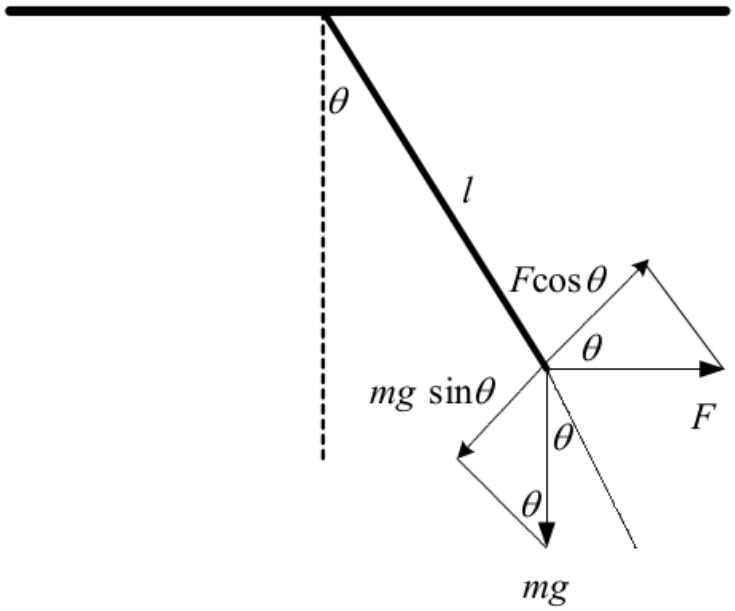
ELEC-E8101 Digital and Optimal Control
Exercise 9
Autumn 2022

Consider the pendulum in the figure below. The stick of the pendulum is massless and the only force that interacts with the mass m is the horizontal force F



1

Determine the (non-linear) dynamics model of the pendulum as a state-space model of the angle θ.

2

Linearize the non-linear state-space model around the equilibrium

$$\left(\theta, \dot{\theta}\right) = (0, 0)$$

3

A state space model

$$\dot{x} = Ax + Bu,\ y = Cx + Du$$

can be discretized with ZOH with sample time T_s as

$$x[k+1] = A_d x[k] + B_d u[k],\ y[k] = Cx[k] + Du[k]$$

where

$$A_d = e^{AT_s}, B_d = A^{-1}(A_d - I)B.$$

Discretize the system assuming sampling time *h* (you can use tables and/or symbolic math software).

4

Assume $g = 9.81,\ l = 9.81, m = 1/9.81$.

Determine if the discretized system is stable.

5 * (extra task for instruction)

Consider controlling the original (non-linear continuous time) system with a PID controller with a filter for D.

Implement the control in Simulink. Implement also the controller for the discretized system from task 2 similarly.

Assume $K_P = K_D = K_I = 2,\ N = 100$.

Simulate the system response for $\theta_{target} = 0.4$ and plot it.

Then plot the response for $\theta_{target} = 1.5$.
Explain the difference.