Mathematics for Economists

Problem Set 6
Due date: Friday 18.11 at 12.15

Exercise 1

Formulate the first order necessary optimality conditions of the below problem. Draw the feasible set and find the solution by geometric argument. Verify that the solution that you find satisfies the first order optimality conditions. Verify also that the non-degeneracy constraint qualification (NDCQ) holds.

$$\begin{aligned} &\max \sqrt{x_1} \\ &\text{s.e. } x_2 \leq 1 - x_1 \\ &x_2 \geq 0. \end{aligned}$$

Exercise 2

Consider the following minimization problem:

$$\begin{aligned} \min_{x,y} \quad & x + y \\ \text{s.t.} \quad & xy \geq 1, \\ x, y \geq 0. \end{aligned}$$

(a) Check if the non-degeneracy constraint qualification (NDCQ) is satisfied.

(b) Form the Lagrangian and find all the points that satisfy the first-order conditions for optimality.

(c) Is any point you found in (b) a solution to this problem? (Tip: concavity and convexity)

Exercise 3

Consider the optimization problem

$$\begin{aligned} \max_{x,y,} &= x^2 y \\ \text{s.t. } & 2x^2 + y^2 \leq a, \\ & y \leq 1, \\ & x, y \geq 0, \end{aligned}$$

where $a \geq 0$ is an exogenous parameter.
a) What does the feasible set look geometrically? Is it convex?
b) Find the solution of the problem as a function of a, i.e. $x(a), y(a)$? Sketch the solution on (x, y)-plane.

Exercise 4

a) Find the solution of the first order conditions of max $f(x; y) = e^{-(x-y)^2}$ where maximization is with respect to x and y is exogenous.
b) Is the solution $x^*(y)$ that you found maximizer of the objective function?
c) Use the envelope theorem to find the derivative of $f(x(y); y)$ with respect to y.

Exercise 5

a) Formulate the monopoly's profit maximization problem over the produced quantity q, when the inverse demand function is $p = a - bq$ and the unit cost of production is c.
b) Find the solution of the monopoly's problem as a function of parameters a, b. (You may assume that the solution is such that $q > 0$)
c) How does the optimal value of monopoly's profit change when a changes? (Tip: use the envelope theorem)