

CS-E4500 Advanced Course in Algorithms

Week 06 – Tutorial

1. The events A_1, A_2, A_3 are pairwise independent if, for all $i \neq j$, A_i is independent of A_j . However, pairwise independence is a weaker statement than mutual independence, which requires the additional condition that $P(A_1, A_2, A_3) = P(A_1)P(A_2)P(A_3)$. Construct an example where three events are pairwise independent but not mutually independent.
2. Let $X = \sum_{i=1}^n X_i$, where the X_i are pairwise independent random variables. Prove that

$$\text{Var}(X) = \sum_{i=1}^n \text{Var}(X_i) .$$

This equality allows us to apply Chebyshev's inequality even when the random variables are only pairwise independent.