Aalto University
School of Science

MS-E2114 Investment Science
Lecture 8: Derivative securities: Forwards, futures and swaps

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Overview

Forward contracts

Swaps

Futures contracts

Hedging

This lecture

- Thus far, we have considered fixed cash flow streams and single-period random cash flow streams
  - These assets have intrinsic value (e.g., bonds, stocks)
- In this and next lectures, we consider derivative securities
- **Derivative security** = A financial instrument whose payoff is determined by (derived from) the value of another variable, typically the price of another ("underlying") asset, e.g.:
  - Option to sell stock
  - Forward contract to purchase raw materials
  - But derivates can be defined by other variables as well (e.g., weather conditions)
- We consider derivative securities whose underlying asset is a financial instrument

Examples of derivatives

- Forward contract
  - Commitment today (31 October 2022) to buy 2000 kg sugar in 6 weeks (12 December 2022) for 0.30 €/kg
  - Value of this contract depends on what the market price of sugar p will be on 12 December
  - If price $p > 0.30$ €/kg on 12 December, the contract has positive value, because the sugar bought with the forward contract can be sold at the market price
  - If price $p < 0.30$ €/kg on 12 December, the contract entails a loss, because it would have been cheaper to buy sugar at the market price

Examples of derivatives

- Option
  - The right (which does not oblige) to buy 100 shares of company A in 6 months (on 31 April) for 25 €
  - If the share price rises to $P > 25$ €, the value in 6 months (on 31 April) is $(P - 25) \cdot 100$€
  - If $P \leq 25$ € on 31 April, the option is worthless, because these shares can be bought from the market at this lower price

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Forward contracts

- **Forward contract**: A commitment to buy or sell a predetermined amount of the underlying asset at a specific price and time
  - E.g. commitment to sell 1000 barrels of oil after 9 months for 100 €/barrel
- **Forward price** F is the price paid at the time of delivery
  - Usually F is determined so that the value of the contract is zero at the time of making the contract
  - Value of the underlying asset is determined in the **spot market**
    - The market in which the underlying asset is traded
    - E.g., the oil spot market for barrels of oil
    - E.g., the stock market for shares of stocks

Forward contracts

- Buyer of the underlying asset is said to have a **long position**
  - If the oil price were to rise to $P > 100$ €/barrel, the buyer has secured the purchase at the lower predetermined price of 100 €
- Seller of the underlying asset is said to have a **short position**
  - If the oil price were to fall to $P < 100$ €/barrel, the seller has secured the sale at the higher predetermined price of 100 €/barrel

Forward price formula

Definition

(**Forward price formula**) Suppose that the asset whose spot price at time $t = 0$ is S can be stored at zero cost and that short selling is possible. Then the theoretical forward price F for delivery at T is

$$F = \frac{S}{d(0, T)} = (1 + s_T)S,$$

where $d(0, T) = \frac{1}{1+s_T}$ is the risk-free discount factor for the time period $[0, T]$, derived from the risk free interest rate s_T .

Forward price formula

Proof: If there are no storage costs, the forward price is implied by forward rates through the following arbitrage arguments:

1. At time 0, take a short position in the forward contract of the asset at the forward price F at time T (i.e., you are committed to sell the asset)
2. At time 0, borrow money equal to asset price S at the risk-free interest rate $s_T = 1/(d(0, T) - 1)$
3. At time 0, buy the asset immediately from the market at price S and store it until time T (for free)
4. At time T, sell the asset at the forward price F
5. At time T, pay back the loan principal S and the accrued interest Ss_T

Forward price formula

Proof (cont'd):

- You had zero cash flow in the beginning and your cash flow at time T is $F - S(1 + s_T) = F - S/d(0, T)$
- In order to avoid arbitrage opportunities, we must have

$$F - S/d(0, T) = 0$$
$$\Rightarrow F = \frac{S}{d(0, T)} \quad \square$$

Forward price formula with carrying costs

- In practice, there may be costs due to storage
- Cost of renting warehouses, insurance,...

Forward price formula with carrying costs

Definition

(**Forward price formula with carrying costs**) Suppose that the holding costs for an asset are $c(k)$ per unit at time k, and that it can be sold short. Suppose the initial price is S. Then the theoretical forward price is

$$F = \frac{S}{d(0,T)} + \sum_{k=0}^{T-1} \frac{c(k)}{d(k,T)},$$

where $d(k,T)$ is the risk-free discount factor from time k to time T.

Forward price formula with carrying costs

Proof: The forward price can again be derived from the no arbitrage assumption using the following logic:

1. At time 0, take a short position in the forward contract of the asset at the forward price F at time T (i.e., you are committed to sell the asset)
2. At time 0, borrow money equal to asset price S at the risk-free interest rate $s_T = 1/(d(0, T) - 1)$
3. At time 0, buy the asset immediately from the market at price S and store it until time T
4. At each time k, borrow money equal to the storage cost $c(k)$ at the risk-free interest rate $f_{k,T} = 1/(d(k, T) - 1)$
5. At each time k, pay the storage cost $c(k)$

Forward price formula with carrying costs

Proof (cont'd):

6. At time T, sell the asset at the forward price F
7. At time T, pay back the loan principal and the accrued interest, which in total is

$$-S(1+s_T) - \sum_{k=0}^{T-1} c(k)(1+f_{k,T})$$

Forward price formula with carrying costs

Proof (cont'd):

- You had zero cash flow in the beginning and at each time k, and your cash flow at time T is

$$
\begin{aligned}
&F - S(1+s_T) - \sum_{k=0}^{T-1} c(k)(1+f_{k,T}) \\
=&F - S/d(0,T) - \sum_{k=0}^{T-1} c(k)/d(k,T)
\end{aligned}
$$

- In order to avoid arbitrage opportunities, we must have

$$
\begin{aligned}
&F - S/d(0,T) - \sum_{k=0}^{T-1} c(k)/d(k,T) = 0 \\
\Rightarrow &F = S/d(0,T) + \sum_{k=0}^{T-1} c(k)/d(k,T) \quad \square
\end{aligned}
$$

Value of forward contract

- Example
  - Price of sugar is 0.17€/kg. What is the forward price for a sugar delivery in 5 months if the linearly annualized interest rate is 9% and if storage costs 0.01€/ month?
  - Monthly interest rate is 9%/12 = 0.75%

$$d(k, T) = \frac{1}{1.0075^{T-k}}$$

$$F = 0.17 \cdot 1.0075^{5} + \sum_{k=0}^{4} 0.01 \cdot 1.0075^{5-k}$$

$$\Rightarrow F = 0.23 \text{ €/kg}$$

- The value of the contract is zero at the time when it is made; but as the price of the (underlying) commodity changes, so does the value of the contract

Value of forward contract

Definition

(**The value of a forward**) Suppose a forward contract for delivery at time T in the future has a delivery price F_0 and that at time $t > 0$ the forward price is F_t. The value of the initial contract at time t is

$$f_t = (F_t - F_0)d(t, T),$$

where $d(t, T)$ is the discount factor over the period from t to T.

Value of forward contract

Proof: Consider the following:

1. At time 0, the investor makes a forward contract to take a long position in the asset at forward price F_0 at time 0
2. At time t, the investor makes a new forward contract to take a short position in the asset at forward price F_t at time t. This contract does not cost anything and can be closed through the long position at time T.
3. At time T, the investor receives a cash flow $F_t - F_0$ for certain, i.e., the asset is delivered from the long position to the short position regardless of the price, fulfilling the obligations for the two positions.

Value of forward contract

- The time-t-present value of this certain cash flow at time T is by definition:

$$(F_t - F_0)d(t, T)$$

- To eliminate arbitrage, this certain value must be equal to the price of the long forward contract f_t at time t, i.e.,

$$f_t = (F_t - F_0)d(t, T) \quad \square$$

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Forward contracts

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Swaps

- A **swap** is an agreement to transform one cash flow stream into another
  - Typically one cash flow stream is random (risky) and the other one is fixed (riskless)
  - E.g. swap a cash flow stream that depends on a variable interest rate (such as EURIBOR) to a fixed cash flow stream
  - The random cash flow stream is called **variable leg**
  - The fixed cash flow stream is called **fixed leg**
- *Plain vanilla swap* = a simple swap
- The size of EUR interest rate swap markets is hundreds of billions of euros

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About EURIBOR®

EURIBOR® is the rate at which wholesale funds in euro could be obtained by credit institutions in the EU and EFTA countries in the unsecured money market. **EURIBOR®** is a critical interest rate benchmark authorised under the **EU Benchmarks Regulation (BMR)**.

The benchmark is of systemic importance for financial stability. In August 2016, **EURIBOR®** was declared a critical benchmark by the **European Commission**. According to the European Commission, **EURIBOR®** is one of most important interest rate benchmarks worldwide. It is estimated that that benchmark underpins more than EUR 180 000 billion worth of contracts. While these contracts are mostly interest rate swaps, the benchmark also covers more than EUR 1 000 billion of retail mortgages. Therefore, the value of financial instruments and financial contracts using that benchmark in the Union exceeds the threshold of EUR 500 billion by far.

EURIBOR® Benchmark Statement

EURIBOR® is grounded, to the extent possible, in euro money market transactions that reflect the Underlying Interest, i.e. the rate at which wholesale funds in euro could be obtained by credit institutions in the EU and EFTA countries in the unsecured money market.

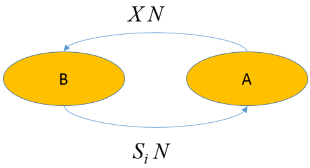
EURIBOR® is published on every TARGET day, at or shortly after 11.00 Brussels time, for each of its Defined Tenors: 1 week, 1 month, 3 months, 6 months and 12 months.

Swaps - Example

- 10-year EUR (plain vanilla) interest rate swap between a bank and an institutional investor
  - The investor pays a fixed semiannual interest rate of 1.65% p.a. to the bank for 10 years
  - The bank pays a 6-month EURIBOR to the investor for 10 years
- If the bank has given loans to mortgage holders at 6-month EURIBOR, the swap with the investor eliminates the interest rate risk the bank is exposed to as a result of the loans it has given to mortgage holders

Value of a commodity swap

- Consider a swap for N units of a commodity with M periods
- In this swap, in each period, party A (e.g., manufacturer):
  1. Receives a payment which equals the spot price S_i of the commodity at time i times N;
  2. Pays a fixed price X for each commodity unit N
- At time i, party A has the cash flow of $(S_i - X)N$
- Party B has the opposite cash flows
- The swap allows party A to buy the commodity effectively for a fixed price



Value of a commodity swap

- Let F_i be the unit forward price for delivery at time i
- Taking a long forward position at this price (at time zero) allows party A to eliminate the risk associated with the spot price while the fixed payment X involves no uncertainties
- The present value of this certain cash flow at time i must therefore be

$$f_i = d(0,i)(F_i - X)N$$

- This yields the total value of the swap for party A as

$$V = \sum_{i=1}^{M} f_i = \sum_{i=1}^{M} d(0,i)(F_i - X)N$$

- X is often selected so that the initial value of the swap is zero

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Futures contracts

- Private contracts such as forwards can be closed only with the original counterparty
  - No competition
  - The counterparty can decline to give a reasonable price
- Another possibility is to make another, opposing forward with another counterparty to remove the price risk of the swap
- However, in this case, you have the risk of default of two counterparties: (1) the original swap counterparty and (2) the opposing swap counterparty
- Closing the swap with the original counterparty would have removed all risk (and they know it), but they may not give a good price (just for that reason)

$\Rightarrow$ Need for a better system

$\Rightarrow$ **Futures contracts**

Futures contracts

- A **futures contract** is a derivative instrument on an asset whose delivery price ("futures price") is changed continuously to the last traded futures price
- Futures contracts are traded on an exchange where every investor has an account, called a **margin account**
- The difference between the earlier delivery price and the new delivery price is credited to or debited from the investor's margin account by the exchange daily
  - If there is only relatively little money left on a margin account, the exchange can issue a **margin call** to force the investor to put more money on the account
- Value of the futures contract is always zero following the adjustment of the delivery price

Futures contracts

- With futures contracts, every investor in the market is holding either a long or short position in exactly the same kind of a derivative instrument (for each maturity)
  - A standardized product
  - Easy to trade, can attract many investors
- Counterparty for the futures contract is the exchange
  - Can be closed out with the exchange, leaving no counterparty risk
- **Note**: A curiosity of futures contracts is that the margin account is debited and credited by the difference in delivery price (which takes place at maturity) directly, not the *present value* of the difference, as one might expect
  - Might be explained by the ease in practice

Futures contracts

- If the futures price increases:
  - Those in the long position receive to their margin account a deposit which corresponds to the increase in the futures price
  - This deposit is taken from those who have a short position
- If the futures price decreases:
  - A deposit is taken from those in long position
  - This deposit is received by those in short position
- The margin account needs an initial deposit for possible compensations (initial margin)
  - Typically about 5-10% of the price of the contract
  - If the margin becomes too small, a margin call is issued to force investor to put more money on the account
  - The account (and the futures position) will be closed if the owner does not make an additional deposit

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MARKET PRICES

NEWS AND NOTICES

Commodities RFQ | History | Market Transactions | Volume Summary

Market Prices

Market: Electricity Nordic Types: Month Product: Futures

Export to Excel | Traded

Aggregated volume (GWh): 141.37

* MW ** HOURS

Updated: 2022-10-30 18:59:50

PRODUCT SERIES	BID	ASK	LAST	+/-	%	HIGH	LOW	ON*	OFF*	VOL*	DAILY FIX	OI	SIZE**	UPD*	MORE...
ENOAFUTBLMNOV-22	144.00	146.00	146.00	6.00	4.29	150.00	145.00	51	14.0	65.00	146.00	4530	720	16:00:00	
ENOAFUTBLMDEC-22	221.00	224.00	223.00	20.00	9.85	225.00	217.50	40	7.0	47.00	223.00	4585	744	16:00:00	
ENOAFUTBLMJAN-23	305.00	330.00	309.00	21.00	7.29	309.00	305.00	9	4.0	13.00	309.00	205	744	16:00:00	
ENOAFUTBLMFEB-23	290.00	310.00	277.00		0.00				1.0	1.0	307.00	95	672	16:00:00	
ENOAFUTBLMMAR-23	165.00	185.00	170.00	19.00	12.58	170.00	167.00	12		12	170.00	222	743	16:00:00	
ENOAFUTBLMAPR-23	150.00	160.00	157.00	14.50	10.18	157.00	157.00	2		2	157.00	70	720	16:00:00	

European Grains Futures Prices

Sun, Oct 30th, 2022 Help ?

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Latest futures price quotes as of Sun, Oct 30th, 2022.

Grains

Symbol	Contract Name	Last	Change	Open	High	Low	Volume	Time	Links
XRX22	Rapeseed (Nov '22)	636.50s	-15.50	644.00	650.25	636.50	447	10/28/22	
XRG23	Rapeseed (Feb '23)	638.75s	-3.00	638.75	640.75	633.00	4,883	10/28/22	
XRK23	Rapeseed (May '23)	637.00s	-4.00	637.75	639.50	633.25	1,606	10/28/22	
LWX22	Feed Wheat (Nov '22)	264.50s	+0.50	0.00	264.50	264.50	0	10/28/22	
LWF23	Feed Wheat (Jan '23)	269.50s	+0.50	0.00	269.50	269.50	0	10/28/22	
LWH23	Feed Wheat (Mar '23)	274.50s	+0.50	273.00	274.50	273.00	10	10/28/22	
MLZ22	Milling Wheat (Dec '22)	337.50s	+1.00	334.75	338.25	334.50	18,827	10/28/22	
MLH23	Milling Wheat (Mar '23)	337.75s	+1.25	334.75	338.50	334.75	7,120	10/28/22	
MLK23	Milling Wheat (May '23)	337.75s	+0.75	335.00	338.50	334.75	3,751	10/28/22	

Example: Futures contract

- A company takes a long position for 5000 bushels of corn in the March futures contract for a futures price of 6.10 €/bushel
- The exchange requires a margin that is initially 800 € and must not fall below 600 €
- The margin is recalculated on a daily basis
- If the futures price drops the next day to 6.07 €, the clearing house withdraws $5000 \cdot 0.03$ € $=150$ €
  - $\Rightarrow$ The balance of margin account becomes 800 - 150 = 650 €
- If the futures price drops the next day additional 0.02 €, the clearing house withdraws $5000 \cdot 0.02$ € $=100$ €
  - $\Rightarrow$ The balance becomes 550 €
- Because 550 € $<$ 600 € the clearing house makes a margin call to the company for an additional payment of 50 € to the margin account

Difference between futures contracts and forwards

- Futures are marked-to-market on a daily basis
- With forwards, cash flows occur only when the contract is terminated

Futures-forward equivalence

Theorem

*(**Futures-forward equivalence**) Suppose that interest rates are known to follow expectations dynamics. Then the theoretical futures and forward prices of respective contracts are identical.*

It is in fact enough that, in each period, the investor can predict the next period's discount factor to maturity. If the margin account were debited and credited by the present value of the change in delivery price, the equivalence would always hold.

Futures-forward equivalence

Proof: Let F_0 = futures price and G_0 = forward price at time $t = 0$. Let there be T periods and let the discount rate from period j to $k > j$ be $d(j, k)$.

Consider the following two strategies.

A Take the following positions in futures

- 0: Go long in $d(1, T)$ units of futures
- 1: Increase position to $d(2, T)$

  ⋮
- k: Increase position to $d(k + 1, T)$

  ⋮
- $T - 1$: Increase position to 1

  - In period $k + 1$, the profit from previous period is $(F_{k+1} - F_k)d(k + 1, T)$.
  - Invest these at the risk-free rate until T

Futures-forward equivalence

A At time T, the value of the investment made at the end of period k is

$$V_k = \frac{(F_{k+1} - F_k)d(k+1, T)}{d(k+1, T)} = F_{k+1} - F_k$$

This no cost strategy yields the profit

$$\sum_{k=0}^{T-1}(F_{k+1} - F_k) = F_T - F_0 = S_T - F_0$$

B Take a long position with a single forward contract
  - No initial cash flow
  - At the end the profit is $S_T - G_0$
- If the strategy is to buy A and sell B then
  - There is no commitment but the profit will be $G_0 - F_0$
  - It follows that $G_0 = F_0$ (or else there would be arbitrage) □

Futures-forward equivalence

Alternative proof:

- F_0 = futures price at time 0
- G_0 = forward price at time 0
- There are T periods
- At time t, discount factor from time i to j is $d_t(i, j)$
- At time t, the amount credited to the margin account (after the futures price has moved from F_{t-1} to F_t) is

$$\Delta_t = F_t - F_{t-1}$$

- No discounting or present value is applied to the cash posted to the margin account

Futures-forward equivalence

Alternative proof (cont'd):

- At time $t-1$, choose your futures position to be long $x_t = d_t(t,T)$ futures contracts
- This requires that this amount is known at time $t-1$ already, which would not be possible if the interest rates are random, hence the assumption that expectations dynamics hold
- At time t, the exchange credits us for

$$x_t \Delta_t = d_t(t,T)(F_t - F_{t-1})$$

- At time t, we can deposit $x_t \Delta_t$ at the risk-free interest rate until time T (this is $1/[d_t(t,T)-1)]$, so that at time T we will have

$$x_t \Delta_t / d_t(t,T) = d_t(t,T)(F_t - F_{t-1})/d_t(t,T) = F_t - F_{t-1}$$

Futures-forward equivalence

Alternative proof (cont'd):

- Thus, in total, we have at time T

$$\sum_{t=1}^{T} x_t \Delta_t / d_t(t,T) = \sum_{t=1}^{T} (F_t - F_{t-1}) = F_T - F_0$$

- If we go short in a respective forward, then we will make $-(S_T - G_0)$ at time T
- Also, the delivery price of the futures contract matches the spot price at maturity, i.e., $F_T = S_T$

Futures-forward equivalence

Alternative proof (cont'd):

- To have no arbitrage, we must have at time T

$$\begin{aligned} & F_T - F_0 - (S_T - G_0) = 0 \\ \Leftrightarrow & S_T - F_0 - S_T + G_0 = 0 \\ \Leftrightarrow & F_0 = G_0 \quad \square \end{aligned}$$

Note that if the margin account were debited and credited by the present value of the change in delivery price, $(F_t - F_{t-1})d_t(t, T)$, x_t could be set to 1 and no advance knowledge of the interest rates would be needed, and the futures-forward equivalence would always hold.

Expected spot price, backwardation, contango

- **Backwardation** occurs when the current spot price of an underlying asset is higher than the prices in the futures market
- **Contango** occurs if the forward curve is upward sloping with higher futures contract price for each successive maturity date
- Expected spot price at maturity (an unknown quantity) and futures price may be different
  - Cannot be really compared, because the expected spot price at maturity cannot be observed from market data
- *Note: There are slightly different interpretations of these terms, some employ the use of expected spot price*

Overview

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Hedging

Perfect hedge

- In a **perfect hedge**, one takes a position that is **equal and opposite** to the investment that one wishes to hedge

$\Rightarrow$ Price risk is eliminated (cf. earlier examples on commodity swaps)

- Perfect hedging may not be possible if futures with suitable terms are not available

Perfect hedge - Example

- A Finnish company delivers goods in 6 months to an American company and receives $ 2 million upon delivery
- The company can protect against foreign exchange rate (FX) risk due to the possible increase in the value of € by taking a short position for $2 million in the euro forward market

$\Rightarrow$ EUR-USD FX risk is completely eliminated (but the counterparty risk of the American company defaulting still remains)

Minimum variance hedge

- Perfect hedging is not always possible
  - Commodity may not have a suitable futures market
  - The available contracts may not have suitable terms (e.g. delivery date)
  - The supply of futures contracts is insufficient
  - Markets are not liquid enough
- **Minimum variance hedge**: Use a hedging instrument (e.g., a futures contract) such that the variance of a portfolio consisting of (i) the investment and (ii) the hedge is minimized

Minimum variance hedge

- Consider a commitment to sell W units of commodity at time T
- At the spot price S_T, the delivery is worth $x = WS_T$
- Hedge this position with h units worth of futures contracts

⇒ At time T, cash flow is

$$y = x + (F_T - F_0)h$$

- Variance of this cash flow is

$$\begin{aligned}\mathrm{Var}[y] &= \mathbb{E}\left[\left(x - \bar{x} + (F_T - \bar{F}_T)h\right)^2\right] \\ &= \mathrm{Var}[x] + 2h\,\mathrm{Cov}[x, F_T] + h^2\,\mathrm{Var}[F_T]\end{aligned}$$

Minimum variance hedge

- Choose $h = h^*$ to minimize variance $\mathrm{Var}[y]$
- Set the derivative with respect to h at $h = h^*$ to 0

$$\frac{d}{dh}\mathrm{Var}[y]\Big|_{h=h^*} = 2\,\mathrm{Cov}[x, F_T] + 2h^*\,\mathrm{Var}[F_T] = 0$$

$$\Rightarrow h^* = -\frac{\mathrm{Cov}[x, F_T]}{\mathrm{Var}[F_T]}$$

- Inserting this into the expression for $\mathrm{Var}[y]$ gives the minimum variance

$$\mathrm{Var}[y]\Big|_{h=h^*} = \mathrm{Var}[x] - \frac{\mathrm{Cov}[x, F_T]^2}{\mathrm{Var}[F_T]}$$

Minimum variance hedge

- When $x = WS_T$, the hedge is

$$h^* = -\frac{\mathrm{Cov}[S_T, F_T]}{\mathrm{Var}[F_T]} W \equiv -\beta W$$

- **Special case**: If the markets for commodities are identical so that $F_T = S_T$, we have $h^* = -W$ and thus

$$\begin{aligned} \mathrm{Var}[y]\big|_{h=h^*} &= \mathrm{Var}[WS_T] - \frac{\mathrm{Cov}[WS_T, F_T]^2}{\mathrm{Var}[F_T]} \\ &= W^2\,\mathrm{Var}[S_T] - \frac{W^2\,\mathrm{Cov}[F_T, F_T]^2}{\mathrm{Var}[F_T]} \\ \Rightarrow \mathrm{Var}[y]\big|_{h=h^*} &= 0 \end{aligned}$$

Optimal hedging in Expected Utility Theory

- Minimum variance hedge does not account for the investor's risk preferences
  - The investor might prefer another hedging strategy
- Risk preferences can be modeled using, for example, Expected Utility Theory (EUT, see Lecture 7)
- In optimal hedging, an EUT investor maximizes the expected utility of a portfolio containing the investment and its hedge:

$$\max_h \mathbb{E}\left[U(y)\right] = \max_h \mathbb{E}\left[U(x + (F_T - F_0)h)\right]$$

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Forward contracts

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Hedging