Exercise and Homework Round 6

These exercises (except for the last) will be gone through on **Friday, November 4, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Friday, November 11 at 12:00**.

Exercise 1. (Ansatz solution of linear ODE)

Consider the spring model with zero force $u(t) = 0$:

$$ma(t) + kp(t) + \eta v(t) = 0.$$

(a) Solve the equation with exponential ansatz $p(t) = C \exp(\lambda\, t)$.

(b) Fix suitable initial conditions, visualize the solution, and discuss effects of the parameters k and η on the solution.

Exercise 2. (Matrix exponential)

(a) Rewrite the spring model in the previous exercise as a state-space model of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_u u(t) \tag{1}$$

(b) With $u(t) = 0$, solve the equation in terms of the matrix exponential $\exp(\mathbf{A}\, t)$.

(c) Visualize the solution and compare it to the solution obtained in the previous exercise.

(d) Generalize the solution to the case with $u(t) \neq 0$.

Exercise 3. (Numerical solution of ODEs)

Consider the state-space ODE in the previous exercise, but now assume that the force is given by the following alternatives:

$$\begin{aligned} u(t) &= 1, \\ u(t) &= \sin(t), \\ u(t) &= t. \end{aligned} \tag{2}$$

(a) Solve the state-space ODE with Euler's method.

(b) Compare the solution to the solution obtained in Exercise 2d.

(c) Use a builtin ODE solver to obtain a numerical solution to the ODE.

Homework 6 (DL Friday, November 11 at 12:00)

Consider the scalar differential equation

$$\dot{x} = f\, x, \qquad x(0) = x_0, \tag{3}$$

with $f = -1/2$ and $x_0 = 3$.

(a) Solve the equation analytically.

(b) Use Euler's method to solve the equation numerically.

(c) Use a builtin ODE solver (e.g. Matlab's ode45 or Python's odeint) to numerically solve the equation.

(d) Visualize the solutions and compare them to each other.