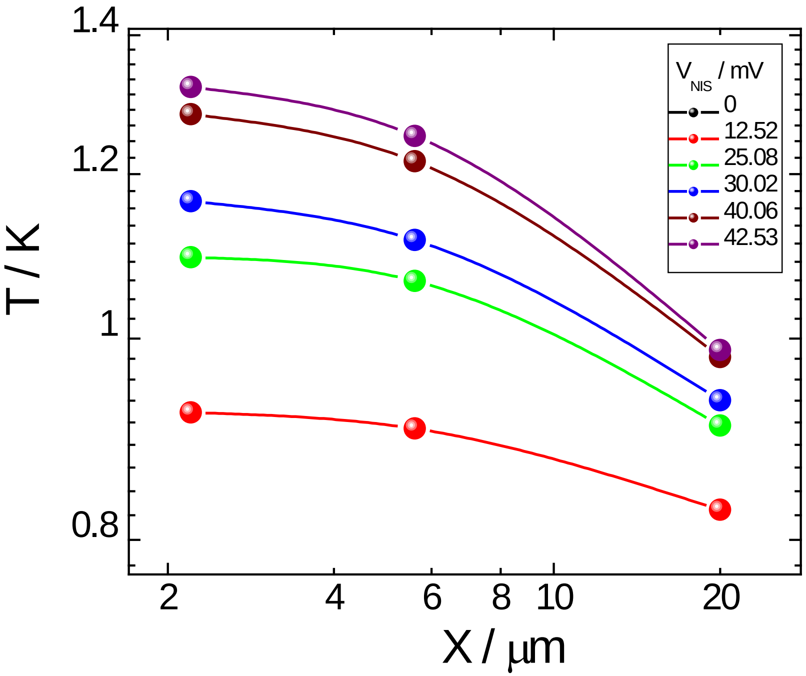


Probing of superconductor temperature by SIS junction ($T_{S1}>T_{S2}$)



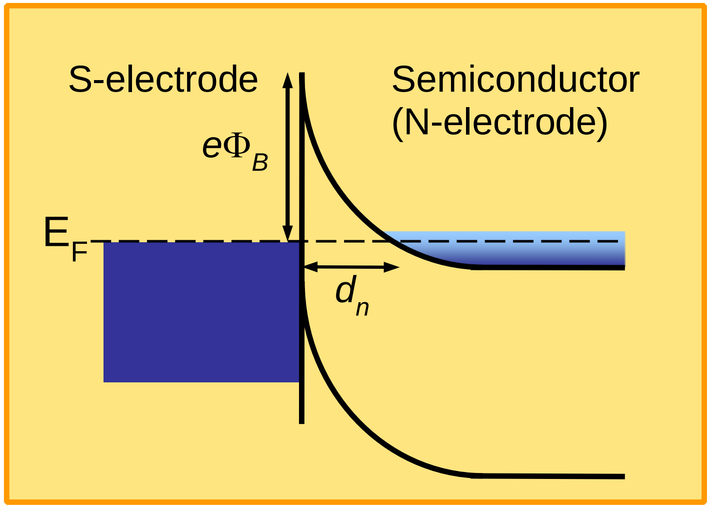
Temperature probed by SIS



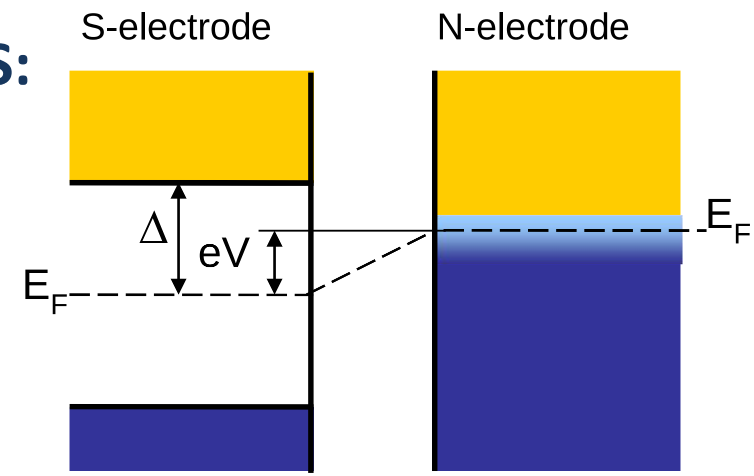


NIS and SmS Thermometry

SmS:

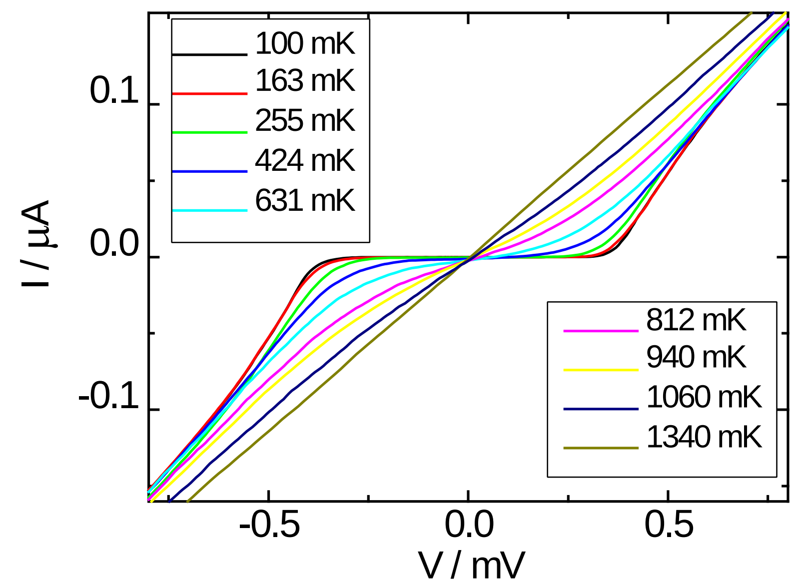


NIS:



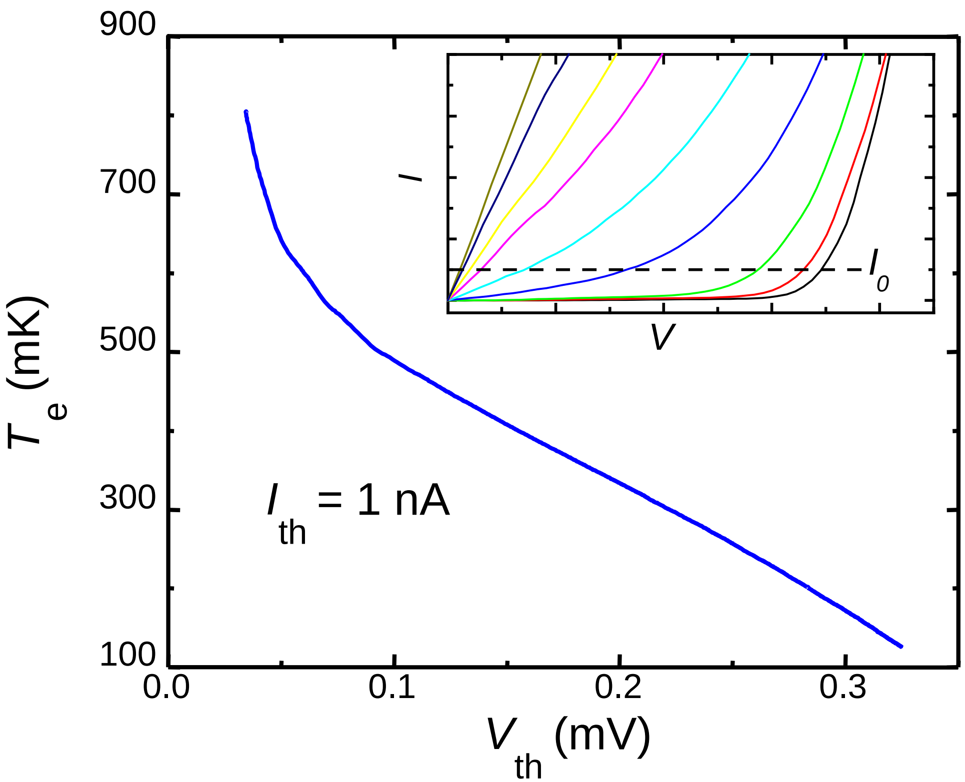
Principle of electron temperature measurement : In a junction biased with voltage $V < \Delta / e$ the tunnelling rate (current) depends on the thermal smearing of the step in the Fermi distribution in the normal metal (only electrons with $(E - E_F) > (\Delta - eV)$ have states to tunnel into superconductor)

Like SINIS structure the current voltage characteristics of S-Sm-S (Al - n^{++}Si - Al) structure strongly depends on the temperature



NIS and SmS Thermometry

Probing temperature of normal electrode N or Sm



Calibration curve of a S-Smc-S thermometer

(Voltage drop over S-Sm-S structure biased with constant current)

Thermometry is based on the temperature dependence of the I-V characteristic of S-Sm-S junction

Tunnelling current in the NIS structure:

$$I(V) = (1/eR_T) \int (f(E,T_n) - f(E-eV,T_s))\, g_S(E-eV)\, dE$$

where R_T is the normal-state tunneling resistance of the junction, f(E,T) is the Fermi distribution function $g_S(E)$ is the density of states in the superconductor, and T_n and T_s are the electronic temperatures of normal and superconducting electrode respectively.

If $k_B T_e \ll \Delta$ and $0 \ll eV < \Delta$, the current through NIS junction is given as

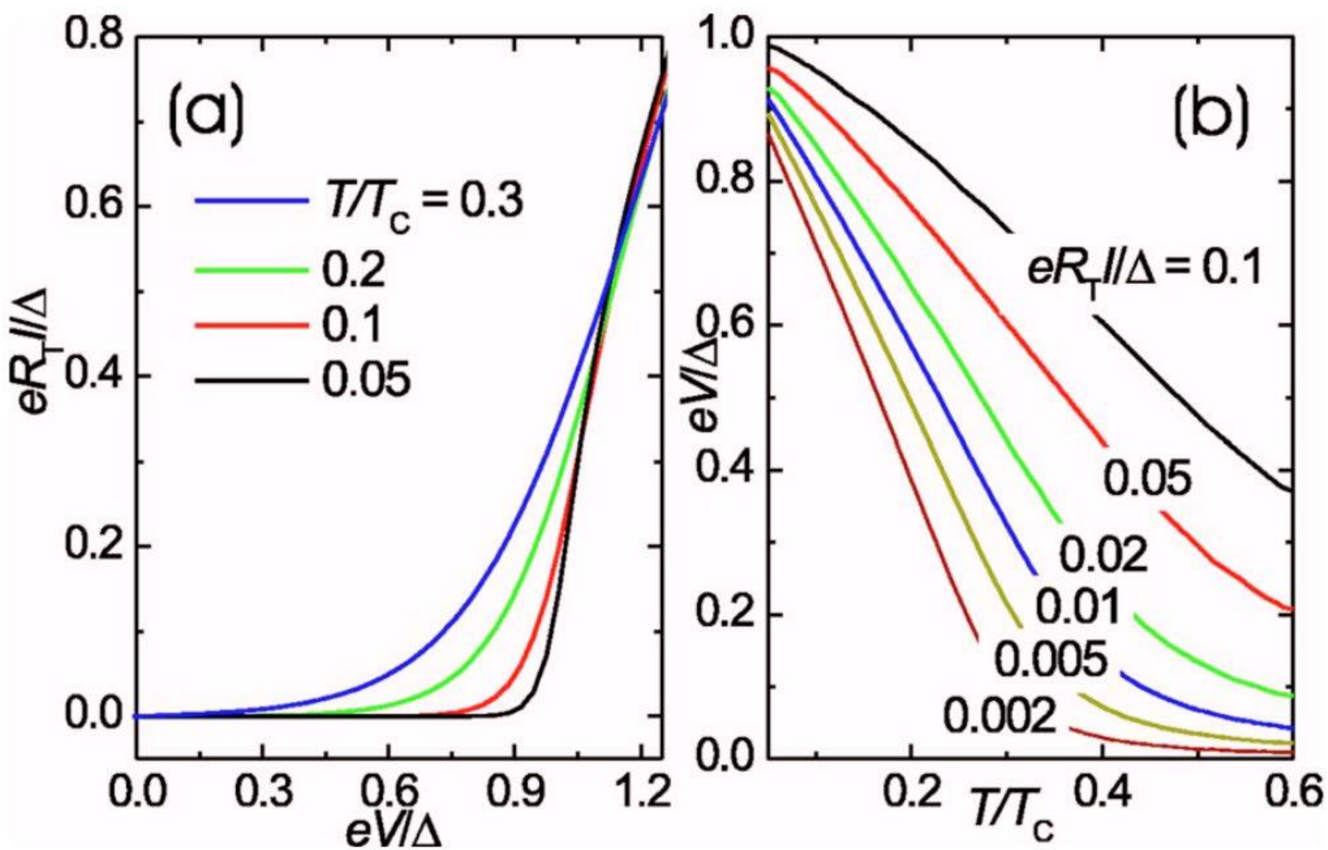
$$I(V) \approx I_a \exp[(eV - \Delta) / k_B T_e],$$
$$I_a = \Delta / e R_T (-k_B T_e / 2\Delta)^{1/2}$$

If junction biased at a constant current:

$$dV/dT_e \approx (k_B / e) \ln (I / I_a)$$

For S-Sm-S: $\mathbf{dV/dT_e \approx 1\ \mu V/mK}$

NIS and SmS Thermometry



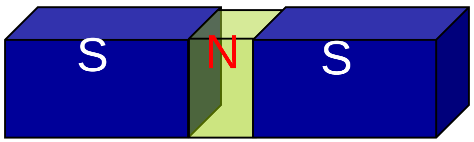
NIS thermometer characteristics.

(a) Calculated *I-V* at various relative temperatures T/T_c.

(b) The corresponding voltage over the junction as a function of temperature when the junction is biased at a constant current.

Temperature dependence of critical current SIS, SsS, SGS, SNS
SNS proximity thermometer

Metallic contact between a normal metal and a superconductor



Supercurrent in diffusive SNS junction*:

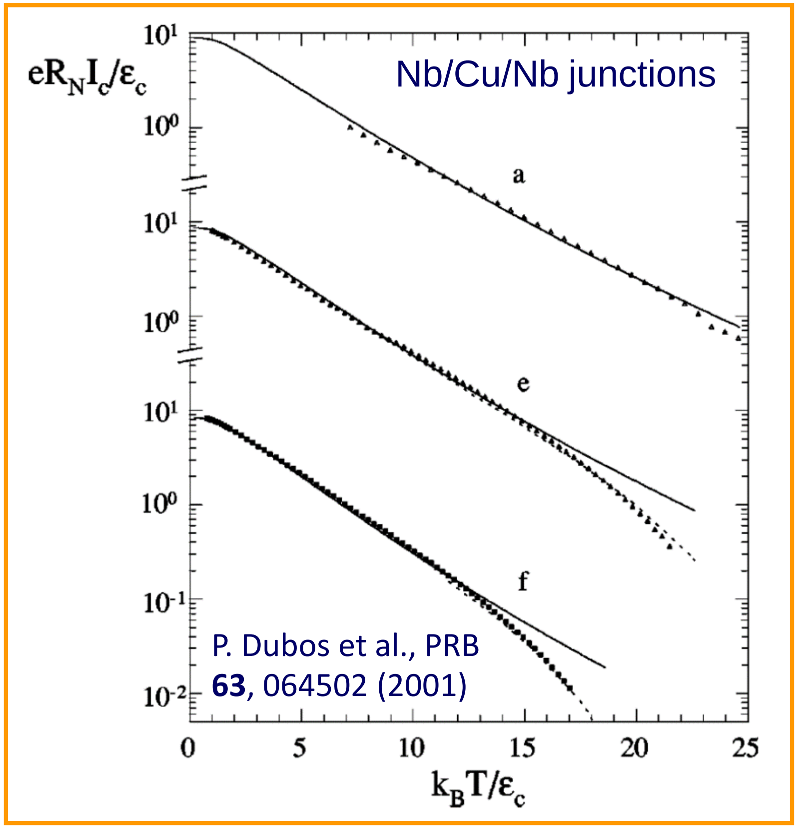
$I_J = (1/2eR_n)\int (f(-E)-f(E))j_s(E,\varphi)dE$

$j_s(E,\varphi)$ – spectrum of supercurrent carrying states

Strong temperature dependence of critical current (long junction ($\Delta >> E_{Th}$) and $E_{Th} << k_BT$):

$I_c \sim T^{3/2}\exp(-(T/T_0)^{1/2})$

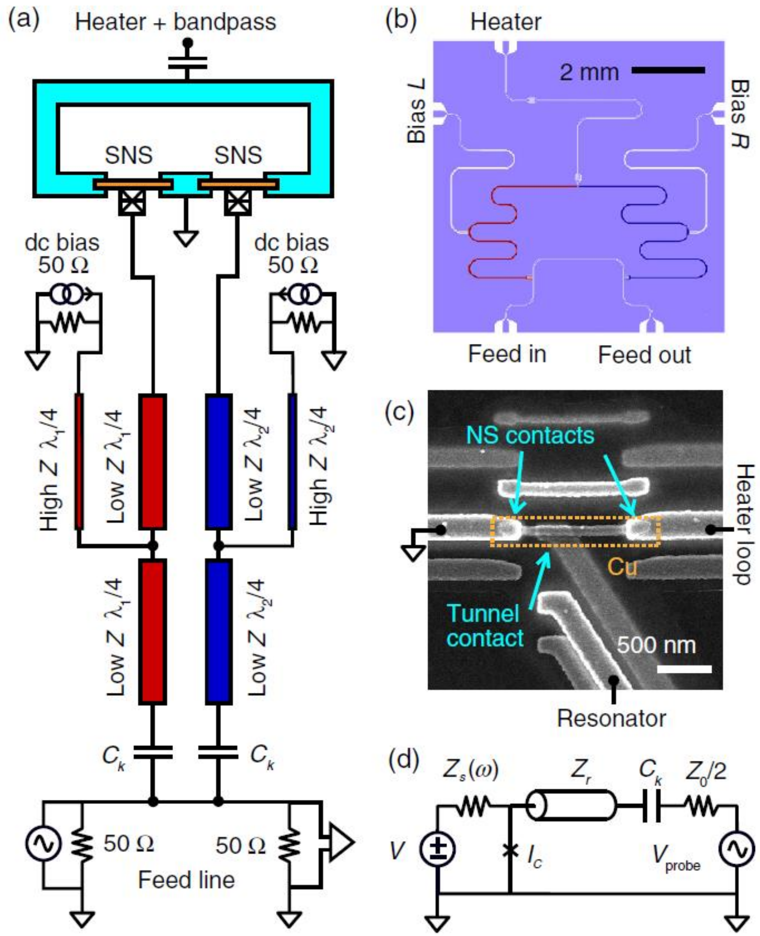
* A.F. Volkov, Phys. Rev. Lett. **74**, 4730 (1995)
* F.K. Wilhelm *et al.*, Phys. Rev. Lett. **81**, 1682 (1998)

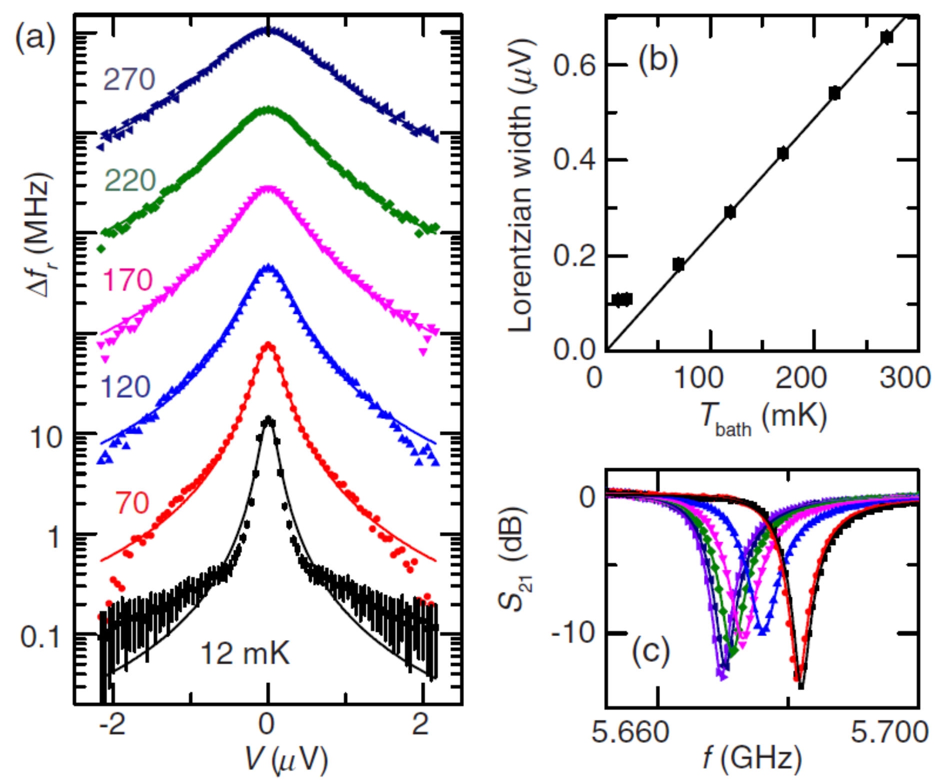


=> Dispersive measurement utilizing Josephson inductance

$$L_j^* = \left(\frac{\Phi_0}{2\pi}\right)^2 \left(\frac{d^2E_0}{d\varphi^2}\right)^{-1}$$

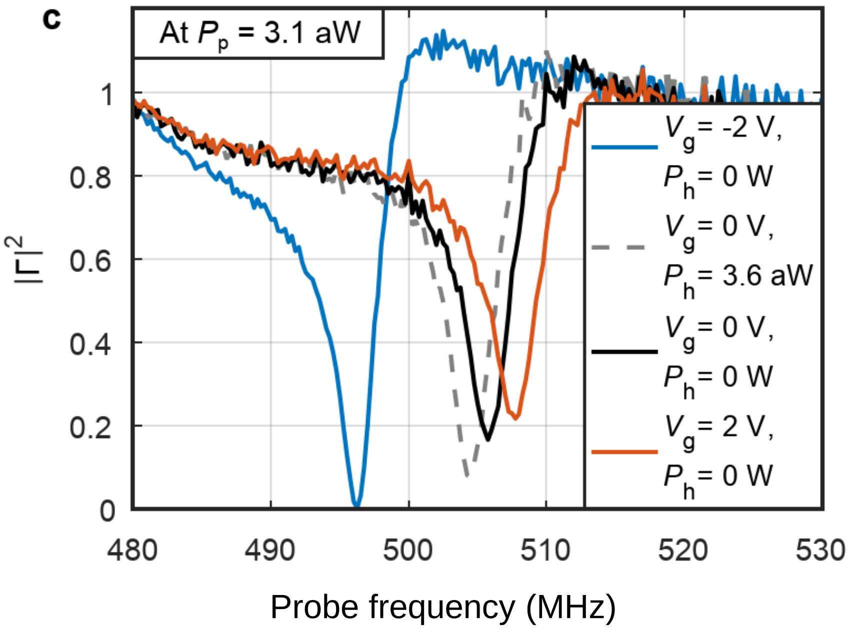
Dispersive Thermometry with a Josephson Junction Coupled to a Resonator (SNS)

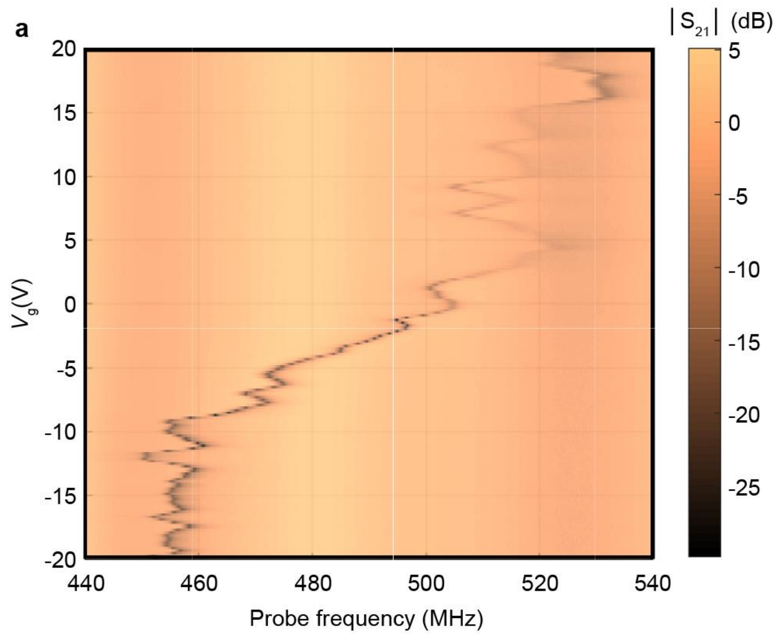




O.-P. Saira, M. Zgirski, K. L. Viisanen, D. S. Golubev, and J. P. Pekola, Phys. Rev. Applied 6, 024005, 2016

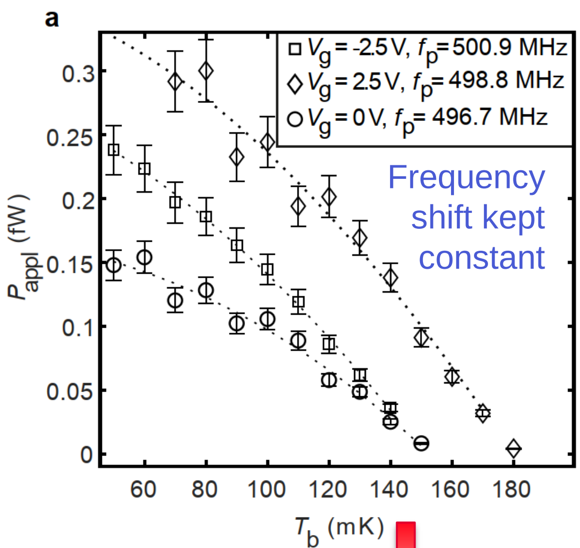
Example: Graphene SNS in microwave cavity





Dirac point around 20 V

Example II: Non-equilibrium state with keeping fixed T_e



$$\tilde{G} = -\frac{\partial P_{e-b}(T_e, T_b)}{\partial T_B}$$

$$G \propto T_b$$

- Photon cooling?
- Electronic diffusion?

Rising edge with τ_r ~100 ns good for circuit QED read-out

NIS primary Thermometry

Current in NIS tunnel junction:

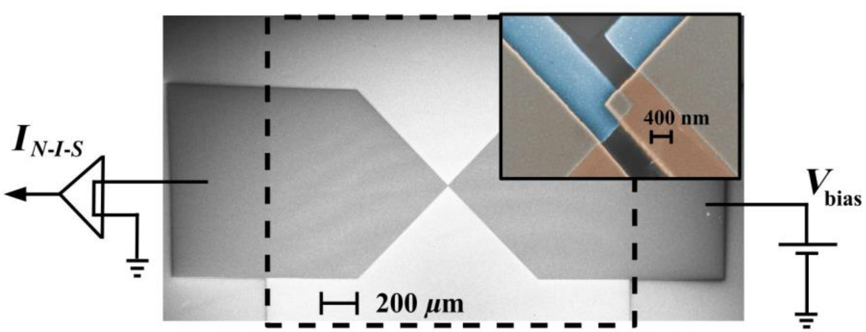
$$I \simeq I_0 e^{-(\Delta - eV)/k_B T_N} + \frac{\gamma V}{R_T \sqrt{1-(eV/\Delta)^2}},$$

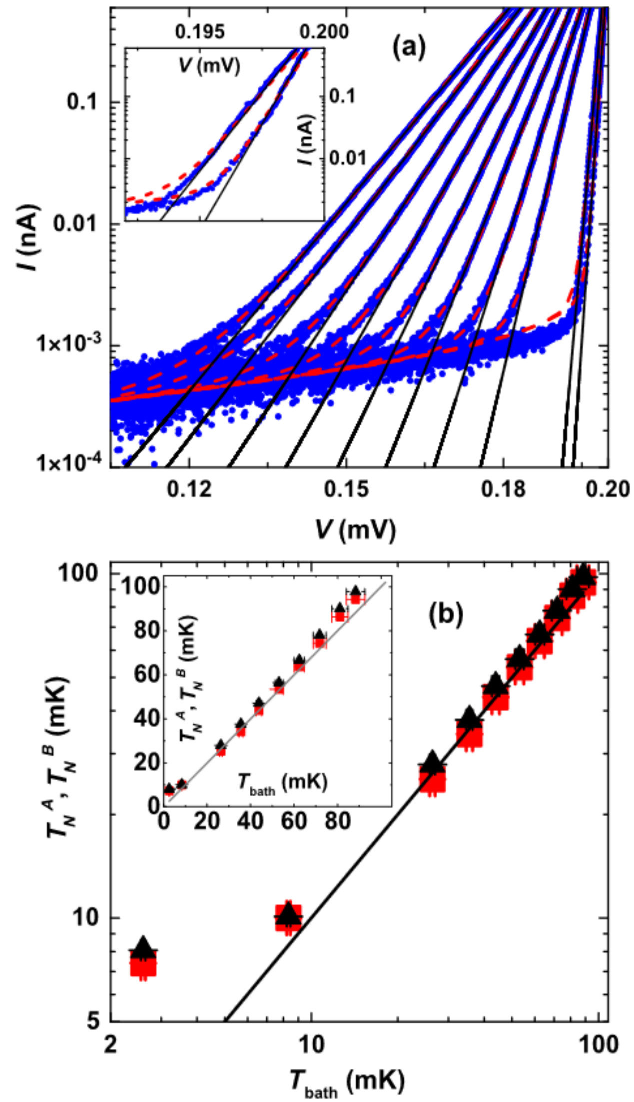
$$I_0 = \sqrt{2\pi \Delta k_B T_N}/2eR_T$$

$$V = \frac{\Delta}{e} + \frac{k_B T_N}{e} \ln(I/I_0)$$

γ small in good quality junctions

$$T_N^B(V) = \frac{e}{k_B} \frac{dV}{d(\ln I)}$$



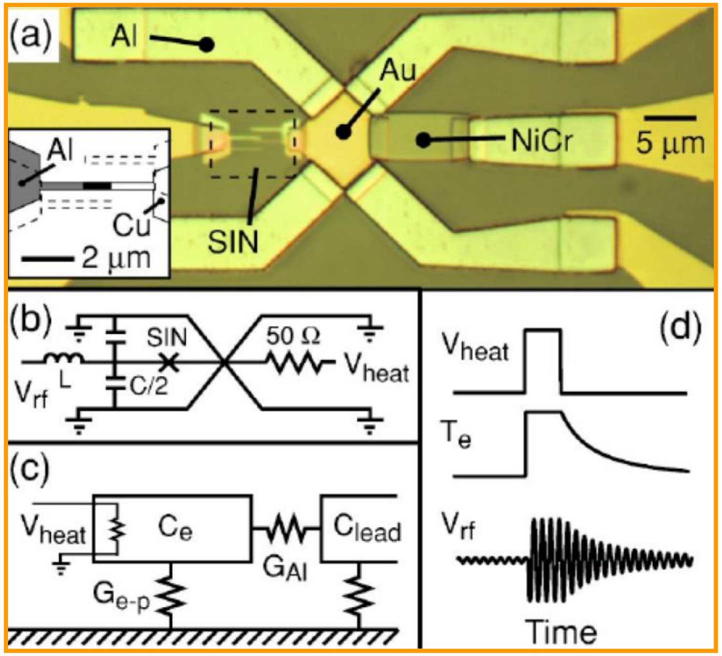


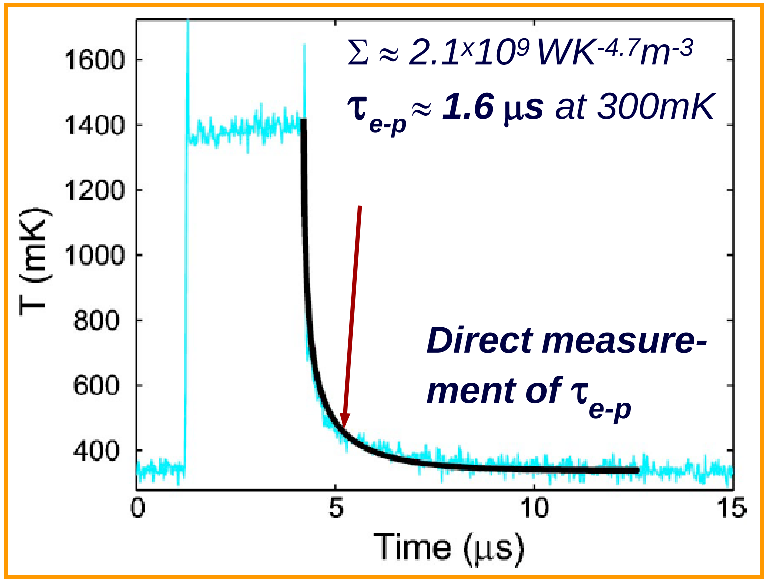
Feshchenko, et al, Phys. Rev. Appl., **4**, 034001 (2015).

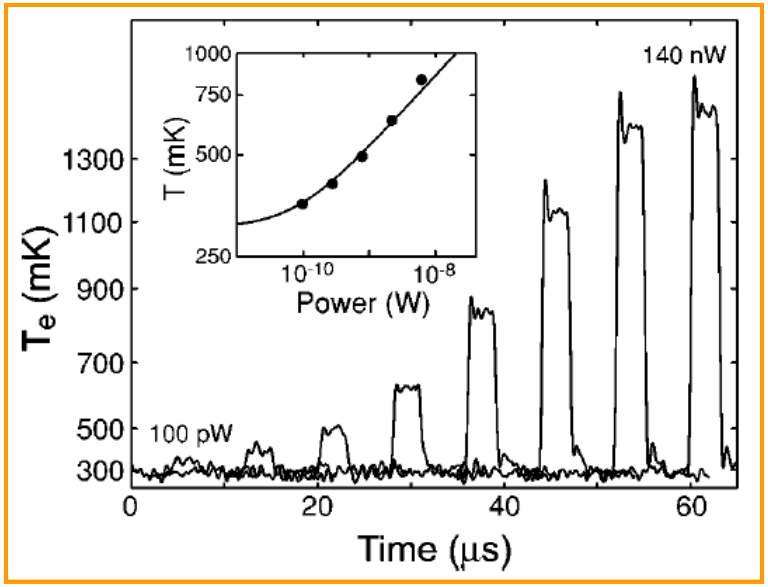
Fast temperature readout (NIS thermometer)

Embedding NIS junction in an LC circuit.
RF-NIS readout (20 MHz thermometer, Q=20).

Reflected power from LC circuit (350 MHz):
at resonance => R_{NIS} => T_e



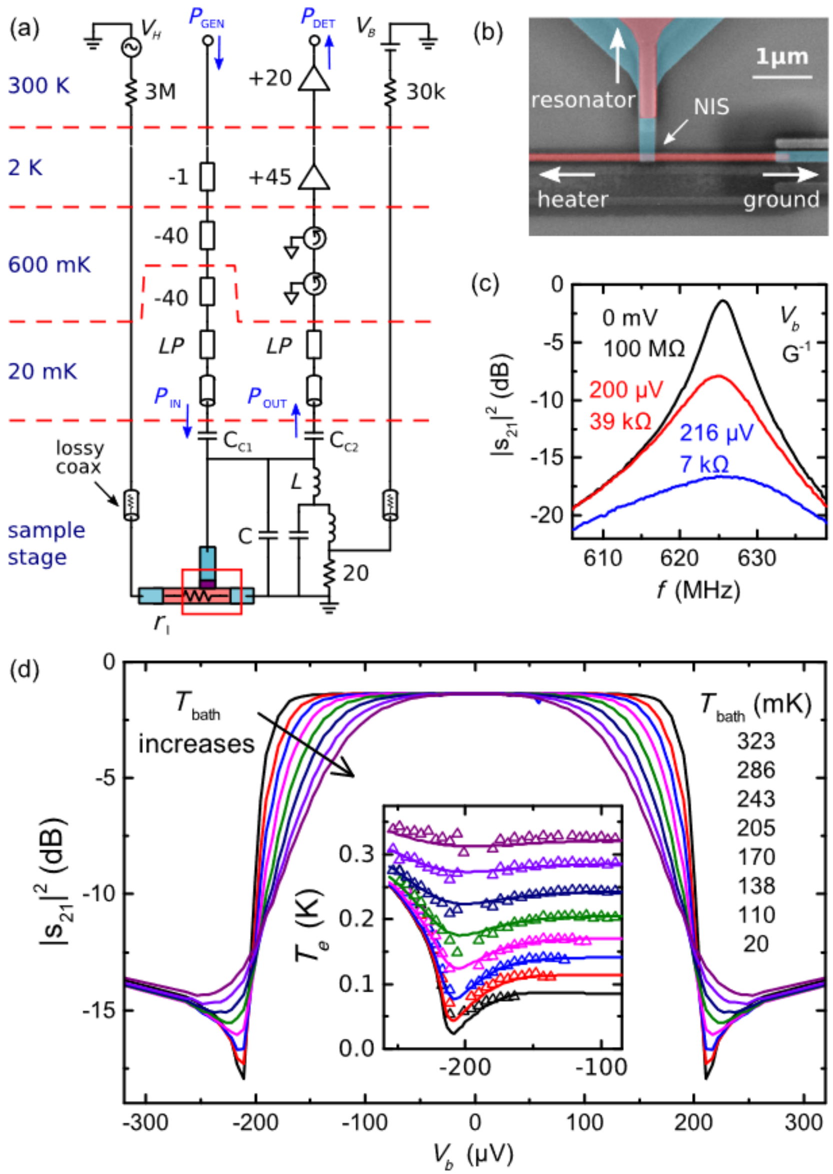




D. R. Schmidt, C. S. Yung, and A. N. Cleland, Phys. Rev. B ***69****, 140301 (2004)*

Fast NIS thermometry for calorimetry

NIS tunnel junction coupled to a resonator with transmission readout $f_0 = 625\ \text{MHz}$



$R_T = 22\,\text{k}\Omega$

Embeded NIS junction in an LC resonant circuit

$$|s_{21}| = 2\kappa \frac{G_0}{G + G_0}$$

$$\kappa = C_{C1}C_{C2}/(C_{C1}^2 + C_{C2}^2)$$

$$G_0 = 4\pi^2(C_{C1}^2 + C_{C2}^2)Z_0 f_0^2$$

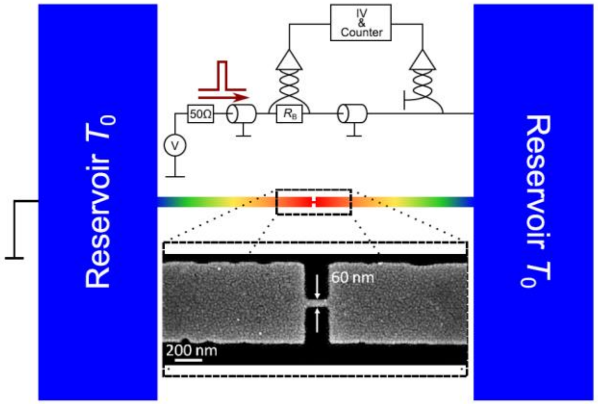
- G_0 for data at $V_b = 0$ and $V_b \gg \Delta/e$

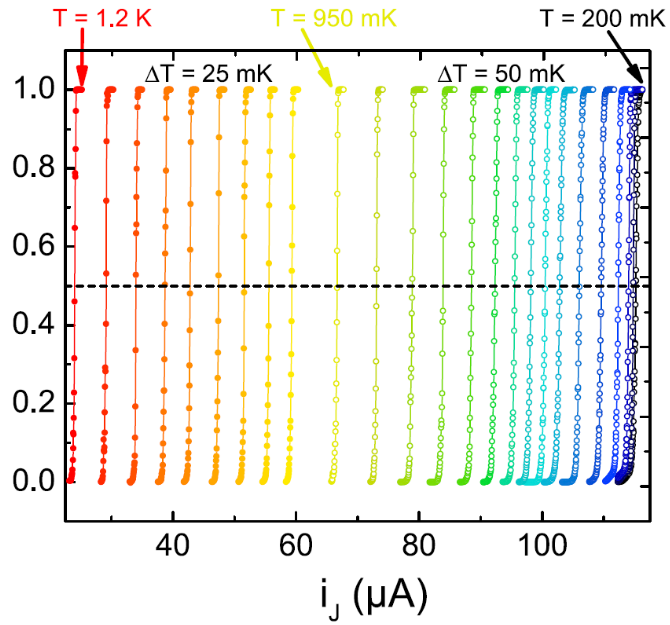
$$G = \frac{1}{R_T k_B T_e}\int dE N_S(E) f(E - eV_b)[1 - f(E - eV_b)]$$

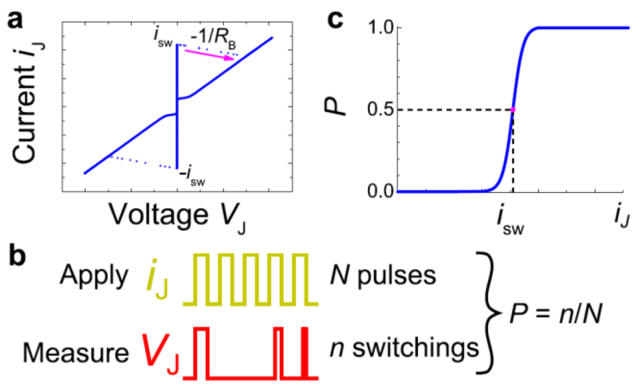
S. Gasparinetti et al., Phys. Rev. Appl. **3**, 014007 (2015)

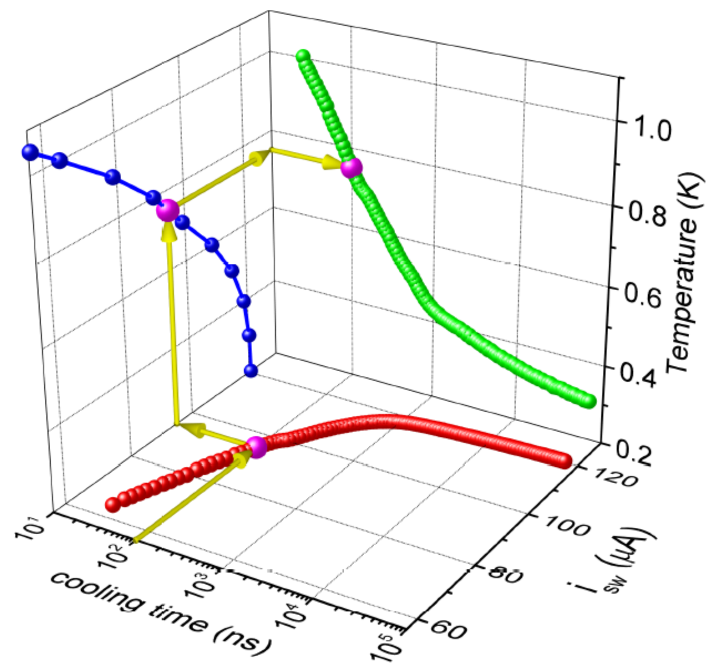
Nanosecond Thermometry with Josephson Junction (SsS)

- Switching current as thermometer
- 10 ns pulses straightforward



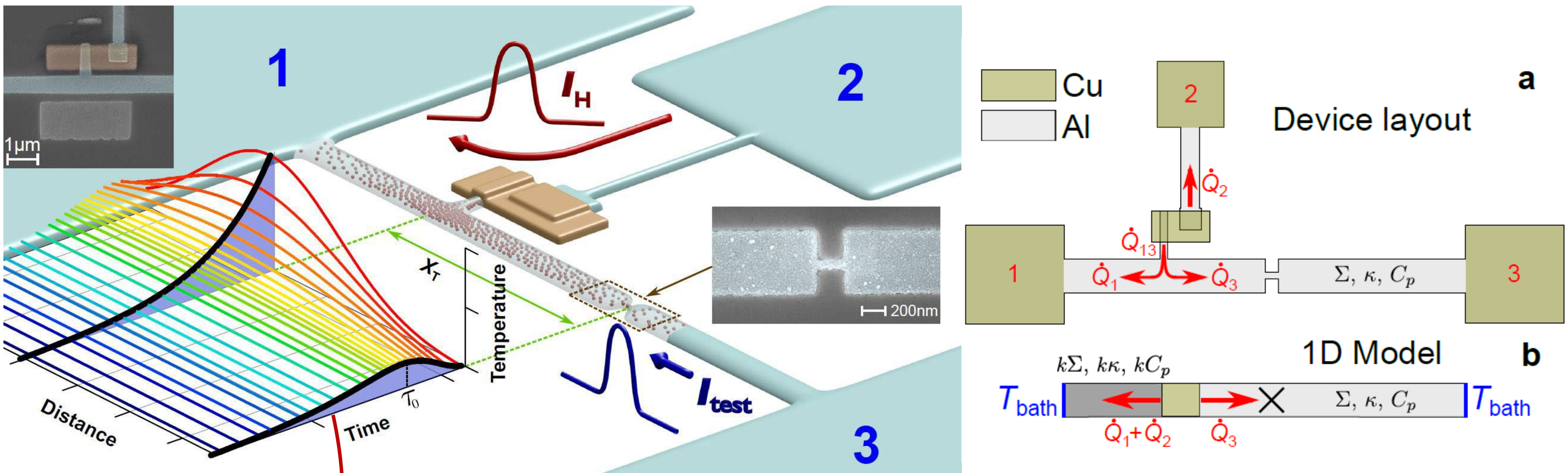


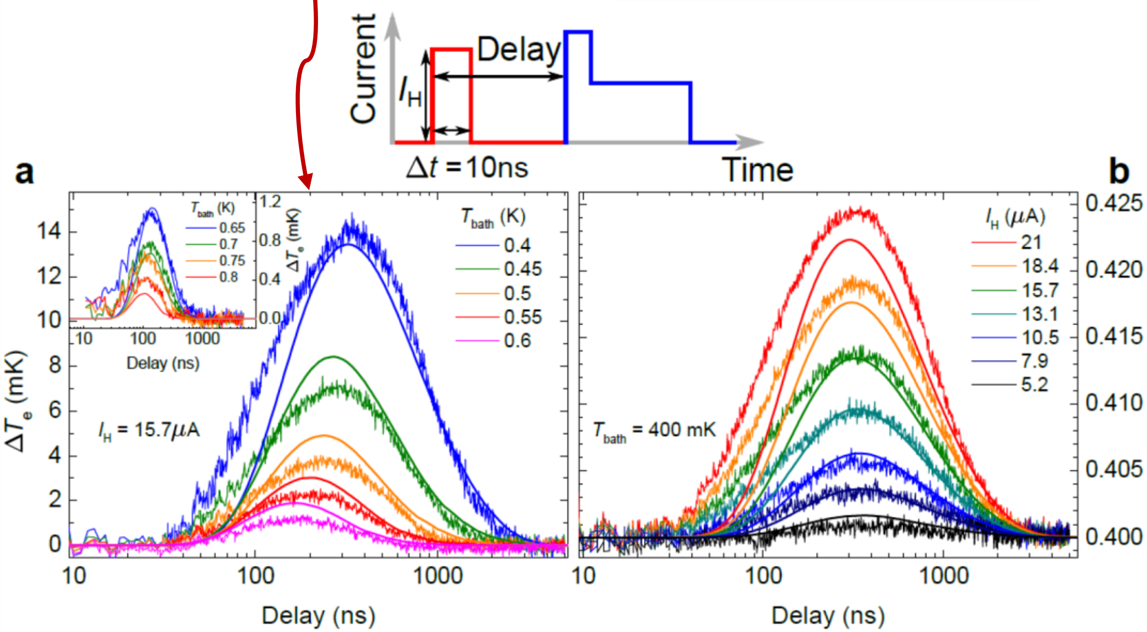




M. Zgirski et al. Phys. Rev. Appl. **10**, 044068 (2018)

Probing of Quasiparticle Diffusion with Nanosecond Resolution





M. Zgirski et al. *Heat hunting in freezer: Direct measurement of quasiparticle diffusion in superconducting nanowire*, Phys. Rev. Appl. **14**, 044024 (2020)