Helmholtzin yhtälö

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

Lähteetön alue

$$(\varepsilon_0, \mu_0)$$

$$\varrho = 0, \quad \bar{J}(\bar{r}) = 0$$

$$\nabla\times\bar{E} = -j\omega\bar{B} \qquad \nabla\times\bar{H} = +j\omega\bar{D}$$

$$\bar{B} = \mu_0\bar{H} \qquad \bar{D} = \varepsilon_0\bar{E}$$

$$\varepsilon_0\nabla\cdot\bar{E} = 0 \rightarrow \nabla\cdot\bar{D} = 0 \qquad \nabla\cdot\bar{B} = 0$$

$$\nabla\times(\nabla\times\bar{E}) = -j\omega\mu_0\nabla\times\bar{H} = -j\omega\mu_0 j\omega\varepsilon_0\bar{E} = \underbrace{\omega^2\mu_0\varepsilon_0}_{k^2}\bar{E}$$

$$= \nabla\underbrace{(\nabla\cdot\bar{E})}_{=0} - \nabla^2\bar{E}$$

$$k = \omega\sqrt{\mu_0\varepsilon_0}$$

$$\boxed{\nabla^2\bar{E}(\bar{r}) + k^2\bar{E}(\bar{r}) = 0}$$

Tasoaaltoratkaisu

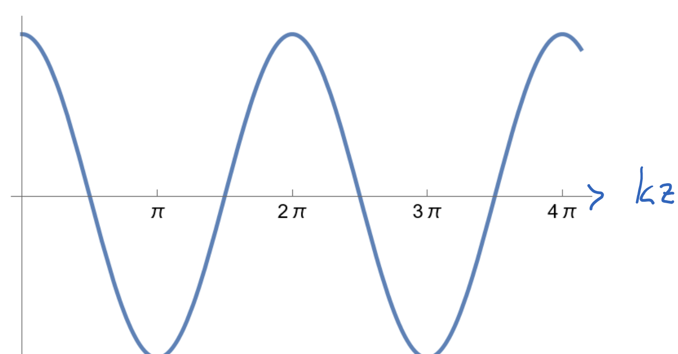
$$\bar{E}(\bar{r}) = \bar{u}\,E(z) \Rightarrow E''(z) + k^2E(z) = 0$$

$$E(z) = E_+e^{-jkz} \Rightarrow E''(z) = (-jk)^2E_+e^{-jkz} = -k^2E_+e^{-jkz} = -k^2E(z)$$

$$E(z) = E_-e^{+jkz} \Rightarrow E''(z) = -k^2E(z)$$

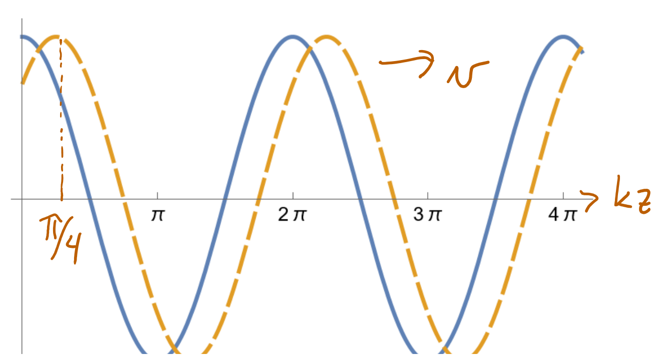
$$\mathrm{Re}\left\{e^{-jkz}\cdot e^{j\omega t}\right\} = \mathrm{Re}\left\{e^{j(\omega t - kz)}\right\} = \cos(\omega t - kz)$$

$$t = 0 \Rightarrow \cos(-kz) = \cos(kz)$$



$$\omega t = \pi/4 \Rightarrow \cos\left(\frac{\pi}{4} - kz\right)$$

$$= \cos\left(kz - \frac{\pi}{4}\right)$$

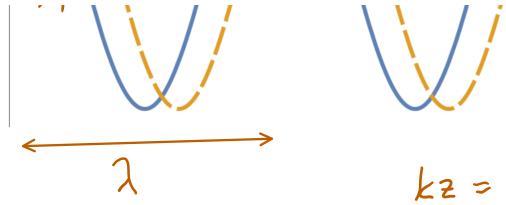


Aallonpituus

$$k\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{k}$$



$$\lambda = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{2\pi}{2\pi f\sqrt{\mu_0\varepsilon_0}} = \frac{c}{f} = \lambda$$

$$kz = \omega t$$

$$z = \frac{\omega}{k} t = \underbrace{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}_{v = c} t$$

Polarisaatio

$$\bar{E}(\bar{r}) = \bar{u} E_+ e^{-jkz}$$

$$\nabla \cdot \bar{E} = 0 \qquad \frac{\partial}{\partial z}\bar{u}_z \cdot \bar{u} E_+ e^{-jkz} = -jk e^{-jkz} \underbrace{\bar{u}_z \cdot \bar{u}}_{=0} E_+ = 0$$

Lineaarinen polarisaatio

$$u = \bar{u}_x$$

$$\bar{u}_y$$

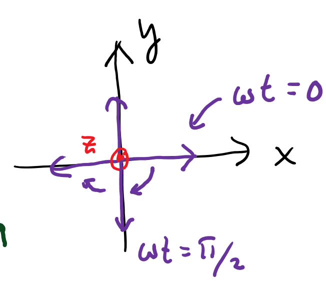
$$\frac{\bar{u}_x + \bar{u}_y}{\sqrt{2}}$$

Ympyräpolarisaatio

$$\frac{\bar{u}_x + j\bar{u}_y}{\sqrt{2}}$$

$$\mathrm{Re}\{(\bar{u}_x + j\bar{u}_y)e^{j\omega t}\} = \bar{u}_x \cos\omega t - \bar{u}_y \sin\omega t$$

Aallon etenemissuunta!
Kätisyys!



Aalto etenee +z-suuntaan (e^{-jkz})

Vasemman käden ympyräpolarisaatio

Vasemman käden ympyräpolarisaatio

Aalto etenee –z-suuntaan $\left(e^{+jkz}\right)$

Oikean käden ympyräpolarisaatio

Elliptinen polarisaatio $\dfrac{\bar{u}_x + j3\bar{u}_y}{\sqrt{10}}$

Tasoaallon yleinen etenemissuunta

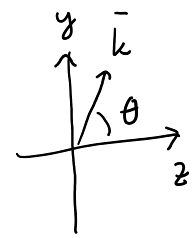
AALTO ETENEE

e^{-jkz} (+z-SUUNTAAN)

e^{+jky} (−y-SUUNTAAN)

$$e^{-k(z\cos\theta + y\sin\theta)} = e^{-j\bar{k}\cdot\bar{r}}$$

$$\bar{k} = k\left(\bar{u}_z\cos\theta + \bar{u}_y\sin\theta\right)$$



AALTO ETENEE VEKTORIN $\bar{k}$ SUUNTAAN!

Tasoaallon magneettikenttä

$$\bar{E}(\bar{r}) = \bar{u}_x E_+ e^{-jkz}$$

$$\nabla\times\bar{E} = -j\omega\mu_0\bar{H} \Rightarrow \bar{H} = \frac{\nabla\times\bar{E}}{-j\omega\mu_0} = \frac{-jk\,e^{-jkz}\,\bar{u}_z\times\bar{u}_x E_+}{-j\omega\mu_0}$$

$$\nabla \to \frac{\partial}{\partial z}\bar{u}_z$$

$$= \bar{u}_y \underbrace{\frac{\omega\sqrt{\mu_0\varepsilon_0}}{\omega\mu_0}}_{\sqrt{\frac{\varepsilon_0}{\mu_0}}} E_+ e^{-jkz}$$

AALTOIMPEDANSSI $\sqrt{\dfrac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega$

$$\bar{H} = \bar{u}_y \frac{E_+}{\eta_0} e^{-jkz}$$

Permittiivisyyden ja permeabilisuuden vaikutus

$$\varepsilon_0, \mu_0 \to \varepsilon, \mu \qquad k = \omega\sqrt{\mu\varepsilon}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Poyntingin vektori ja etenevä teho

$$\bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^* = \frac{1}{2}\bar{u}_x E_+ e^{-jkz} \times \left(\bar{u}_y \frac{E_+}{\eta} e^{-jkz}\right)^*$$

$$= \frac{1}{2}\bar{u}_z E_+ e^{-jkz} \frac{E_+^*}{\eta} e^{+jkz} \qquad E_+ E_+^* = |E_+|^2$$

$$= \bar{u}_z \frac{|E_+|^2}{2\eta} \qquad [\bar{S}] = \frac{V^2}{m^2}\frac{1}{V/A} = \frac{VA}{m^2} = \frac{W}{m^2}$$

$$\bar{E} = \bar{u} E_+ e^{-jkz} \Rightarrow \bar{H} = \frac{\bar{u}_z\times\bar{u}}{\eta} E_+ e^{-jkz}$$

$$\bar{S} \sim \bar{u}\times(\bar{u}_z\times\bar{u}) = \bar{u}_z(\bar{u}\cdot\bar{u}) - \bar{u}\underbrace{(\bar{u}_z\cdot\bar{u})}_{=0} = \bar{u}_z$$

$$\bar{u}\times(\bar{u}_z\times\bar{u})^* = \bar{u}_z\underbrace{(\bar{u}\cdot\bar{u}^*)}_{1} - \bar{u}^*\underbrace{(\bar{u}_z\cdot\bar{u})}_{=0} = \bar{u}_z$$

$$\bar{E}(\bar{r}) = \bar{u} E_+ e^{-jkz} \qquad k = \omega\sqrt{\mu\varepsilon} \qquad \cos(kz)$$

$$\lambda \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad \lambda = \frac{2\pi}{k}$$

$$\nabla\cdot\bar{E} = 0 \Rightarrow \bar{u}_z\frac{\partial}{\partial z}\cdot\bar{u}E_0 e^{-jkz}$$

$$= -jk\underbrace{\bar{u}_z\cdot\bar{u}}_{=0} E_0 e^{-jkz} = 0$$

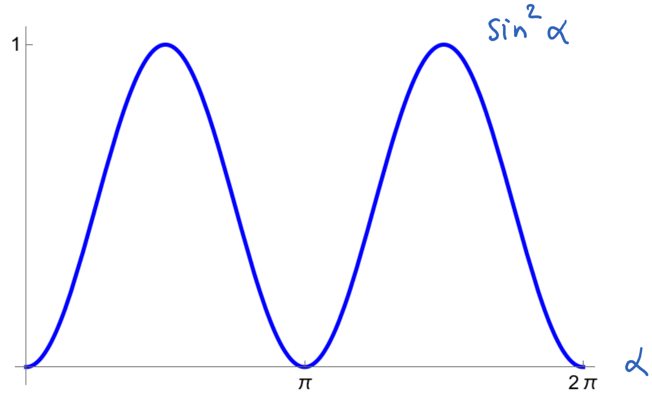
$$\bar{H}(\bar{r}) = \frac{\nabla\times\bar{E}}{-j\omega\mu} = \frac{-jk\,\bar{u}_z\times\bar{E}}{-j\omega\mu} = \frac{1}{\eta}\bar{u}_z\times\bar{E}$$

$$\bar{S}(\bar{r}) = \frac{1}{2}\bar{E}\times\bar{H}^* = \frac{1}{2}\bar{u}_x E_+ e^{-jkz} \times \left(\frac{1}{\eta}\overbrace{\bar{u}_z\times\bar{u}_x}^{\bar{u}_y} E_+ e^{-jkz}\right)^*$$

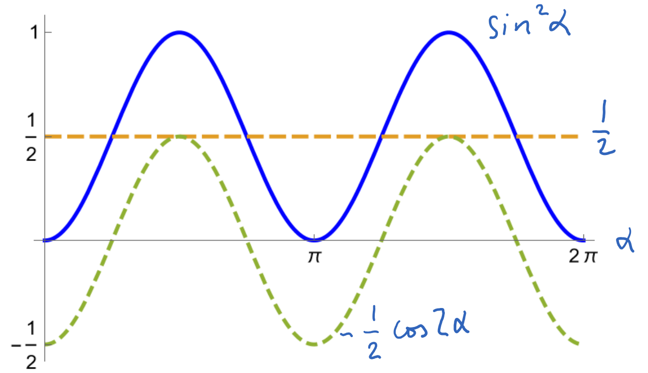
$$= \frac{1}{2} \frac{E_+ E_+^*}{\eta} \bar{u}_z = \frac{|E_+|^2}{2\eta} \bar{u}_z$$

$$\frac{1}{2\pi} \int_0^{2\pi} \sin^2\alpha \, d\alpha = \frac{1}{2}$$

$$\sin^2\alpha = 1 - \cos^2\alpha$$
$$= 1 - \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$
$$= \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$



$$\left(\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \right.$$
$$\left. = 2\cos^2\alpha - 1 \right)$$



↓

TEM-aaltojohdot
(Transversal Electric Magnetic)

TEM $\quad \bar{u}_z \cdot \bar{E} = 0$

$\bar{u}_z \cdot \bar{H} = 0$

T | ⊥ $\qquad \underline{\parallel} \qquad e^{-j\beta z}$

$\longrightarrow z$

$$\bar{E}(\bar{r}) = \bar{E}_T(x,y)\, e^{-j\beta z} \qquad \bar{H}(\bar{r}) = \bar{H}_T(x,y)\, e^{-j\beta z}$$

$$\nabla \times \bar{E} = \nabla \times \bar{E}_T \, e^{-j\beta z}$$

$\parallel \qquad \perp$

$$\nabla \times \bar{E} = \nabla \times \bar{E}_T e^{-j\beta z}$$

$$= \left(\nabla_T + \bar{u}_z \frac{\partial}{\partial z}\right) \times \bar{E}_T e^{-j\beta z} = \overbrace{\left(\nabla_T \times \bar{E}_T\right) e^{-j\beta z}}^{\parallel} \overbrace{- j\beta\, \bar{u}_z \times \bar{E}_T e^{-j\beta z}}^{\perp}$$

$$= \underbrace{-j\omega\mu \bar{H}_T e^{-j\beta z}}_{\perp}$$

$$\nabla_T \times \bar{E}_T = 0$$

$$-j\beta\, \bar{u}_z \times \bar{E}_T = -j\omega\mu \bar{H}_T \Rightarrow \bar{H}_T = \frac{\beta}{\omega\mu} \bar{u}_z \times \bar{E}_T$$

$$\nabla \times \bar{H} = \left(\nabla_T + \bar{u}_z \frac{\partial}{\partial z}\right) \times \bar{H}_T e^{-j\beta z} = \left(\nabla_T \times \bar{H}_T\right) e^{-j\beta z} - j\beta\, \bar{u}_z \times \bar{H}_T e^{-j\beta z}$$

$$= +j\omega\varepsilon \bar{E}_T e^{-j\beta z}$$

$$\bar{E}_T = -\frac{\beta}{\omega\varepsilon} \bar{u}_z \times \bar{H}_T$$

$$= -\frac{\beta}{\omega\varepsilon} \cdot \frac{\beta}{\omega\mu} \underbrace{\bar{u}_z \times \left(\bar{u}_z \times \bar{E}_T\right)}_{\bar{u}_z \underbrace{(\bar{u}_z \cdot \bar{E}_T)}_{0} - (\bar{u}_z \cdot \bar{u}_z)\bar{E}_T = -\bar{E}_T}$$

$$\bar{E}_T = \frac{\beta^2}{\omega^2 \mu\varepsilon} \bar{E}_T \Rightarrow \beta^2 = \omega^2 \mu\varepsilon \Rightarrow \beta = \pm \omega\sqrt{\mu\varepsilon} = k$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

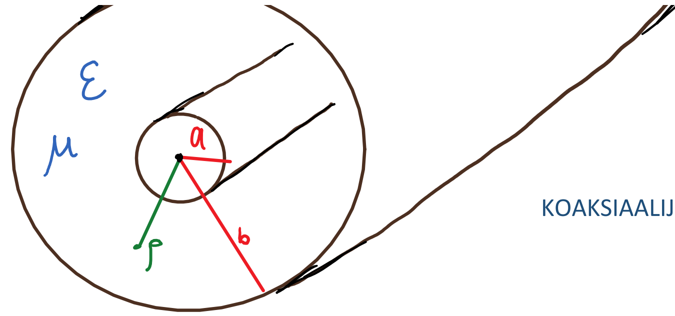
$$\bar{H}_T = \frac{\beta}{\omega\mu} \bar{u}_z \times \bar{E}_T = \frac{\bar{u}_z \times \bar{E}_T}{\eta}$$

$$\nabla_T \times \bar{E}_T = 0 \Rightarrow \bar{E}_T(x,y) = -\nabla\phi_T(x,y)$$

$$a \le \rho \le b \qquad \nabla_T^2 \phi_T(\rho,\varphi) = 0$$
$$0 \le \varphi \le 2\pi$$

$$\nabla^2 \phi_T(\rho) = 0$$

$$\nabla \varphi_T(\rho) = 0$$



KOAKSIAALIJOHTO

$$\frac{1}{\rho}\underbrace{\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right)}_{0} = 0$$

$$\rho\frac{\partial\phi}{\partial\rho} = A$$

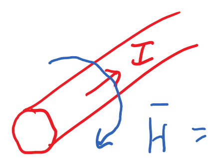
$$\frac{\partial\phi_T}{\partial\rho} = \frac{A}{\rho} \Rightarrow \phi_T(\rho) = A\ln\rho + \overset{A\ln c}{B} = A\underbrace{\ln(c\rho)}_{\ln(\rho/b)}$$

$$\phi_T(b) = 0 = A\ln(\underbrace{cb}_{1}) \qquad c = \frac{1}{b}$$

$$\phi_T(a) = U = A\ln\frac{a}{b} \Rightarrow A = \frac{U}{\ln\frac{a}{b}} \qquad \Rightarrow \phi_T(\rho) = \frac{U}{\ln\frac{a}{b}}\ln\frac{\rho}{b}$$

$$\bar{E}_T = -\nabla_T\phi_T = -\bar{u}_\rho\frac{\partial}{\partial\rho}\left(\frac{U}{\ln\frac{a}{b}}\ln\frac{\rho}{b}\right) = -\bar{u}_\rho\frac{U}{\rho\underbrace{\ln\frac{a}{b}}_{-\ln\frac{b}{a}}} = \bar{u}_\rho\frac{U}{\rho\ln\frac{b}{a}}$$

$$\bar{H}_T = \frac{\bar{u}_z\times\bar{E}_T}{\eta} = \bar{u}_\varphi\underbrace{\frac{U}{\eta\rho\ln\frac{b}{a}}}_{\frac{I}{2\pi\rho}}$$



$$\bar{H} = \bar{u}_\varphi\frac{I}{2\pi\rho}$$

$$I = \frac{2\pi}{\eta\ln\frac{b}{a}}U \Rightarrow U = \underbrace{\eta\frac{\ln\frac{b}{a}}{2\pi}}_{Z_0}I$$

$$P_{et} = \frac{U^2}{2Z_0}$$