Exercise and Homework Round 7

These exercises (except for the last) will be gone through on **Friday, November 11, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Friday, November 18 at 12:00**.

Exercise 1. (Analytical solution of a nonlinear ODE)

Consider the following logistic differential equation:

$$\dot{x} = \lambda \, x \, (1 - x),$$

with the initial condition $x(0) = x_0$.

(a) Check that the differential equation is not linear.

(b) Solve the differential equation by using separation of variables.

Exercise 2. (Numerical solution of a nonlinear ODE)

Consider the nonlinear ODE in the previous exercise with $\lambda = 1$, $x_0 = 1/10$.

(a) Solve the ODE numerically with Euler method.

(b) Solve the ODE numerically with Runge–Kutta method of order 4.

(c) Use a builtin ODE solver to obtain a numerical solution to the ODE.

In each of the above, compare the solutions to the solution obtained in Exercise 1b.

Exercise 3. (Numerical solution of robot dynamics)

Consider the following 2D dynamic model of a robot platform:

$$\begin{aligned}
\dot{p}^x(t) &= v(t)\cos(\varphi(t)) + w_1(t), \\
\dot{p}^y(t) &= v(t)\sin(\varphi(t)) + w_2(t), \\
\dot{\varphi}(t) &= \omega_{\text{gyro}}(t) + w_3(t),
\end{aligned}$$

where p^x, p^y is the position, φ is the orientation angle, v is the speed input, ω_{gyro} is the gyroscope reading, and w_1, w_2, w_3 are white noise processes. Assume that we start from $p^x(0) = 0$, $p^y(0) = 0$, and $\varphi(0) = 0$.

(a) Rewrite the equation in a canonical form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{B}_w(\mathbf{x}(t))\mathbf{w}(t). \tag{1}$$

(b) Consider the following input signals:

$$v(t) = \begin{cases} t, & t \in [0,1), \\ 1, & t \in [1,4), \\ 5-t, & t \in [4,5), \end{cases} \qquad \omega_{\text{gyro}}(t) = \begin{cases} 0, & t \in [0,2), \\ \pi/2, & t \in [2,3), \\ 0, & t \in [3,5). \end{cases} \tag{2}$$

Explain what kind of physical situation this corresponds to and explain what the solution should look like. You can assume that the noises are zero.

(c) Numerically, using Euler method, solve the differential equations with the inputs above. Visualize the solution and compare to the explanation that you came up with above.

(d) Include some noise into your simulation (Euler–Maruyama) and visualize and discuss its effect on the solutions.

Homework 7 (DL Friday, November 18 at 12:00)

Consider the noise-free 2D robot dynamics equations

$$\begin{aligned}\dot{p}^x(t) &= v(t)\cos(\varphi(t)),\\ \dot{p}^y(t) &= v(t)\sin(\varphi(t)),\\ \dot{\varphi}(t) &= \omega_{\text{gyro}}(t),\end{aligned}$$

where p^x, p^y is the position, φ is the orientation angle, v is the speed input, and ω_{gyro} is the gyroscope reading.

(a) Assume that the robot starts at time $t = 0$ from origin, heading upwards, that is, towards the positive y values. What should be the initial conditions $p^x(0), p^y(0), \varphi(0)$ corresponding to this?

(b) Construct speed and gyroscope signals which correspond to the following movement:

- The speed is constant $v(t) = 2$ for the time interval $t \in [0, 10]$ and zero otherwise.
- The orientation of the robot is upwards (and thus it moves up) in all time moments except during $t \in [3, 7)$ when it does a 360-degree turn clockwise.

(c) Numerically, using Euler method, solve the differential equations with the inputs that you constructed above. Visualize the solution and check that it is what you expected.