PHYS-E055102, Low Temperature Physics, Exercise 9 DL 16.11.2022

1. (a) Derive the following expression for the precooling time of the demagnetization stage in a constant magnetic field:

$$t = \left(\frac{1}{a} + 2R\right)\frac{c}{T_0^2}\left[\frac{1}{2T_0}\ln\left(\frac{T+T_0}{T-T_0}\right) - \frac{1}{T}\right], \tag{1}$$

   when the precooling is done using a dilution refrigerator, whose power is $\mathrm{d}Q/\mathrm{d}t = a\left(T_{\mathrm{mc}}^2 - T_0^2\right)$ with a base temperature T_0 and it is connected to the nuclear stage by a heat switch with a thermal resistance R.

   (b) How long would it take to precool the nuclear stage (100 mol copper) to 10 mK in a constant magnetic field of 9 T, when the dilution flow is 1 mmol/s, $T_0 = 3\,\mathrm{mK}$, and $R(T) = R_0/T = \left(10\,\mathrm{K}^2/\mathrm{W}\right)/T$?

   (c) Argue why a switching factor of 10^3 is not sufficient compared to 10^6 ?

2. Indium has a nuclear spin $I = 9/2$ and its lattice structure is tetragonal. The energies of the quadrupole states are obtained from

$$E_m = P\left[3m^2 - I\left(I+1\right)\right], \tag{2}$$

   where $P = -5.9 \cdot 10^{-28}\,\mathrm{J}$ and $m = -I, -I+1, \ldots, I$. An indium nuclear stage is precooled to an initial state with $B = 8\,\mathrm{T}$ and $T_i = 20\,\mathrm{mK}$, after which it is demagnetized to zero field. How long does the temperature stay below 1 mK?

3. Changes in magnetic field induce eddy currents in metallic objects. Derive an expression for the heating caused by this in a cylindrical rod, whose diameter is d and whose rotational axis is parallel to the magnetic field. Do the same for a plate, whose width and thickness in the plane perpendicular to the magnetic field are W and d. Apply the results in the case, where a nuclear stage with 35 mol of copper is demagnetized at a rate of 0.5 T/h. Use a RRR of copper of 500. Let the nuclear stage be

   (a) a cylinder with diameter 40 mm.

   (b) made of 2 mm thick insulated wires.

   (c) Compare the previous values to a RRR 5000. What are the differences?

4. In ideal nuclear demagnetization, B/T remains constant during the process. Show that the demagnetization losses are given by

$$\Delta\left(\frac{B}{T}\right) = a\left[\ln\left(\frac{B_i}{B_f}\right)\left(q_{ns}\frac{t_{dm}}{B_i - B_f} + \gamma\frac{B_i - B_f}{t_{dm}}\right) + bt_{dm}\right], \qquad (3)$$

when the demagnetization is done linearly in time t_{dm} and a field dependent background heat leak $\mathrm{d}Q/\mathrm{d}t = q_{\mathrm{ns}} + bB$ and eddy current heating $\mathrm{d}Q/\mathrm{d}t = \gamma\,(\mathrm{d}B/\mathrm{d}t)^2$ are taken into consideration. The factors a and b are constants.

5. In the NMR picture calculate the electron temperature T_e and its final field B_f value at the minimum of T_e for a magnetic field dependent heat load $\dot{Q}(B) = Q_0 + aB$, where Q_0 and a are constants.