Model Solutions 7

1. (a) We can solve for the p^* that maximizes joint profits by thinking as if the pricing decision was made by a profit-maximizing monopoly. The demand it would face is the sum of the demands of both firms. Since the problem is symmetric, we know that $p_1^* = p_2^* = p^*$.

$$Q^D(p) = Q_1(p,p) + Q_2(p,p) = 2 \times (20 - 2p + p) = 40 - 2p$$

Let's then formulate the profit function:

$$\begin{aligned}\Pi(p) &= Q^D(p)(p - MC)\\ &= (40 - 2p)(p - 5)\end{aligned}$$

And differentiate wrt. p to get the optimal collusion price p^*:

$$\begin{aligned}\frac{\partial \Pi(p)}{\partial p} &= 40 - 4p + 10 = 0\\ \implies p^* &= 12.5 \text{ €/l}\end{aligned}$$

(b) We know that the Nash equilibrium price is the same that we solved for the stage-game in PS 6.4a, $p^N = 10$. Let's then solve for p^C from the best-response function we solved in PS 6.4a:

$$\begin{aligned}BR_1(p^*) = \frac{30 + p^*}{4} &= \frac{30 + 12.5}{4}\\ &= 10.625 \text{ €/l}\end{aligned}$$

Now that we have the three prices, let's calculate the payoffs of different price combinations for OneGulp (by symmetry, they will be the same for TwoSips) by using the payoff function from PS 6.4a:

p_1	p_2	$\Pi(p_1, p_2) = (p_1 - MC)Q_1(p_1, p_2)$
10	10	$(10 - 5)(20 - 20 + 10) = 50$ €k
10	10.625	$(10 - 5)(20 - 20 + 10.625) \approx 53$ €k
10	12.5	$(10 - 5)(20 - 20 + 12.5) \approx 63$ €k
10.625	10	$(10.625 - 5)(20 - 21.25 + 10) \approx 49$ €k
10.625	10.625	$(10.625 - 5)(20 - 21.25 + 10.625) \approx 53$ €k
10.625	12.5	$(10.625 - 5)(20 - 21.25 + 12.5) \approx 63$ €k
12.5	10	$(12.5 - 5)(20 - 25 + 10) \approx 38$ €k
12.5	10.625	$(12.5 - 5)(20 - 25 + 10.625) \approx 42$ €k
12.5	12.5	$(12.5 - 5)(20 - 25 + 12.5) \approx 56$ €k

The payoff matrix becomes:

TwoSips

OneGulp: €k	P^N	P^C	P^*
P^N	50,50	53,49	63,38
P^C	49,53	53,53	63,42
P^*	38,63	42,63	56,56

(c) The best chance for sustaining collusion is the Grim strategy, where a player colludes as long as the other player colludes but switches to playing "the stage-game Nash" for forever if the other player ever deviates from the collusion price. The "punishment" action is to choose P^N, the collusion action is to choose P^* and the "cheating" action is to choose P^C. There are three relevant payoffs: $\Pi^N = 50$, $\Pi^* = 56$ and $\Pi^C = 63$. We need to check that no player has an incentive to deviate from either the punishment state $\{P^N, P^N\}$ or the collusion state $\{P^*, P^*\}$. We know that no player has an incentive to deviate from the punishment state, because it is the stage-game Nash equilibrium. So we only need to verify that a player doesn't have an incentive to deviate from the collusion state.

$$\underbrace{56 + \frac{56}{0.05}}_{\text{Present value of cooperating}} \geq \underbrace{63 + \frac{50}{0.05}}_{\text{Present value of cheating}}$$

$$1176 \text{ €k} \geq 1063 \text{ €k}$$

This verifies that collusion can be sustained with r = 5%.

(d) Let's solve for the highest r that makes collusion sustainable:

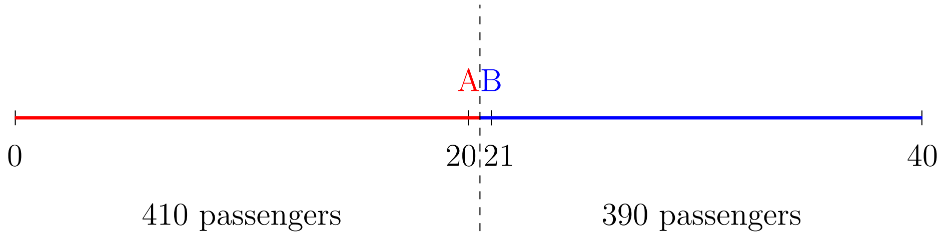
$$56 + \frac{56}{r} \geq 63 + \frac{50}{r}$$
$$\frac{6}{r} \geq 7$$
$$r \leq \frac{6}{7} \approx 86\%$$

When r is below 86 %, collusion is sustainable.

2. (a) This is a Hotelling line problem, where the airlines choose their departure times and passengers choose to fly with the airline that departs closest to their preferred departure time. Let's denote the departure time by $t \in \{0, 40\}$, where the 08:00 departure time is $t = 0$ and and the last departure time 18:00 is $t = 40$. The consumer surplus for a passenger whose preferred departure time is i is $CS_i = 400 - 10 \times |i - t| - 200$ dollars. This means that if the departure time differs from consumer i's preferred departure time by more than 20 time units (or five hours), she will choose not to fly. Note that in this market, it is profitable to serve all customers since the additional revenue from a second flight is 300 passengers × 200 dollars per ticket = \$60 000 which is higher than

the additional fixed cost and also efficient to serve all customers because each of the 800 potential passengers gets at least a weakly positive consumer surplus.

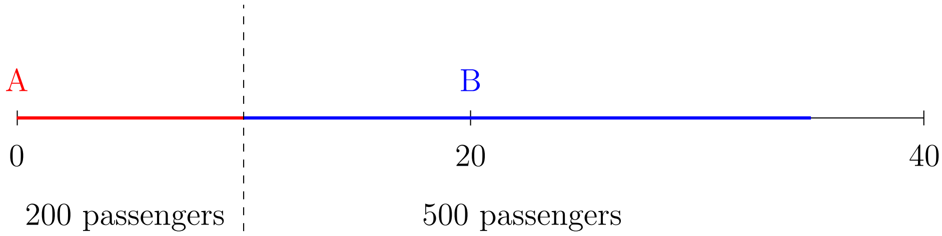
In equilibrium, AcmeAir (A) locates at t = 20 and BonkWings (B) either at t = 19 or t = 21, which is shown graphically below. If A places somewhere else than in the middle, B will want to locate in the middle and that would leave A with fewer passengers. B's best response to A locating in the middle is to locate as close to the middle as possible. The red line shows the share of customers that choose A and the blue line those that choose B.



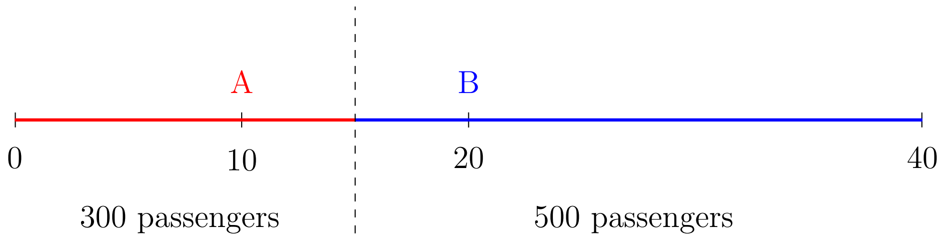
The profits of the airlines are:

$$\Pi_A(t_A = 20, t_B = 21) = 410 \times 200 - 40\ 000 = \$42\ 000$$
$$\Pi_B(t_A = 20, t_B = 21) = 390 \times 200 - 40\ 000 = \$38\ 000$$

To see why A wants to locate in the middle, let's consider a case where A would locate at t = 0. In this case, B would choose its position so that it gets the maximum capacity of 500 passengers, which is achieved by locating eg. at t = 20.



A could thus improve by moving towards the middle. If A chooses for example t = 10, it will get more passengers than by locating at t = 0. B would still want to locate in the middle and would get more passengers than A.



(b) The total revenues to be earned in the market are $800 \times 200 = \$160\ 000$. Since the fixed cost is \$40,000, there can be at most four flights departing in the market. Let's solve the exercise by guessing a potential equilibrium schedule and verifying that it indeed is an equilibrium. An obvious candidate is a symmetric equilibrium where the distance between each neighbouring flight is the same. One such equilibrium is to have A's flights located at $t = 5$ and $t = 25$ and B's flights at $t = 15$ and $t = 35$. All flights would get 200 passengers and earn zero profits. This is an equilibrium, since neither of the firms can increase its profits by increasing or decreasing its amount of flights.

There are also many other equilibria. In one such equilibrium, A has two flights at $t = 5$ and $t = 35$ and B one flight at $t = 20$

(c) Maximal profits are earned with a schedule that has two flights and that covers the whole market. This is accomplished for example with a schedule that has flights at $t = 10$ and $t = 30$. Profits are $800 \times 200 - 2 \times 40\ 000 = \$80\ 000$. It is not profitable to increase the number of flights, since it would not bring any additional passengers.

(d) In a social optimum, flights need to be scheduled so that the average distance from preferred departure times is minimized. With two flights, this is achieved with flights at t = 10 and t = 30, resulting in an average waiting time of 5 time units. The total surplus (TS) is:

$$\begin{aligned} TS(2 \text{ flights}) &= CS(2 \text{ flights}) + \Pi(2 \text{ flights}) \\ &= 800 \times (400 - 10 \times 5 - 200) + 80\ 000 = \$200\ 000 \end{aligned}$$

Having three flights placed optimally would decrease the average waiting time to $3\frac{1}{3}$ time units. Saving on average $5 - 3\frac{1}{3} = 1\frac{2}{3}$ time units of waiting would increase consumer surplus by $800 \times 10 \times 1\frac{2}{3} \approx \$13\ 300$, which is less than the increase in fixed costs. Thus, $\{t = 10,\ t = 30\}$ is the optimum.

3. (a) Let's start by aggregating the demand from hipsters and normies. Since the price at which demand is zero is the same for both customer groups (p = 24), we can simply sum up the individual demands:

$$\begin{aligned} Q^D(p) &= N_H Q_H^D(p) + N_N Q_N^D(p) = 100(24 - p) + 200(12 - 0.5p) \\ &= 100(24 - p) + 200(12 - 0.5p) = 3600 - 150p \end{aligned}$$

The profit function of Warre's Buffet is:

$$\begin{aligned} \Pi(p) &= (p - \text{MC})Q^D(p) - \text{FC} \\ &= (p - 4)(3600 - 150p) - 10\ 000 \end{aligned}$$

The optimal price is:

$$\begin{aligned} \frac{\partial \Pi(p)}{\partial p} &= 4200 - 300p = 0 \\ &\implies p^* = 14 \end{aligned}$$

(b) Since we have two customer groups and we can track which units are consumed by the same buyer, we can maximize profits by designing a two-part tariff pricing scheme with an entry fee F and a unit price P for the buffet. To design an optimal two-part tariff scheme, let's follow the steps outlined in the lecture slides. The low-type customers are the normies, since their demand curve is always below the demand curve of the hipsters.

Let's set the entrance fee so that it extracts all of normies' CS:

$$F(p) = (24 - p)Q_N^D(P)\frac{1}{2} = (24 - p)(12 - 0.5p)\frac{1}{2} = \frac{1}{4}(24 - p)^2$$

Let's then formulate the profit function of Warre's Buffet wrt. the unit price p:

$$\begin{aligned}\Pi(p) &= (N_H Q_H^D(p) + N_N Q_N^D(p))(p - \text{MC}) + (N_H + N_N)(F(p)) - \text{FC} \\ &= (100(24 - p) + 200(12 - 0.5p))(p - 4) + 300(\frac{1}{4}(24 - p)^2) - 10000 \\ &= -125p^2 + 2000p + 14000\end{aligned}$$

Let's differentiate wrt. p to get the optimal unit price:

$$\frac{\partial \Pi(p)}{p} = -250p + 2000 = 0$$

$$\implies p^* = 8$$

The optimal entrance fee is $F(8) = \frac{1}{4}(24 - 8)^2 = 64$ euros and profits are $\Pi(8) = -125 \times 8^2 + 2000 \times 8 + 14000 = 22\,000$ euros. Let's verify that Warre is not better of by serving only hipsters by setting the entrance fee equal to their consumer surplus at the marginal cost. The entrance fee would be $F_H(4) = (24 - 4)(24 - 4)\frac{1}{2} = 200$ euros and profits:

$$\Pi_H(p) = N_H F_H(4) - FC = 100 \times 200 - 10000 = 10\,000$$

The profit-maximizing pricing strategy is to serve both groups.

(c) Let's calculate the consumer surpluses of hipsters and normies under optimal simple pricing:

$$CS_N(p^* = 14) = N_N(24 - 14)Q_N^D(14)\frac{1}{2} = 200 \times (24 - 14)(12 - 7)\frac{1}{2} = 5\,000\ €$$

$$CS_H(p^* = 14) = N_H(24 - 14)Q_H^D(14)\frac{1}{2} = 100 \times (24 - 14)(24 - 14)\frac{1}{2} = 5\,000\ €$$

The CS of normies under two-part tariffs is zero. The consumer surplus of hipsters is:

$$CS_H(F = 64, p^* = 8) = 100 \times ((24 - 8)(24 - 8)\frac{1}{2} - 64) = 100 \times 64 = 6\,400\ €$$

As a result of two-part tariffs, normies' aggregate CS drops from 5 000 euros to zero and hipsters' aggregate CS increases from 5 000 euros to 6 400 euros.

(d) Since serving only hipsters was not profitable even with the lower marginal cost, it clearly won't be with the higher marginal cost either - the fee will be smaller while the fixed costs stay the same. Thus the profit-maximizing pricing scheme will follow the same strategy as previously. The fee is also still the same as a function of p, since it is still targeted at extracting the surplus from the normies.

With two different marginal costs, the profit function looks slightly different:

$$\begin{aligned}\Pi(p) &= (N_H Q_H^D(p))(p - \mathrm{MC}_H) + (N_N Q_N^D(p))(p - \mathrm{MC}_N) + (N_H + N_N)(F(p)) - \mathrm{FC} \\ &= 100(24 - p)(p - 6) + 200(12 - 0.5p)(p - 4) + 300(\frac{1}{4}(24 - p)^2) - 10000 \\ &= -125p^2 + 2200p + 9200\end{aligned}$$

The optimal unit price is:

$$\frac{\partial \Pi(p)}{p} = -250p + 2200 = 0$$
$$\implies p^* = 8.8$$

The optimal entrance fee is $F(8.8) = \frac{1}{4}(24 - 8.8)^2 = 57.78$ euros. So the optimal price is higher and the optimal entrance fee lower than before. Profits will be lower than before due to the increased marginal cost of serving hipsters.

4. (a) An example of two-part tariffs is the purchase of a Nestlé Nescafé capsule coffee maker. You pay an "entrance fee" (around 200 euros) for the coffee machine and a unit price (around 0.5 euros) for the capsules

(b) Coca-Cola is often sold with a quantity discount. 2x1.5 litres costs less than double (€3.89/pack) what a single 1.5 litre bottle costs (€2.25/bottle).

(c) One example is four friends that have a regular time for badminton doubles practice. In the "coordination" state, everybody comes to the practice. This maximizes joint payoffs, since badminton doubles requires all four players at the court. In the "stage-game" state, there is no regular time for practice and it is difficult to organize a time that suits everyone. If someone is lazy and decides to skip the practice (too often) for short-term gain and leaving everyone else worse off, the "punishment" action is to stop having the regular practice time and end up in the "stage-game" equilibrium.

Team sports is also an example of network externalities, where the consumption value of a good depends on the number of other users (players). Network externalities will be discussed later in the course.