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2 CABLING

This chapter is devoted to cabling and cable shielding, and Chapter 3 covers grounding. Since the subjects of cable shielding and grounding are closely related, these two chapters should be studied together. Chapter 2, for example, shows that a cable shield used to suppress electric fields should be grounded, but Chapter 3 explains where that ground should be made.

Cables are important because they are the longest parts of a system and therefore act as efficient antennas that pick up and/or radiate noise. This chapter covers the coupling mechanisms that occur between fields and cables, and between cables (crosstalk). Both unshielded and shielded cables are considered.

In this chapter we assume the following:

1. Shields are made of nonmagnetic materials and have a thickness much less than a skin depth at the frequency of interest.*
2. The receptor is not coupled so tightly to the source that it loads down the source.
3. Induced currents in the receptor circuit are small enough not to distort the original field. (This does not apply to a shield around the receptor circuit.)
4. Cables are short compared to a wavelength.

Since cables are assumed short compared to a wavelength, the coupling between circuits can be represented by lumped capacitance and inductance between the conductors. The circuit can then be analyzed by normal network theory.

Three types of couplings are considered. The first is capacitive or electric coupling, which results from the interaction of electric fields between circuits. This type of coupling is commonly identified in the literature as electrostatic coupling, an obvious misnomer since the fields are not static.

The second is inductive, or magnetic, coupling, which results from the interaction between the magnetic fields of two circuits. This type of coupling is commonly described as electromagnetic, again misleading terminology since no electric fields are involved. The third is a combination of electric and magnetic fields and is appropriately called electromagnetic coupling or

*If the shield is thicker than a skin depth, some additional shielding is present besides that calculated by methods in this chapter. The effect is discussed further in Chapter 6.

radiation. The techniques developed to cope with electric coupling are also appropriate for the electromagnetic case. For analysis in the near field, we normally consider the electric and magnetic fields separately, whereas the electromagnetic field case is considered when the problem is in the far field.* The circuit causing the interference is called the source, and the circuit being affected by the interference is called the receptor.

CAPACITIVE COUPLING

A simple representation of capactive coupling between two conductors is shown in Fig. 2-1. Capacitance C_{12} is the stray capacitance between conductors 1 and 2. Capacitance C_{1G} is the capacitance between conductor 1 and ground, C_{2G} is the total capacitance between conductor 2 and ground, and R is the resistance of circuit 2 to ground. The resistance R results from the circuitry connected to conductor 2 and is not a stray component. Capacitance C_{2G} consists of both the stray capacitance of conductor 2 to ground and the effect of any circuitry connected to conductor 2.

The equivalent circuit of the coupling is also shown in Fig. 2-1. Consider the voltage V_1 on conductor 1 as the source of interference and conductor 2 as the affected circuit or receptor. Any capacitance connected directly across the source, such as C_{1G} in Fig. 2-1, can be neglected since it has no effect on the noise coupling. The noise voltage V_N produced between conductor 2 and ground can be expressed as follows:

$$V_N = \frac{j\omega[C_{12}/(C_{12}+C_{2G})]}{j\omega + 1/R(C_{12}+C_{2G})} V_1 . \qquad (2\text{-}1)$$

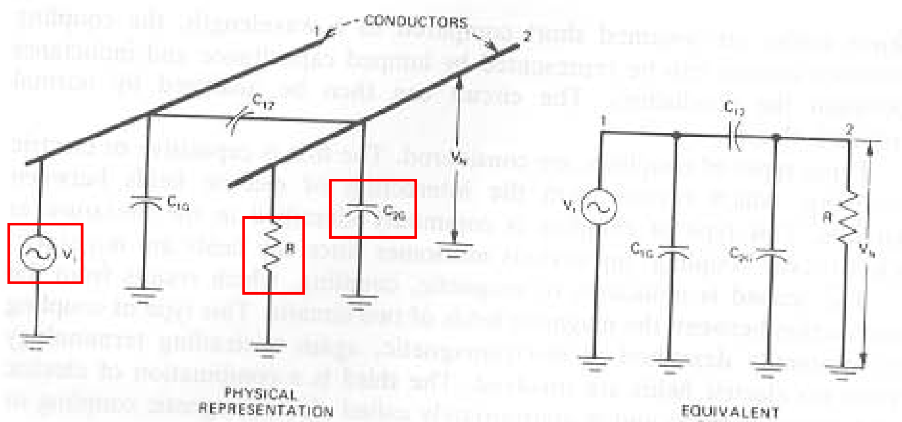

Figure 2-1. *Capacitive coupling between two conductors.*

*See Chapter 6 for definitions of near and far fields.

Equation 2-1 does not show clearly how the pickup voltage depends on the various parameters. Equation 2-1 can be simplified for the case when R is a lower impedance than the impedance of the stray capacitance C_{12} plus C_{2G}. In most practical cases this will be true. Therefore, for

$$R \ll \frac{1}{j\omega(C_{12}+C_{2G})},$$

Eq. 2-1 can be reduced to the following:

$$\boxed{V_N = j\omega R C_{12} V_1 .} \qquad (2\text{-}2)$$

Electric field (capacitive) coupling can be modeled as a current generator, connected between the receptor circuit and ground, with a magnitude of $j\omega C_{12}V_1$. This is shown in Fig. 2-9A.

Equation 2-2 is the most important equation describing the capacitive coupling between two conductors, and it clearly shows how the pickup voltage depends on the parameters. Equation 2-2 shows that the noise voltage is directly proportional to the frequency ($\omega = 2\pi f$) of the noise source, the resistance R of the affected circuit to ground, the capacitance C_{12} between conductors 1 and 2, and the magnitude of the voltage V_1.

Assuming that the voltage and frequency of the noise source cannot be changed, this leaves only two remaining parameters for reducing capacitive coupling. The receiver circuit can be operated at a lower resistance level, or capacitance C_{12} can be decreased. Capacitance C_{12} can be decreased by proper orientation of the conductors, by shielding (described in the next section), or by physically separating the conductors. If the conductors are moved farther apart, C_{12} decreases, thus decreasing the induced voltage on conductor 2.* The effect of conductor spacing on capacitive coupling is shown in Fig. 2-2. As a reference, 0 dB is the coupling when the conductors are separated by three times the conductor diameter. As can be seen in the figure, little additional attenuation is gained by spacing the conductors a distance greater than 40 times their diameter (1 in. in the case of 22-gauge wire).

If the resistance from conductor 2 to ground is large, such that

$$R \gg \frac{1}{j\omega(C_{12}+C_{2G})},$$

then Eq. 2-1 reduces to

$$\boxed{V_N = \left(\frac{C_{12}}{C_{12}+C_{2G}}\right)V_1 .} \qquad (2\text{-}3)$$

*The capacitance between two parallel conductors of diameter d and spaced D apart is $C_{12} = \pi\epsilon/\cosh^{-1}(D/d)$, (F/m). For $D/d > 3$, this reduces to $C_{12} = \pi\epsilon/\ln(2D/d)$, (F/m), where $\epsilon = 8.85 \times 10^{-12}$ farads per meter (F/m) for free space.

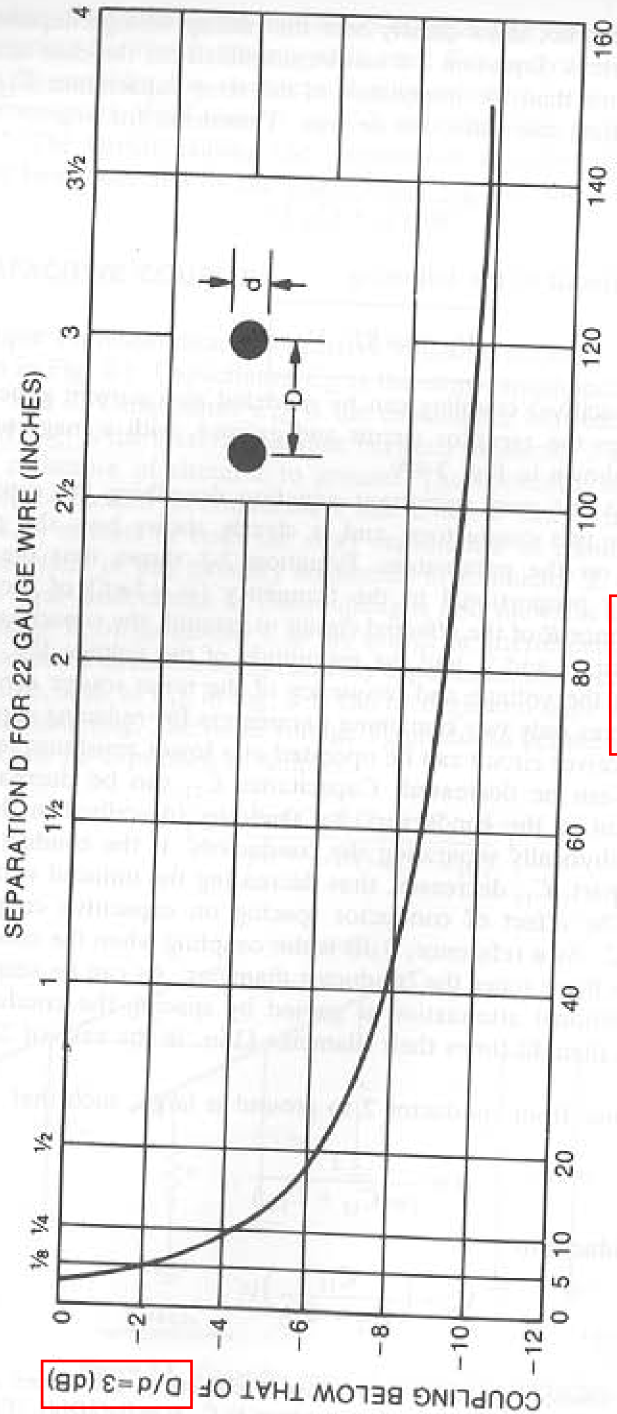

Figure 2-2. *Effect of conductor spacing on capacitive coupling. In the case of 22-gauge wire, most of the attenuation occurs in the first inch of separation.*

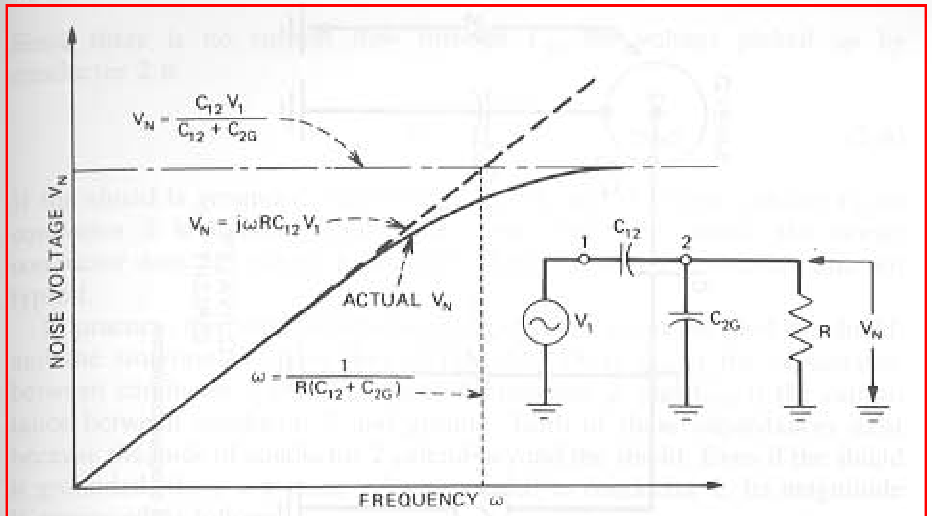

Figure 2-3. *Frequency response of capacitive coupled noise voltage.*

Under this condition the noise voltage produced between conductor 2 and ground is due to the capacitive voltage divider C_{12} and C_{2G}. The noise voltage is independent of frequency and is of a larger magnitude than when R is small.

A plot of Eq. 2-1 versus ω is shown in Fig. 2-3. As can be seen, the maximum noise coupling is given by Eq. 2-3. The figure also shows that the actual noise voltage is always less than or equal to the value given by Eq. 2-2. At a frequency of

$$\omega = \frac{1}{R(C_{12}+C_{2G})} \tag{2-4}$$

Equation 2-2 gives a value of noise that is 1.41 times the actual value. In almost all practical cases, the frequency is much less than this, and Eq. 2-2 applies.

EFFECT OF SHIELD ON CAPACITIVE COUPLING

First consider the case where the receptor (conductor 2) has infinite resistance to ground. If a shield is placed around conductor 2, the configuration becomes that of Fig. 2-4. An equivalent circuit of the capacitive coupling between conductors is included. The voltage picked up by the shield is

$$V_S = \left(\frac{C_{1S}}{C_{1S}+C_{SG}}\right)V_1 . \tag{2-5}$$

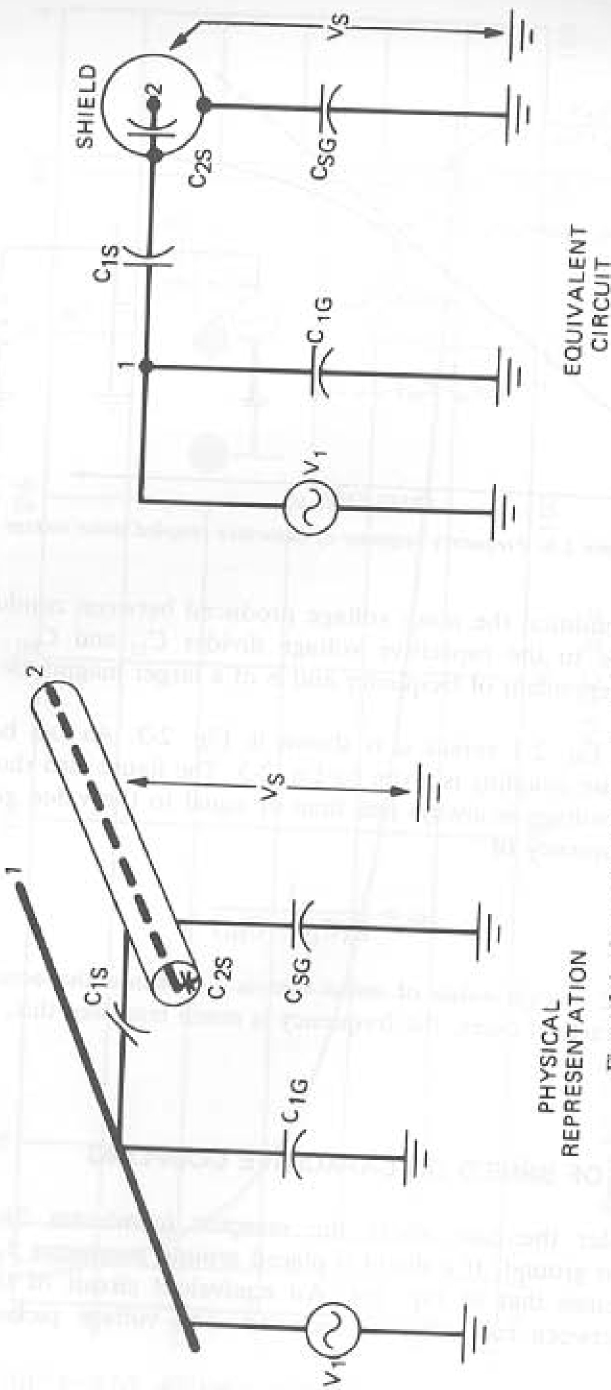

Figure 2-4. *Capacitive coupling with shield placed around receptor conductor.*

Since there is no current flow through C_{2S} the voltage picked up by conductor 2 is

$$V_N = V_S . \tag{2-6}$$

If the shield is grounded, the voltage $V_S = 0$, and the noise voltage V_N on conductor 2 is likewise reduced to zero. This case—where the center conductor does not extend beyond the shield—is an ideal situation and not typical.

In practice, the center conductor normally does extend beyond the shield, and the situation becomes that of Fig. 2-5. There C_{12} is the capacitance between conductor 1 and the shielded conductor 2, and C_{2G} is the capacitance between conductor 2 and ground. Both of these capacitances exist because the ends of conductor 2 extend beyond the shield. Even if the shield is grounded, there is a noise voltage coupled to conductor 2. Its magnitude is expressed as follows:

$$V_N = \frac{C_{12}}{C_{12} + C_{2G} + C_{2S}} V_1 . \tag{2-7}$$

The value of C_{12}, and hence V_N, in Eq. 2-7 depends on the length of conductor 2 that extends beyond the shield.

For good electric field shielding, it is therefore necessary (1) *to minimize the length of the center conductor that extends beyond the shield and* (2) *to provide a good ground on the shield.* A single ground connection makes a good shield ground, provided the cable is not longer than one-twentieth of a wavelength. On longer cables multiple grounds may be necessary.

If in addition the receiving conductor has finite resistance to ground, the arrangement is that shown in Fig. 2-6. If the shield is grounded, the equivalent circuit can be simplified as shown in the figure. Any capacitance directly across the source can be neglected since it has no effect on the noise coupling. The simplified equivalent circuit can be recognized as the same circuit analyzed in Fig. 2-1, provided C_{2G} is replaced by the sum of C_{2G} and C_{2S}. Therefore, if

$$R \ll \frac{1}{j\omega(C_{12} + C_{2G} + C_{2S})} ,$$

which is normally true, the noise voltage coupled to conductor 2 is

$$V_N = j\omega R C_{12} V_1 . \tag{2-8}$$

This is the same as Eq. 2-2, which is for an unshielded cable, except that C_{12} is greatly reduced by the presence of the shield. Capacitance C_{12} now consists primarily of the capacitance between conductor 1 and the un-

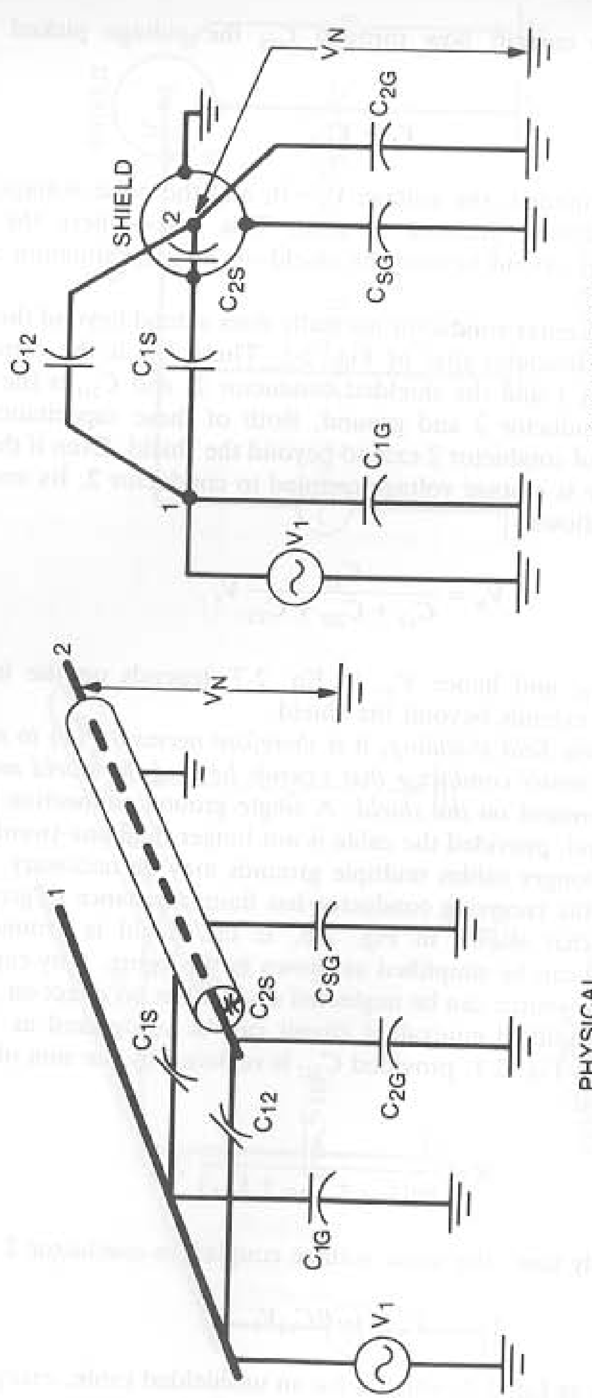

Figure 2-5. *Capacitive coupling when center conductor extends beyond shield; shield grounded at one point.*

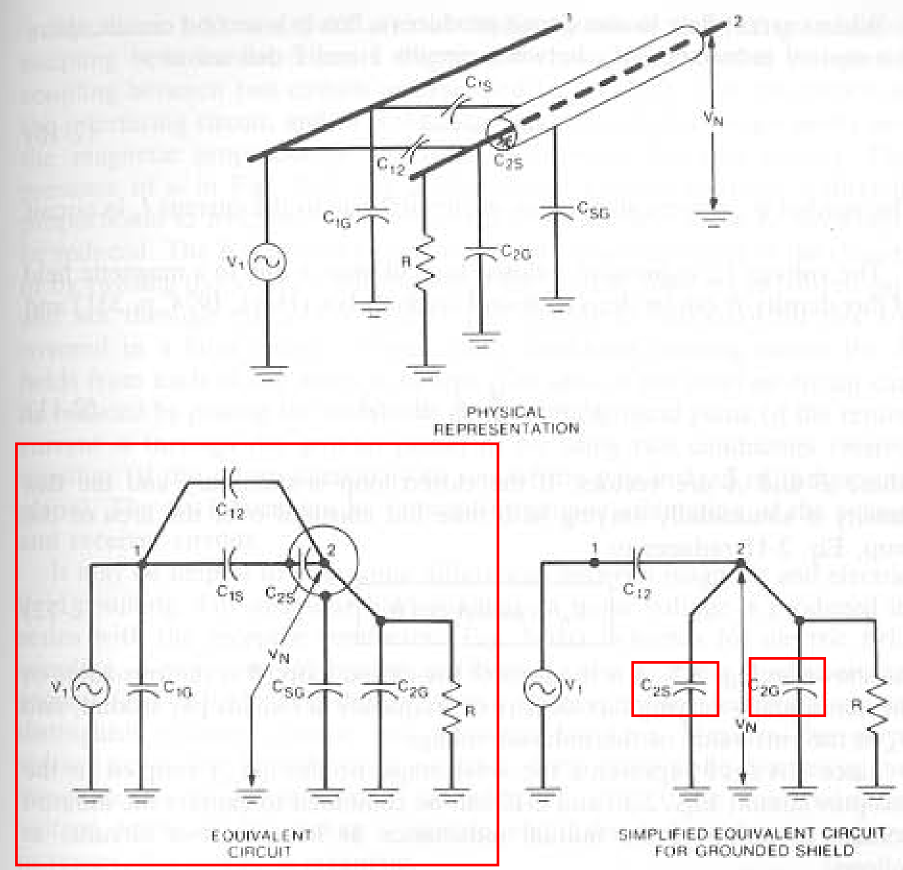

Figure 2-6. *Capacitive coupling when receptor conductor has resistance to ground.*

shielded portions of conductor 2. If the shield is braided, any capacitance that exists from conductor 1 to 2 through the holes in the shield must also be included in C_{12}.

INDUCTIVE COUPLING

When a current I flows in a closed circuit, it produces a magnetic flux ϕ which is proportional to the current. The constant of proportionality is called the inductance L, hence we can write

$$\phi = LI. \tag{2-9}$$

The inductance value depends on the geometry of the circuit and the magnetic properties of the medium containing the field.

When current flow in one circuit produces a flux in a second circuit, there is a mutual inductance M_{12} between circuits 1 and 2 defined as

$$M_{12} = \frac{\phi_{12}}{I_1}. \tag{2-10}$$

The symbol ϕ_{12} represents the flux in circuit 2 due to the current I_1 in circuit 1.

The voltage V_N induced in a closed loop of area $\bar{A}$ due to a magnetic field of flux density $\bar{B}$ can be derived from Faraday's law (Hayt, 1974, p. 331) and is

$$V_N = -\frac{d}{dt}\int_A \bar{B}\cdot d\bar{A}, \tag{2-11}$$

where $\bar{B}$ and $\bar{A}$ are vectors. If the closed loop is stationary and the flux density is sinusoidally varying with time but constant over the area of the loop, Eq. 2-11 reduces to

$$\boxed{V_N = j\omega BA\cos\theta.}\,* \tag{2-12}$$

As shown in Fig. 2-7, A is the area of the closed loop, B is the rms value of the sinusoidally varying flux density of frequency ω radians per second, and V_N is the rms value of the induced voltage.

Since $BA\cos\theta$ represents the total magnetic flux (ϕ_{12}) coupled to the receptor circuit, Eqs. 2-10 and 2-12 can be combined to express the induced voltage in terms of the mutual inductance M between two circuits, as follows:

$$V_N = j\omega MI_1 = M\frac{di_1}{dt}. \tag{2-13}$$

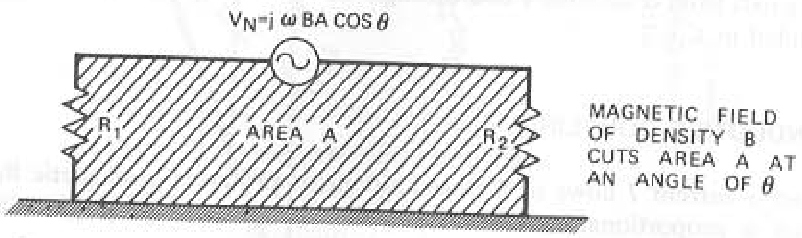

Figure 2-7. *Induced noise depends on the area enclosed by the disturbed circuit.*

*Equation 2-12 is correct when the MKS system of units is being used. Flux density B is in webers per square meter (or tesla), and area A is in square meters. If B is expressed in gauss and A is in square centimeters (the CGS system), the right side of Eq. 2-12 must be multiplied by 10^{-8}.

Equations 2-12 and 2-13 are the basic equations describing inductive coupling between two circuits. Figure 2-8 shows the inductive (magnetic) coupling between two circuits as described by Eq. 2-13. I_1 is the current in the interfering circuit, and M is the term that accounts for the geometry and the magnetic properties of the medium between the two circuits. The presence of ω in Eqs. 2-12 and 2-13 indicates that the coupling is directly proportional to frequency. To reduce the noise voltage, B, A, or $\cos\theta$ must be reduced. The B term can be reduced by physical separation of the circuits or by twisting the source wires, provided the current flows in the twisted pair and not through the ground plane. The conditions necessary for this are covered in a later section. Under these conditions twisting causes the B fields from each of the wires to cancel. The area of the receiver circuit can be reduced by placing the conductor closer to the ground plane (if the return current is through the ground plane) or by using two conductors twisted together (if the return current is on one of the pair instead of the ground plane). The $\cos\theta$ term can be reduced by proper orientation of the source and receiver circuits.

It may be helpful to note some differences between magnetic and electric field coupling. For magnetic field coupling, a noise voltage is produced in series with the receptor conductor (Fig. 2-9B), whereas for electric field coupling, a noise current is produced between the receptor conductor and ground (Fig. 2-9A). This difference can be used in the following test to distinguish between electric and magnetic coupling. Measure the noise voltage across the impedance at one end of the cable while decreasing the impedance at the opposite end of the cable (Fig. 2-9). If the measured noise voltage decreases, the pickup is electric, and if the measured noise voltage increases, the pickup is magnetic.

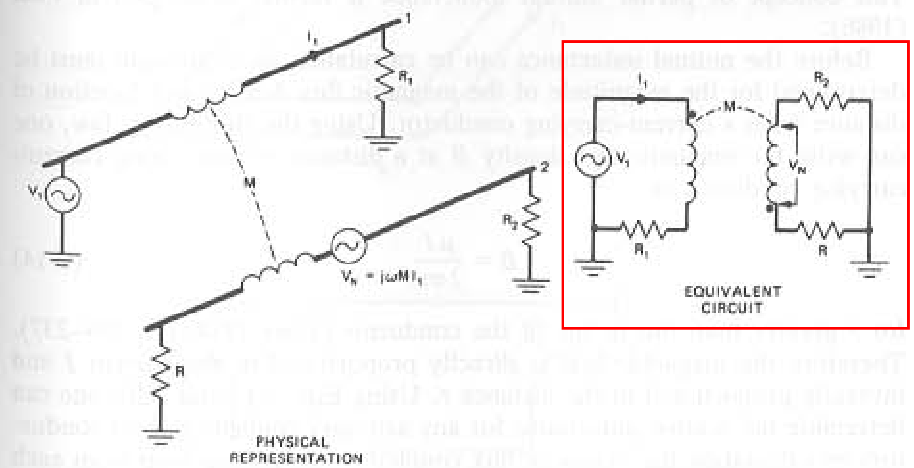

Figure 2-8. *Magnetic coupling between two circuits.*

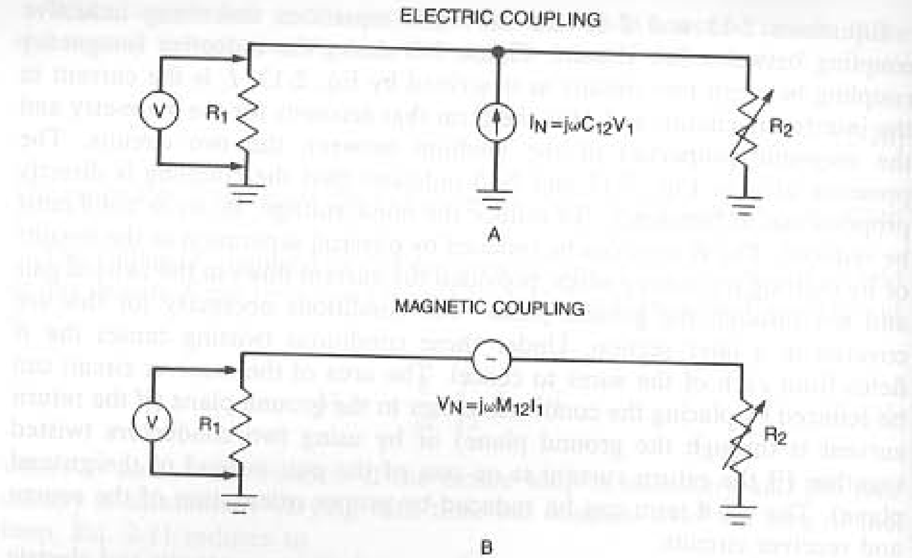

Figure 2-9. *(A) Equivalent circuit for electric field coupling; (B) equivalent circuit for magnetic field coupling.*

MUTUAL INDUCTANCE CALCULATIONS

To evaluate the expression in Eq. 2-13, the mutual inductance between the source and receptor circuit must be known. Most texts do not pay much attention to mutual inductance calculations for practical circuit geometries. Grover (1973), however, provides an extensive treatment of the subject, and Ruehli (1972) develops the useful concept of partial mutual inductance. This concept of partial mutual inductance is further developed in Paul (1986).

Before the mutual inductance can be calculated, an expression must be determined for the magnitude of the magnetic flux density as a function of distance from a current-carrying conductor. Using the Biot-Savart law, one can write the magnetic flux density B at a distance r from a long current-carrying conductor as

$$B = \frac{\mu I}{2\pi r}, \tag{2-14}$$

for r greater than the radius of the conductor (Hayt 1974, pp. 235–237). Therefore the magnetic field is directly proportional to the current I and inversely proportional to the distance r. Using Eqs. 2-14 and 2-10, one can determine the mutual inductance for any arbitrary configuration of conductors by calculating the magnetic flux coupled to the pickup loop from each current-carrying conductor individually, and then superimposing all the results to obtain the total flux coupling.

Example 2.1. Calculate the mutual inductance between the two nested coplaner loops shown in Fig. 2-10A, assuming that the sides of the loop are much longer than the ends (i.e., the coupling contributed by the end conductors can be neglected). Conductors 1 and 2 are carrying a current I_1 which induces a voltage V_N into the loop formed by conductors 3 and 4. Figure 2-10B is a cross-sectional view showing the spacing between the conductors. The magnetic flux produced by the current in conductor 1 crossing the loop between conductors 3 and 4 is

$$\theta_{12} = \int_a^b \frac{\mu I_1}{2\pi r}\, dr = \frac{\mu I_1}{2\pi} \ln\left(\frac{b}{a}\right). \tag{2-15}$$

Conductor 2 also contributes an equal flux due to the symmetry of the conductors. This flux is in the same direction as the flux produced by the current in conductor 1. Therefore the total flux coupled to the loop formed by conductors 3 and 4 is twice that given by Eq. 2-15, or

$$\theta_{12} = \left[\frac{\mu}{\pi} \ln\left(\frac{b}{a}\right)\right] I_1. \tag{2-16}$$

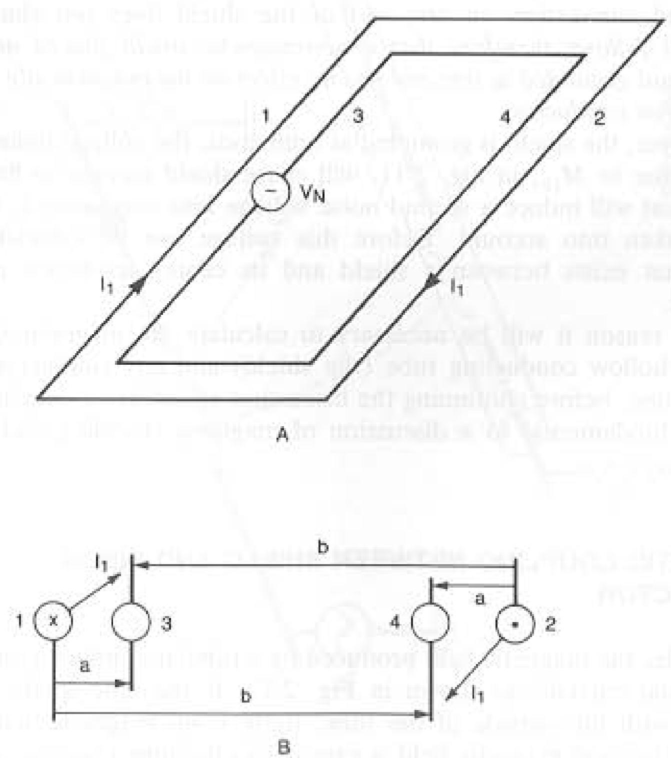

Figure 2-10. *(A) Nested coplanar loops; (B) cross-sectional view of A.*

Dividing Eq. 2-16 by I_1 and substituting $4\pi \times 10^{-7}$ H/m for μ, we obtain as the mutual inductance

$$M = 4 \times 10^{-7} \ln\left(\frac{b}{a}\right). \tag{2-17}$$

The voltage coupled between the two loops can be calculated by substituting the result from Eq. 2-17 into Eq. 2-13.

EFFECT OF SHIELD ON MAGNETIC COUPLING

If an ungrounded and nonmagnetic shield is now placed around conductor 2, the circuit becomes that of Fig. 2-11, where M_{1S} is the mutual inductance between conductor 1 and the shield. Since the shield has no effect on the geometry or magnetic properties of the medium between circuits 1 and 2, it has no effect on the voltage induced into conductor 2. The shield does, however, pick up a voltage due to the current in conductor 1:

$$V_S = j\omega M_{1S} I_1 . \tag{2-18}$$

A ground connection on one end of the shield does not change the situation. *It follows therefore that a nonmagnetic shield placed around a conductor and grounded at one end has no effect on the magnetically induced voltage in that conductor.*

If, however, the shield is grounded at both ends, the voltage induced into the shield due to M_{1S}, in Fig. 2-11, will cause shield current to flow. The shield current will induce a second noise voltage into conductor 2, and this must be taken into account. Before this voltage can be calculated, the coupling that exists between a shield and its center conductor must be determined.

For this reason it will be necessary to calculate the magnetic coupling between a hollow conducting tube (the shield) and any conductor placed inside the tube, before continuing the discussion of inductive coupling. This concept is fundamental to a discussion of magnetic shielding and will be needed later.

MAGNETIC COUPLING BETWEEN SHIELD AND INNER CONDUCTOR

First consider the magnetic field produced by a tubular conductor carrying a *uniform* axial current, as shown in Fig. 2-12. If the hole in the tube is concentric with the outside of the tube, there is no magnetic field in the cavity and the total magnetic field is external to the tube (Smythe, p. 278).

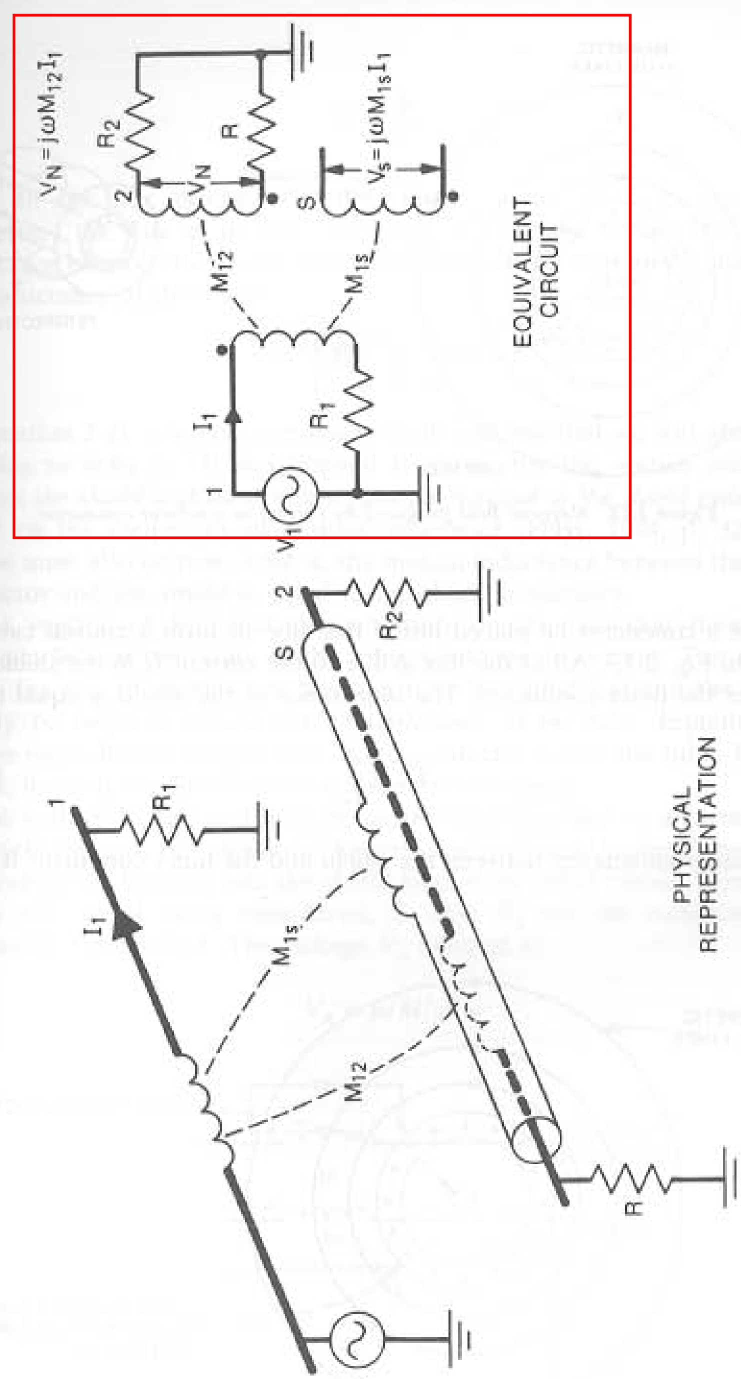

Figure 2-11 *Magnetic coupling when a shield is placed around the receptor conductor.*

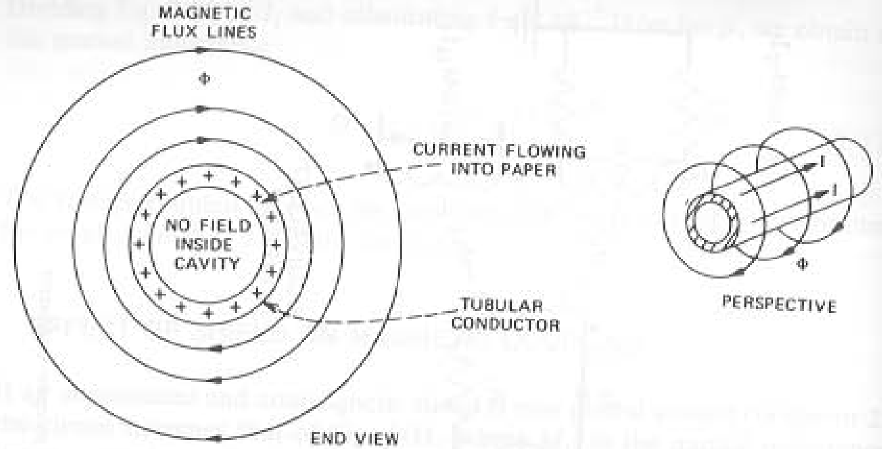

Figure 2-12. *Magnetic field produced by current in a tubular conductor.*

Now let a conductor be placed inside the tube to form a coaxial cable, as shown in Fig. 2-13. All of the flux ϕ due to the current I_S in the shield tube encircles the inner conductor. The inductance of the shield is equal to

$$L_S = \frac{\phi}{I_S}. \tag{2-19}$$

The mutual inductance between the shield and the inner conductor is equal

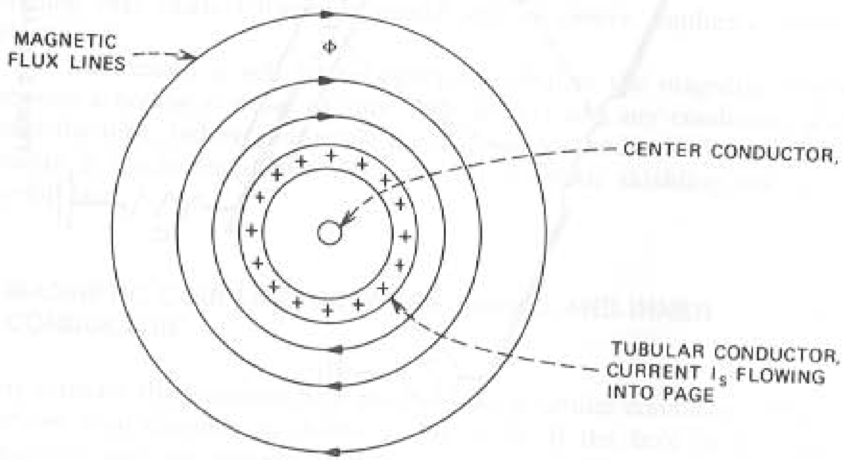

Figure 2-13. *Coaxial cable with shield current flowing.*

to

$$M = \frac{\phi}{I_S}. \tag{2-20}$$

Since all the flux produced by the shield current encircles the center conductor, the flux ϕ in Eqs. 2-19 and 2-20 is the same. The mutual inductance between the shield and center conductor is therefore equal to the self inductance of the shield

$$\boxed{M = L_S.} \tag{2-21}$$

Equation 2-21 is a most important result and one that we will often have occasion to refer to. It was derived to show that the *mutual inductance between the shield and the center conductor is equal to the shield inductance*. Based on the reciprocity of mutual inductance (Hayt, 1974, p. 321), the inverse must also be true. That is, the mutual inductance between the center conductor and the shield is equal to the shield inductance.

The validity of Eq. 2-21 depends only on the fact that there is no magnetic field in the cavity of the tube due to shield current. The requirements for this to be true are that the tube be cylindrical and the current density be *uniform around the circumference of the tube*. Equation 2-21 applies regardless of the position of the conductor within the tube. In other words, the two conductors do not have to be coaxial.

The voltage V_N induced into the center conductor due to a current I_S in the shield can now be calculated. Assume that the shield current is produced by a voltage V_S induced into the shield from some other circuit. Figure 2-14 shows the circuit being considered; L_S and R_S are the inductance and resistance of the shield. The voltage V_N is equal to

$$V_N = j\omega M I_S. \tag{2-22}$$

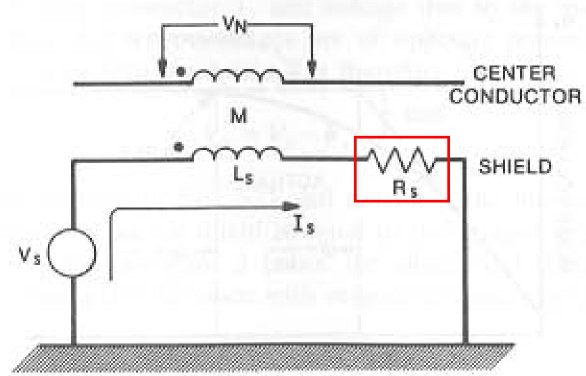

Figure 2-14. *Equivalent circuit of shielded conductor.*

The current I_S is equal to

$$I_S = \frac{V_S}{L_S}\left(\frac{1}{j\omega + R_S/L_S}\right). \tag{2-23}$$

Therefore

$$V_N = \left(\frac{j\omega M V_S}{L_S}\right)\left(\frac{1}{j\omega + R_S/L_S}\right). \tag{2-24}$$

Since $L_S = M$ (from Eq. 2-21),

$$V_N = \left(\frac{j\omega}{j\omega + R_S/L_S}\right)V_S. \tag{2-25}$$

A plot of Eq. 2-25 is shown in Fig. 2-15. The break frequency for this curve is defined as the shield cutoff frequency (ω_c) and occurs at

$$\omega_c = \frac{R_S}{L_S}, \quad \text{or} \quad f_c = \frac{R_S}{2\pi L_S}. \tag{2-26}$$

The noise voltage induced into the center conductor is zero at dc and increases to almost V_S at a frequency of $5R_S/L_S$ rad/s. Therefore, if shield current is allowed to flow, a voltage is induced into the center conductor that nearly equals the shield voltage at frequencies greater than five times the shield cutoff frequency.

This is a very important property of a conductor inside a shield. Measured values of the shield cutoff frequency and five times this frequency are tabulated in Table 2-1 for various cables. For most cables, five times the shield cutoff frequency is near the high end of the audio-frequency band. The aluminum-foil-shielded cable listed has a much higher shield cutoff

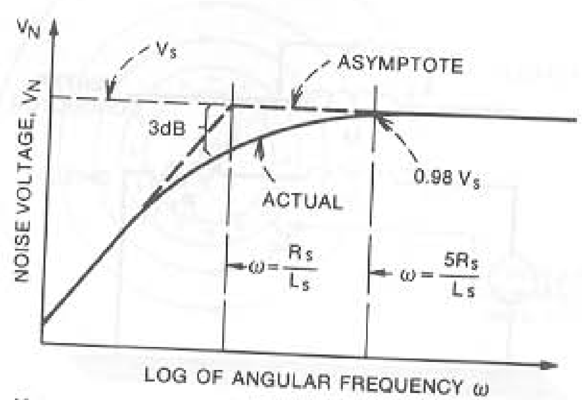

Figure 2-15. *Noise voltage in center conductor of coaxial cable due to shield current.*

Table 2-1 Measured Values of Shield Cutoff Frequency (f_c)

Cable	Impedance (Ω)	Cutoff Frequency (kHz)	Five Times Cutoff Frequency (kHz)	Remarks
Coaxial cable				
RG-6A	75	0.6	3.0	Double shielded
RG-213	50	0.7	3.5	
RG-214	50	0.7	3.5	Double shielded
RG-62A	93	1.5	7.5	
RG-59C	75	1.6	8.0	
RG-58C	50	2.0	10.0	
Shielded twisted pair				
754E	125	0.8	4.0	Double shielded
24 Ga.	—	2.2	11.0	
22 Ga.[a]	—	7.0	35.0	Aluminum-foil shield
Shielded single				
24 Ga.	—	4.0	20.0	

[a]One pair out of an 11-pair cable (Belden 8775).

frequency than any other. This is due to the increased resistance of its thin aluminum-foil shield.

Magnetic Coupling—Open Wire to Shielded Conductor

Figure 2-16 shows the magnetic couplings that exist when a nonmagnetic shield is placed around conductor 2 and the shield is grounded at both ends. In this figure the shield conductor is shown separated from conductor 2 to simplify the drawing. Since the shield is grounded at both ends, the shield current flows and induces a voltage into conductor 2. Therefore there are two components of the voltage induced into conductor 2: voltage due to direct induction from conductor 1, and voltage due to the induced shield current. Note that these two voltages are of opposite polarity. The total noise voltage induced into conductor 2 is therefore

$$V_N = V_2 - V_c. \tag{2-27}$$

If we use the identity of Eq. 2-21 and note that the mutual inductance M_{1S} from conductor 1 to the shield is equal to the mutual inductance M_{12} from conductor 1 to conductor 2 (since the shield and conductor 2 are located in the same place in space with respect to conductor 1), Eq. 2-27 becomes

$$V_N = j\omega M_{12} I_1\left[\frac{R_S/L_S}{j\omega + R_S/L_S}\right]. \tag{2-28}$$

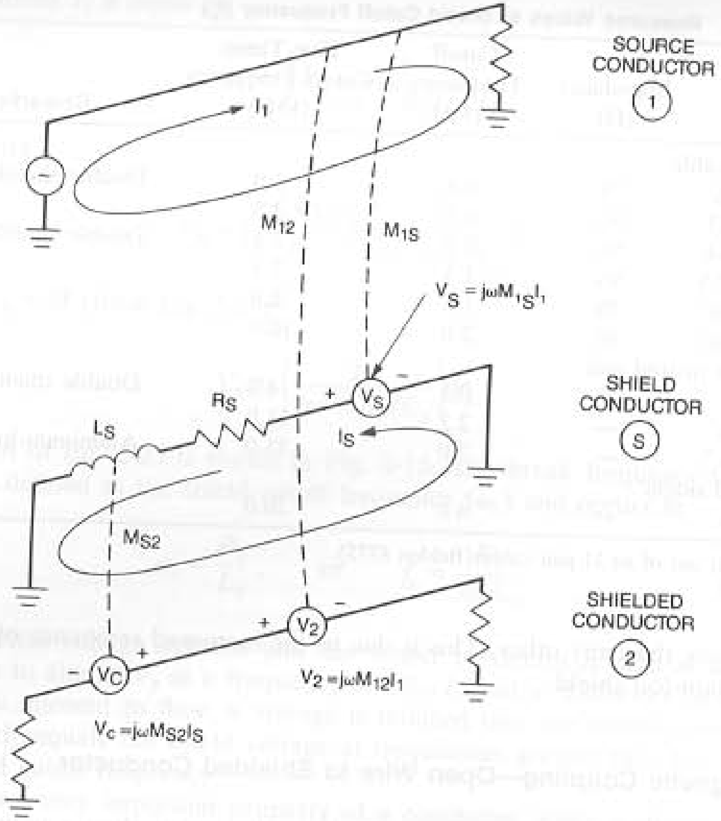

Figure 2-16. *Magnetic coupling to a shielded cable with the shield grounded at both ends.*

If ω is small in Eq. 2-28, the term in brackets equals 1, and the noise voltage is the same as for the unshielded cable. If ω is large, Eq. 2-28 reduces to

$$V_N = M_{12} I_1 \left(\frac{R_S}{L_S}\right). \tag{2-29}$$

Equation 2-28 is plotted in Fig. 2-17. At low frequencies the noise pickup in the shielded cable is the same as for an unshielded cable; however, at frequencies above the shield cutoff frequency the pickup voltage stops increasing and remains constant. The shielding effectiveness (shown cross-hatched in Fig. 2-17) is therefore equal to the difference between the curve for the unshielded cable and for the shielded cable.

Figure 2-18 shows a transformer analogy equivalent circuit for the configuration of Fig. 2-16. As can be seen, the shield acts as a shorted turn in the transformer to short out the voltage in winding 2.

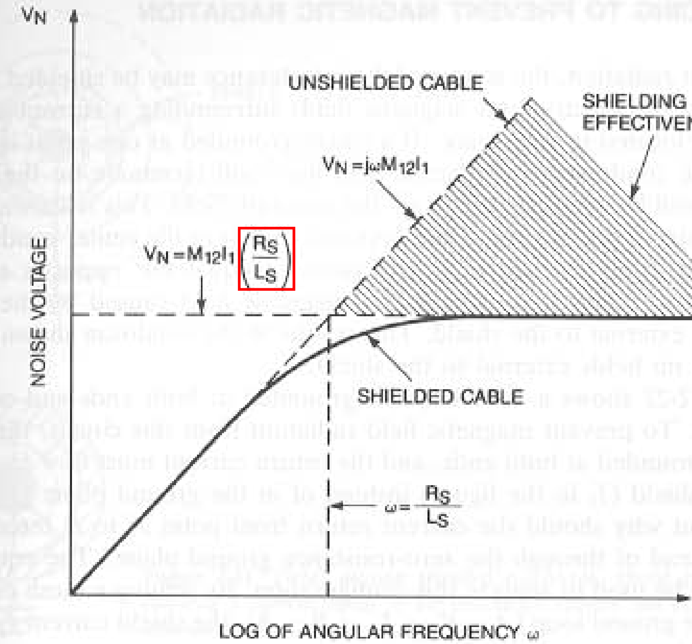

Figure 2-17. *Magnetic field coupled noise voltage for an unshielded and shielded cable (shield grounded at both ends) versus frequency.*

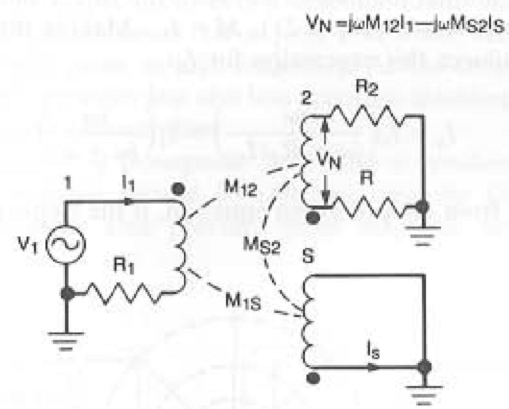

Figure 2-18. *Transformer analogy of magnetic field coupling to a shielded cable when shield is grounded at both ends (M_{S2} is much larger than M_{12} or M_{1S}).*

SHIELDING TO PREVENT MAGNETIC RADIATION

To prevent radiation, the source of the interference may be shielded. Figure 2-19 shows the electric and magnetic fields surrounding a current-carrying conductor located in free space. If a shield grounded at one point is placed around the conductor, the electric field lines will terminate on the shield, but there will be very little effect on the magnetic field. This is shown in Fig. 2-20. If a shield current equal and opposite to that in the center conductor is made to flow on the shield, it generates an equal but opposite external magnetic field. This field cancels the magnetic field caused by the center conductor external to the shield. This results in the condition shown in Fig. 2-21, with no fields external to the shield.

Figure 2-22 shows a circuit that is grounded at both ends and carries a current I_1. To prevent magnetic field radiation from this circuit, the shield must be grounded at both ends, and the return current must flow from A to B in the shield (I_S in the figure) instead of in the ground plane (I_G in the figure). But why should the current return from point A to B through the shield instead of through the zero-resistance ground plane? The equivalent circuit can be used to analyze this configuration. By writing a mesh equation around the ground loop ($A - R_S - L_S - B - A$), the shield current I_S can be determined:

$$0 = I_S(j\omega L_S + R_S) - I_1(j\omega M), \tag{2-30}$$

where M is the mutual inductance between the shield and center conductor and as previously shown (Eq. 2-21), $M = L_S$. Making this substitution and rearranging produces this expression for I_S:

$$I_S = I_1\left(\frac{j\omega}{j\omega + R_S/L_S}\right) = I_1\left(\frac{j\omega}{j\omega + \omega_c}\right). \tag{2-31}$$

As can be seen from the preceding equation, if the frequency is much above

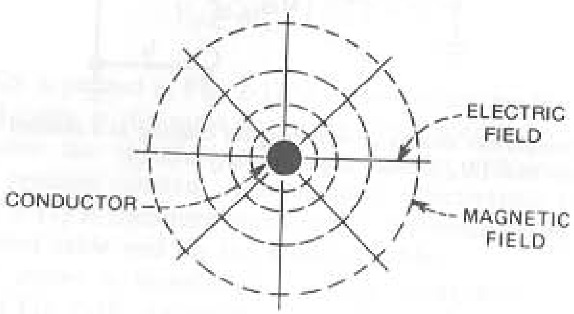

Figure 2-19. *Fields around a current-carrying conductor.*

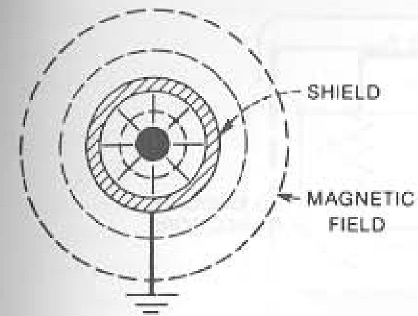

Figure 2-20. *Fields around shielded conductor; shield grounded at one point.*

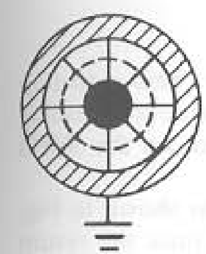

Figure 2-21. *Fields around shielded conductor; shield grounded and carrying a current equal to the conductor current but in the opposite direction.*

the shield cutoff frequency ω_c, the shield current approaches the center conductor current. Because of the mutual inductance between the shield and center conductor, a coaxial cable acts a common-mode choke (see Chapter 3), and the shield provides a return path with lower total circuit inductance than the ground plane at high frequency. As the frequency decreases below $5\omega_c$, the cable provides less and less magnetic shielding as more of the current returns via the ground plane.

To prevent radiation of a magnetic field from a conductor grounded at both ends, the conductor should be shielded, and the shield should be grounded at both ends. This provides good magnetic field shielding at

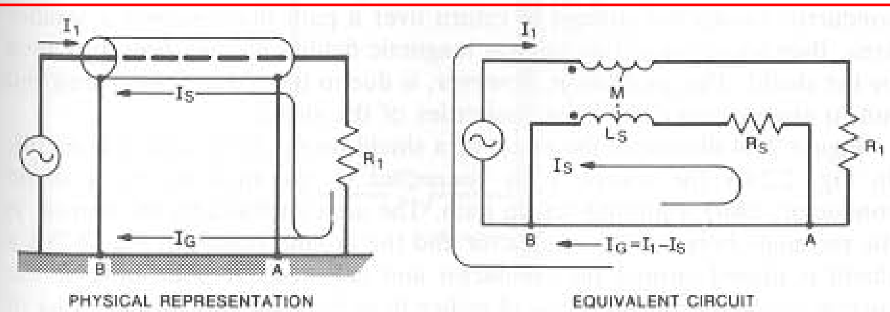

Figure 2-22. *Division of current between shield and ground plane.*

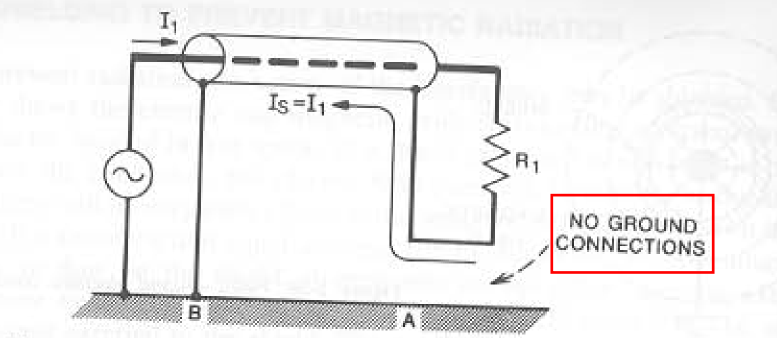

Figure 2-23. *Without ground at far end, all return current flows through shield.*

frequencies considerably above the shield cutoff frequency. This reduction in the radiated magnetic field is not because of the magnetic shielding properties of the shield as such. Rather, the return current on the shield generates a field that cancels the conductor's field.

If the ground is removed from one end of the *circuit*, as shown in Fig. 2-23, then the shield should not be grounded at that end since the return current must now all flow on the shield. This is true especially at frequencies less than the shield cutoff frequency. Grounding both ends of the shield, in this case, reduces the shielding since some current would return via the ground plane.

SHIELDING A RECEPTOR AGAINST MAGNETIC FIELDS

The best way to protect against magnetic fields at the receptor is to decrease the area of the receptor loop. The area of interest is the total area enclosed by current flow in the receptor circuit. An important consideration is the path taken by the current in returning to the source. Quite often the current returns by a path other than the one intended by the designer, and therefore the area of the loop changes. If a nonmagnetic shield placed around a conductor causes the current to return over a path that encloses a smaller area, then some protection against magnetic fields will have been provided by the shield. This protection, however, is due to the reduced loop area and not to any magnetic shielding properties of the shield.

Figure 2-24 illustrates the effect of a shield on the loop area of a circuit. In Fig. 2.24A the source V_s is connected to the load R_L by a single conductor, using a ground return path. The area enclosed by the current is the rectangle between the conductor and the ground plane. In Fig. 2-24B a shield is placed around the conductor and grounded at both ends. If the current returns through the shield rather than the ground plane, the area of the loop is decreased, and a degree of magnetic protection is provided. The

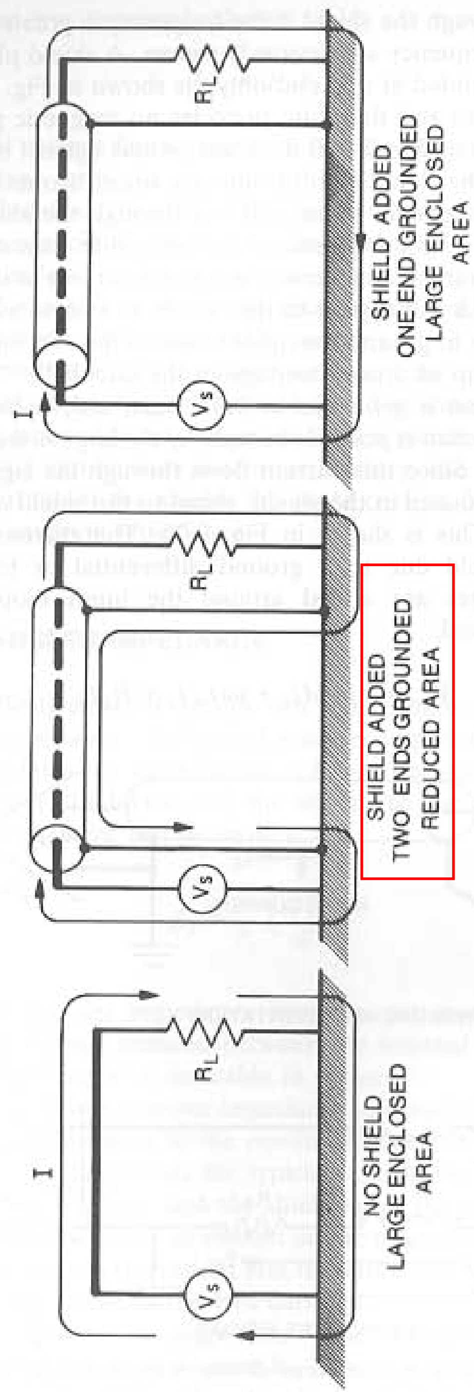

Figure 2-24. *Effect of shield on receptor loop area.*

current returns through the shield if the frequency is greater than five times the shield cutoff frequency as previously shown. A shield placed around the conductor and grounded at one end only, as shown in Fig. 2-24C, does not change the loop area and therefore provides no magnetic protection.

The arrangement of Fig. 2-24B does not protect against magnetic fields at frequencies below the shield cutoff frequency since then most of the current returns through the ground plane and not through the shield. This circuit should be avoided at low frequencies for two other reasons: (1) since the shield is one of the circuit conductors, any noise current in it will produce an *IR* drop in the shield and appear to the circuit as a noise voltage, and (2) if there is a difference in ground potential between the two ends of the shield, this too will show up as a noise voltage in the circuit.*

Whenever a circuit is grounded at both ends, only a limited amount of magnetic field protection is possible because of the large noise current induced in the ground loop. Since this current flows through the signal conductor, a noise voltage is produced in the shield, equal to the shield current times the shield resistance. This is shown in Fig. 2-25. The current I_S is the noise current in the shield due to a ground differential or to external noise coupling. If voltages are added around the input loop, the following expression is obtained:

$$V_{IN} = -j\omega M I_S + j\omega L_S I_S + R_S I_S \,. \tag{2-32}$$

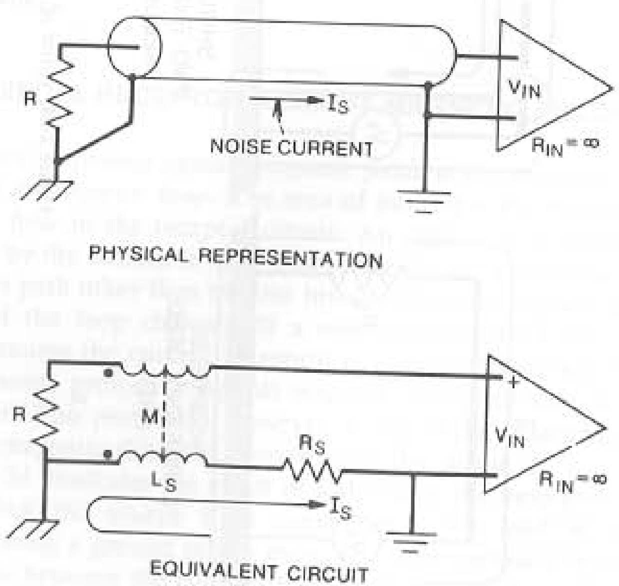

Figure 2-25. *Effect of noise current flowing in the shield of a coaxial cable.*

*See Chapter 3, for further discussion of a shielded cable grounded at both ends.

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \,.$$

Since $L_S = M$, as was previously shown

$$V_{IN} = R_S I_S \,. \tag{2-33}$$

Whenever shield current flows, a noise voltage is produced in the shield due to the $I_S R_S$ voltage drop.

Even if the shield is grounded at only one end, shield noise currents may still flow due to capacitive coupling to the shield. *Therefore, for maximum noise protection at low frequencies, the shield should not be one of the signal conductors, and one end of the circuit must be isolated from ground.*

At high frequencies a coaxial cable contains three isolated conductors: (1) the center conductor, (2) the inner surface of the shield conductor, and (3) the outer surface of the shield conductor. The inner and outer surfaces of the shield are isolated from each other by skin effect. Therefore the noise coupling discussed earlier does not occur; because the signal current flows on the inside surface of the shield, the noise current flows on the outside, and there is no common impedance.

SHIELD TRANSFER IMPEDANCE

The shielding effectiveness of a cable can also be expressed in terms of the shield transfer impedance. The shield transfer impedance is a property of the shield that relates the open circuit voltage (per unit length) developed between the center conductor and the shield, to the shield current. The shield transfer impedance can be written as

$$Z_T = \frac{1}{I_S}\left(\frac{dV}{dl}\right), \tag{2-34}$$

where Z_T is the transfer impedance in ohms per meter, I_S is the shield current, V is the voltage induced between the internal conductors and the shield, and l is the length of the cable in meters.

At low frequencies the transfer impedance is equal to the dc resistance of the shield. This is equivalent to the results obtained in Eq. 2-33. At higher frequencies (above a megahertz for typical cables), the transfer impedance decreases due to skin effect, and the shielding of the cable increases. Skin effect causes the noise current to remain on the outside surface of the shield, and the signal current on the inside, and therefore it eliminates the common impedance coupling between the two currents.

Figure 2-26 is a plot of the magnitude of the transfer impedance (normalized to the value of the dc resistance R_{dc}) for a *solid* tubular shield. If the shield is braided, not solid, the transfer impedance will increase with frequency above about 1 MHz, as shown in Fig. 2-32.

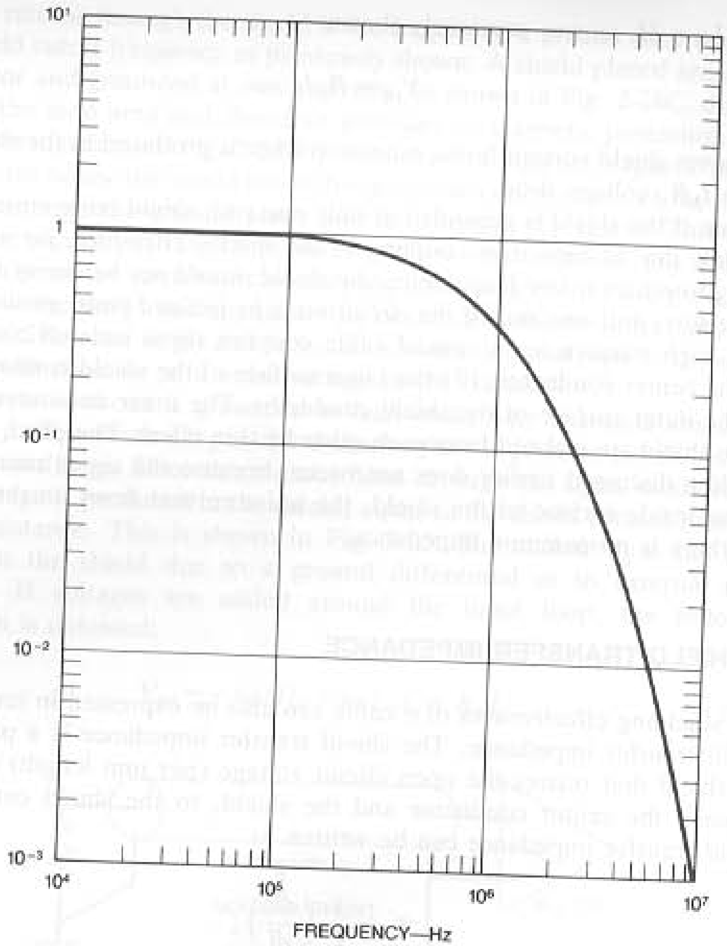

Figure 2-26. *Magnitude of normalized transfer impedance for a* solid *shield.*

EXPERIMENTAL DATA

The magnetic field shielding properties of various cable configurations were measured and compared. The test setup is shown in Fig. 2-27, and the test results are tabulated in Figs. 2-28 and 2-29. The frequency (50 kHz) is greater than five times the shield cutoff frequency for all the cables tested. The cables shown in Figs. 2-28 and 2-29 represent tests cables shown as $L2$ in Fig. 2-27.

In circuits A through F (Fig. 2-28), both ends of the circuit are grounded. They provide much less magnetic field attenuation than do circuits G through K (Fig. 2-29), where only one end is grounded.

Circuit A in Fig. 2-28 provides essentially no magnetic field shielding. The

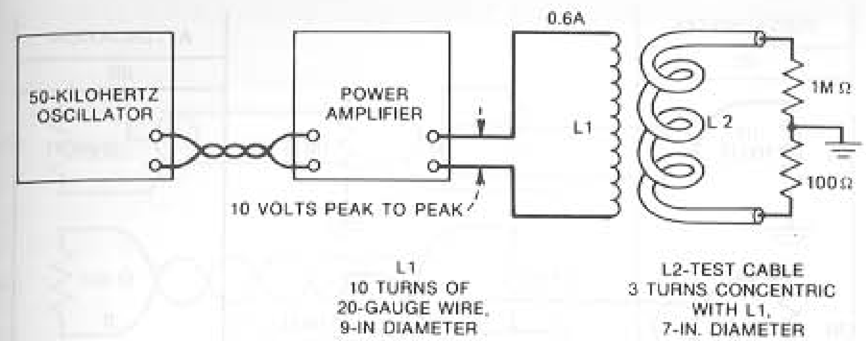

Figure 2-27. *Test setup of inductive coupling experiment.*

actual noise voltage measured across the one megohm resistor in this case was 0.8 V. The pickup in configuration A is used as a reference and is called 0 dB, to compare the performance of all the other circuits. In circuit B, the shield is grounded at one end; this has no effect on the magnetic shielding. Grounding the shield at both ends as in configuration C provides some magnetic field protection because the frequency is above the shield cutoff frequency. The protection would be even greater if it were not for the ground loop formed by grounding both ends of the circuit. The magnetic field induces a large noise current into the low-impedance ground loop consisting of the cable shield and the two ground points. The shield noise current then produces a noise voltage in the shield, as was shown in the preceding section.

Use of a twisted pair as in circuit D should provide much greater magnetic field noise reduction, but its effect is defeated by the ground loop formed by circuit grounds at both ends. This can clearly be seen by comparing the attenuation of circuit H to that of circuit D. Adding a shield with one end grounded, to the twisted pair as in E has no effect. Grounding the shield at both ends as in F provides additional protection, since the low-impedance shield shunts some of the magnetically induced ground-loop current away from the signal conductors. In general, however, none of the circuit configurations in Fig. 2-28 provide good magnetic field protection because of the ground loops. If the circuit must be grounded at both ends, configurations C or F should be used.

Circuit G shows a significant improvement in magnetic field shielding. This is due to the very small loop area formed by the coaxial cable and the fact that there is no ground loop to defeat the shielding. The coax provides a very small loop area since the shield can be represented by an equivalent conductor located on its center axis. This effectively locates the shield at or very near the axis of the center conductor.

It was expected that the twisted pair of circuit H would provide considerably more shielding than the 55 dB shown. The reduced shielding is due to

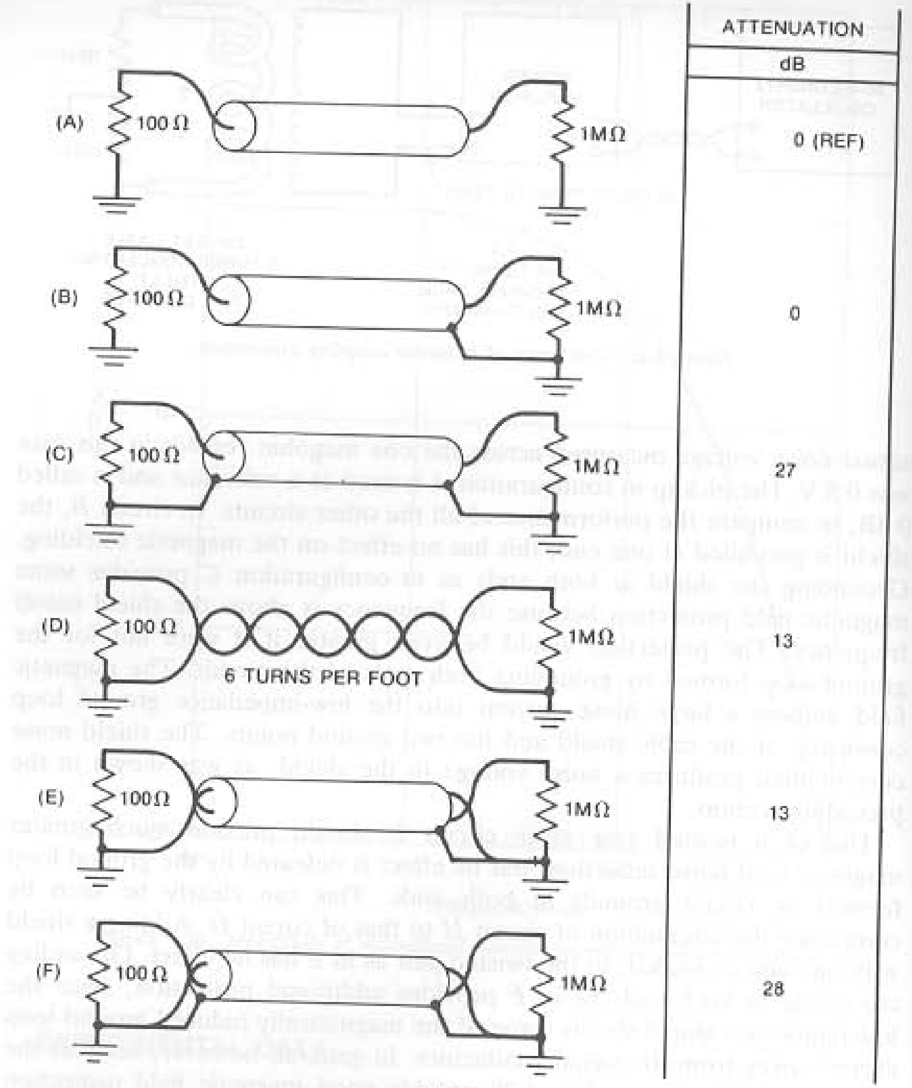

Figure 2-28. *Results of inductive coupling experiment; all circuits grounded at both ends.*

the fact that some electric field coupling is now beginning to show up. This can be seen in circuit *I*, where attenuation increases to 70 dB by placing a shield around the twisted pair. The fact that attenuation in circuit *G* is better than in *I* indicates that in this case the particular coaxial cable presents a smaller loop area to the magnetic field than does the twisted pair. This, however, is not necessarily true in general. Increasing the number of turns

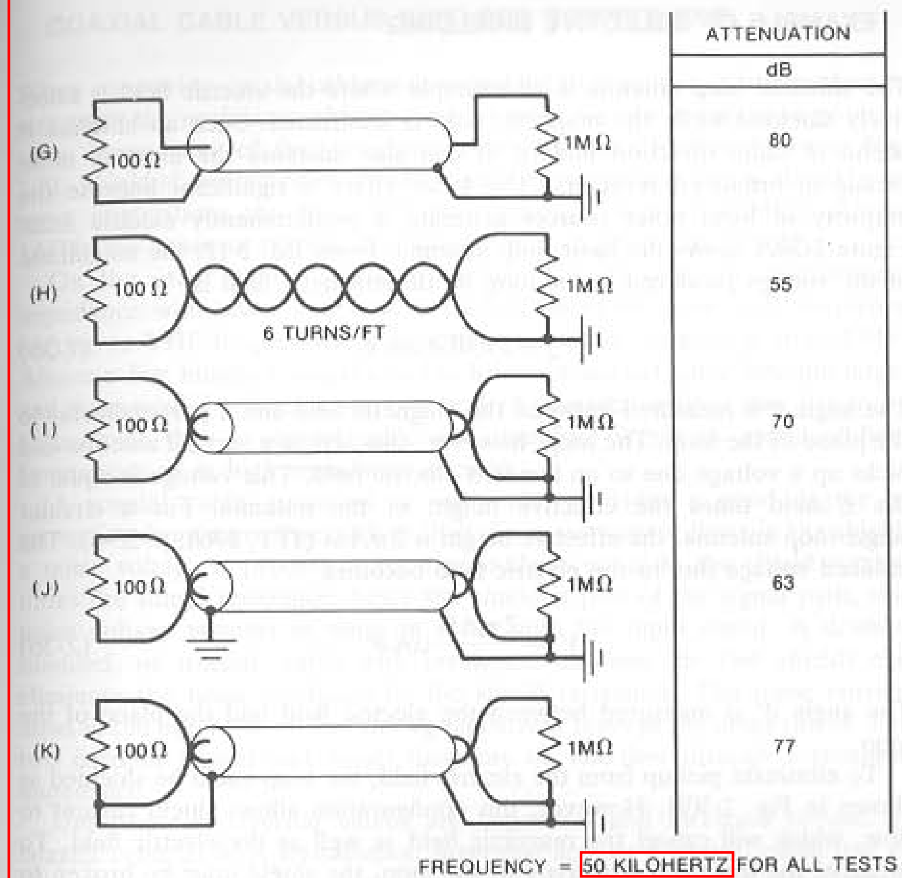

Figure 2-29. *Results of inductive coupling experiment; all circuits grounded at one end only.*

per foot for either of the twisted pairs (*H* or *I*) would reduce the pickup. In general, circuit *I* is preferred to circuit *G* for low-frequency magnetic shielding since in *I* the shield is not also one of the signal conductors.

Grounding both ends of the shield as in circuit *J* decreases the shielding slightly. This is due to the high shield current in the ground loop formed by the shield inducing unequal voltages in the two center conductors. Circuit *K* provides more shielding than *I* since it combines the features of the coax *G* with those of the twisted pair *I*. Circuit *K* is not normally desirable since any noise voltages or currents that do get on the shield can flow down the signal conductor. It is almost always better to connect the shield and signal conductors together at just one point. That point should be such that noise current from the shield does not have to flow down the signal conductor to get to ground.

EXAMPLE OF SELECTIVE SHIELDING

The shielded loop antenna is an example where the electric field is selectively shielded while the magnetic field is unaffected. Such an antenna is useful in radio direction finders. It can also decrease the antenna noise pickup in broadcast receivers. The latter effect is significant because the majority of local noise sources generate a predominantly electric field. Figure 2-30A shows the basic loop antenna. From Eq. 2-12, the magnitude of the voltage produced in the loop by the magnetic field is

$$V_m = 2\pi f B A \cos\theta \,. \tag{2-35}$$

The angle θ is measured between the magnetic field and a perpendicular to the plane of the loop. The loop, however, also acts as a vertical antenna and picks up a voltage due to an incident electric field. This voltage is equal to the E field times the effective height of the antenna. For a circular single-loop antenna, the effective height is $2\pi A/\lambda$ (ITT, 1968, p. 25-6). The induced voltage due to the electric field becomes

$$V_e = \frac{2\pi A E}{\lambda} \cos\theta' \,. \tag{2-36}$$

The angle θ' is measured between the electric field and the plane of the loop.

To eliminate pickup from the electric field, the loop could be shielded as shown in Fig. 2-30B. However, this configuration allows shield current to flow, which will cancel the magnetic field as well as the electric field. To preserve the magnetic sensitivity of the loop, the shield must be broken to prevent the flow of shield current. This can be done as shown in Fig. 2-30C by breaking the shield at the top. The resulting antenna responds only to the magnetic field component of an applied wave.

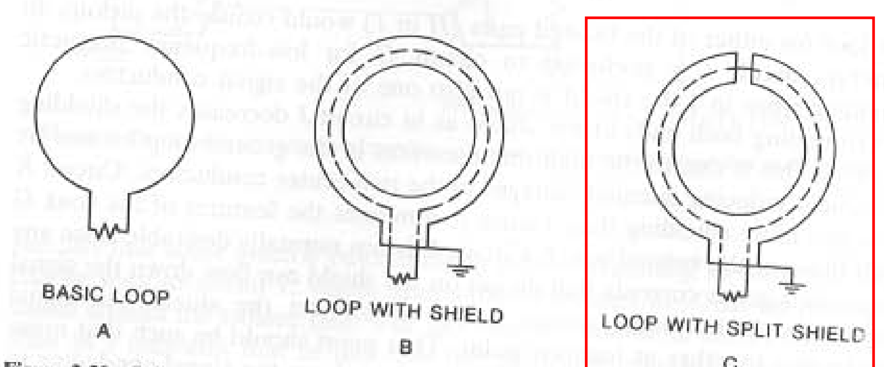

Figure 2-30. *Split shield on loop antenna selectively reduces electric field while passing magnetic field.*

COAXIAL CABLE VERSUS SHIELDED TWISTED PAIR

When comparing coaxial cable with a shielded twisted pair, it is important to recognize the usefulness of both types of cable from a propagation point of view, irrespective of their shielding characteristics. This is shown in Fig. 2-31. Shielded twisted pairs are very useful at frequencies below 100 kHz. In some applications the frequency may reach as high as 10 MHz. Above 1 MHz the losses in the shielded twisted pair increase considerably.

On the other hand, coaxial cable has a more uniform characteristic impedance with lower losses. It is useful, therefore, from zero frequency (dc) up to VHF frequencies, with some applications extending up to UHF. Above a few hundred megahertz the losses in coaxial cable become large, and waveguide becomes more practical. A shielded twisted pair has more capacitance than a coaxial cable and therefore is not as useful at high frequencies or in high-impedance circuits.

A coaxial cable grounded at one point provides a good degree of protection from capacitive pickup. But if a noise current flows in the shield, a noise voltage is produced. Its magnitude is equal to the shield current times the shield resistance. Since the shield is part of the signal path, this noise voltage appears as noise in series with the input signal. A double-shielded, or triaxial, cable with insulation between the two shields can eliminate the noise produced by the shield resistance. The noise current flows in the outer shield, and the signal current flows in the inner shield. The two currents (signal and noise) therefore do not flow through a common impedance.

Unfortunately, triaxial cables are expensive and awkward to use. A coaxial cable at high frequencies, however, acts as a triaxial cable due to skin effect. For a typical shielded cable, skin effect becomes important at

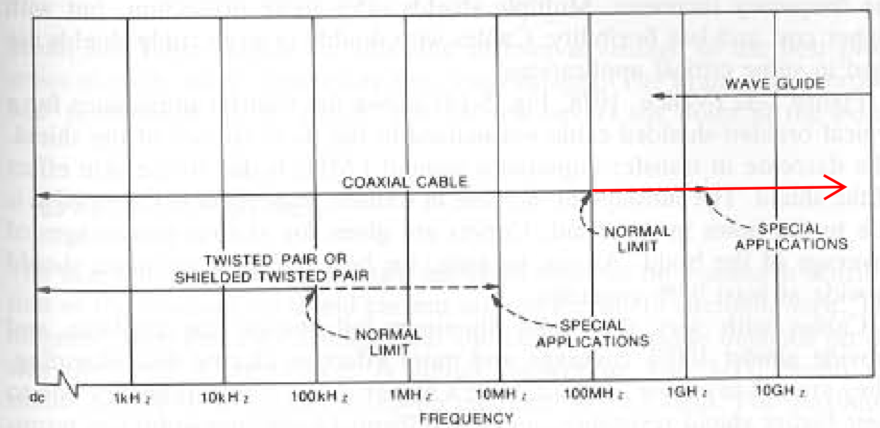

Figure 2-31. *Useful frequency range for various transmission lines.*

about 1 MHz. The noise current flows on the outside surface of the shield while the signal current flows on the inside surface. For this reason a coaxial cable is better for use at high frequencies.

A shielded twisted pair has characteristics similar to a triaxial cable and is not as expensive or awkward. The signal current flows in the two inner conductors, and any noise currents flow in the shield. Common-resistance coupling is eliminated. In addition any shield current is coupled equally into both inner conductors by mutual inductance, and the voltages therefore cancel.

An unshielded twisted pair, unless its terminations are balanced, provides very little protection against capacitive pickup, but it is very good for protection against magnetic pickup. The shielded twisted pair provides the best shielding for low-frequency signals, in which magnetic pickup is the major problem. The effectiveness of twisting increases as the number of twists per unit length increases.

BRAIDED SHIELDS

Most cables are actually shielded with braid rather than with a solid conductor. The advantages of braid are flexibility, durability, strength, and long flex life. Braids, however, typically provide only 60–98% coverage and are less effective as shields than solid conductors. Braided shields usually provide just slightly reduced electric field shielding (except at UHF frequencies) but greatly reduced magnetic field shielding. The reason is that braid distorts the uniformity of the shield current. A braid is typically from 5 to 30 dB less effective than a solid shield for protecting against magnetic fields.

At higher frequencies the effectiveness of the braid decreases further. This is because the braid holes become larger compared to a wavelength, as the frequency increases. Multiple shields offer more protection, but with higher cost and less flexibility. Cables with double or even triple shields are used in some critical applications.

Figure 2-32 (Vance, 1978, Fig. 5-14) shows the transfer impedance for a typical braided-shielded cable normalized to the dc resistance of the shield. The decrease in transfer impedance around 1 MHz is due to the skin effect of the shield. The subsequent increase in transfer impedance above 1 MHz is due to the holes in the braid. Curves are given for various percentages of coverage of the braid. As can be seen, for best shielding the braid should provide at least 95% coverage.

Cables with very thin solid aluminum-foil shields are available and provide almost 100% coverage and more effective electric field shielding. They are not as strong as braid, have a higher shield cutoff frequency due to their higher shield resistance, and are difficult (if not impossible) to terminate properly. Shields are also available that combine a foil shield with a

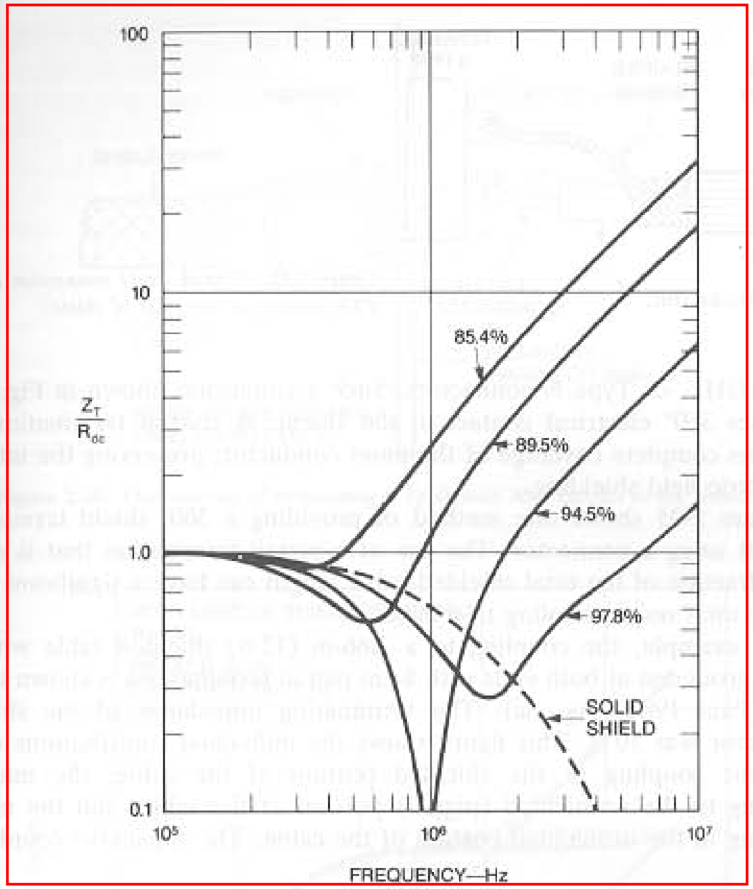

Figure 2-32. *Normalized transfer impedance of a braided-wire shield, as a function of percent braid coverage (from Vance, 1978, © Wiley).*

braid, and these shields are intended to take advantage of the best properties of each, while minimizing the disadvantages. The braid allows proper 360° termination of the shield, and the foil covers the holes in the braid.

EFFECT OF PIGTAILS

The magnetic shielding previously discussed depends on a uniform distribution of the longitudinal shield current around the shield circumference. The magnetic shielding effectiveness near the ends of the cable depends on the way the braid is terminated. A pigtail connection, Fig. 2-33, causes the shield current to be concentrated on one side of the shield. For maximum protection, the shield should be terminated uniformly around its cross section. This can be accomplished by using a coaxial connector such as the

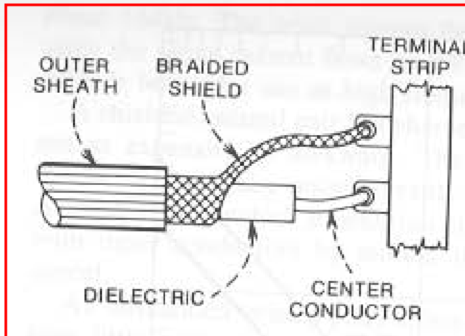

Figure 2-33. *Pigtail shield connection concentrates current on one side of shield.*

BNC, UHF, or Type N connectors. Such a connector, shown in Fig. 2-34, provides 360° electrical contact to the shield. A coaxial termination also provides complete coverage of the inner conductor, preserving the integrity of electric field shielding.

Figure 2-35 shows one method of providing a 360° shield termination without using a connector. The use of a pigtail termination that is only a small fraction of the total shielded cable length can have a significant effect on the total noise coupling to a cable.

For example, the coupling to a 3.66-m (12 ft) shielded cable with the shield grounded at both ends with 8-cm pigtail terminations is shown in Fig. 2-36 (Paul 1980, Fig. 8a). The terminating impedance of the shielded conductor was 50 Ω. This figure shows the individual contributions of the magnetic coupling to the shielded portion of the cable, the magnetic coupling to the unshielded (pigtail) portion of the cable, and the electric coupling to the unshielded portion of the cable. The capacitive coupling to

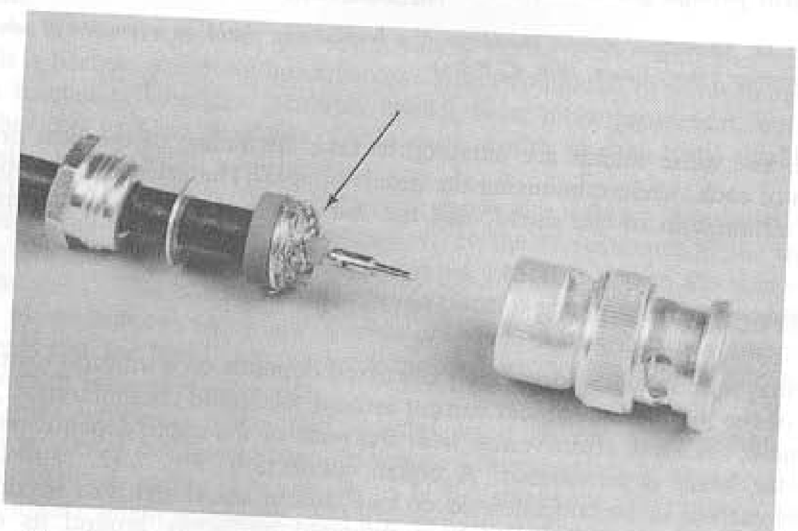

Figure 2-34. *Disassembled BNC connector showing a 360° contact to shield*

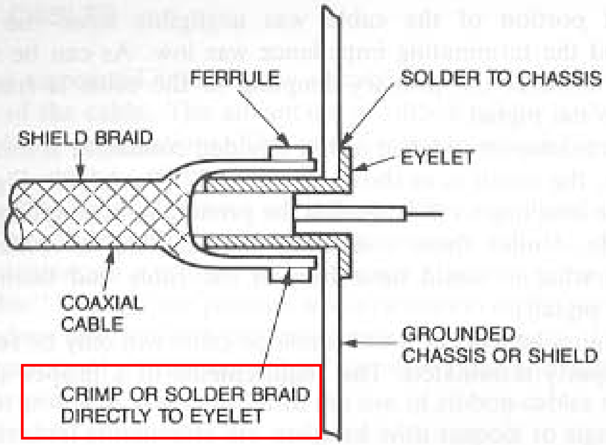

Figure 2-35. *One method of terminating a cable with 360° contact to the shield.*

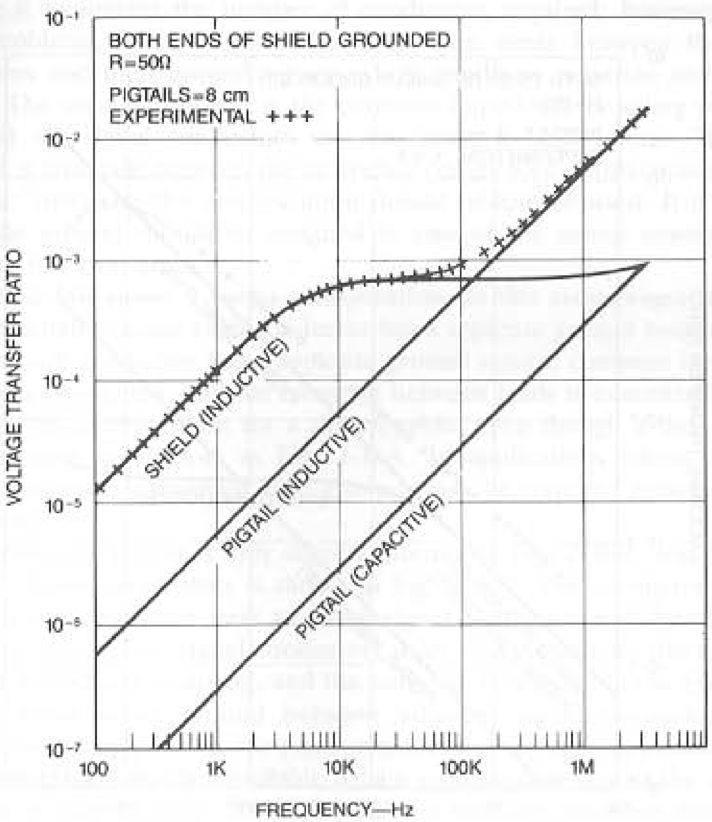

Figure 2-36. *Coupling to a 3.7-m shielded cable with an 8-cm pigtail termination. Circuit termination equals 50 Ω (from Paul, 1980, © IEEE).*

the shielded portion of the cable was negligible since the shield was grounded and the terminating impedance was low. As can be seen in Fig. 2-36, above 100 kHz the primary coupling to the cable is from inductive coupling into the pigtail.

If the terminating impedance of the shielded conductor is increased from 50 to 1000 Ω, the result is as shown in Fig. 2-37 (Paul 1980, Fig. 8b). Here the capacitive coupling to the pigtail is the predominant coupling mechanism above 10 kHz. Under these conditions the coupling at 1 MHz is 40 dB greater than what it would have been if the cable had been completely shielded (no pigtail).

The maximum benefits of a well-shielded cable will only be realized if the shield is properly terminated. The requirements of a proper shield termination are

1. A very low impedance ground connection
2. A 360° contact with the shield

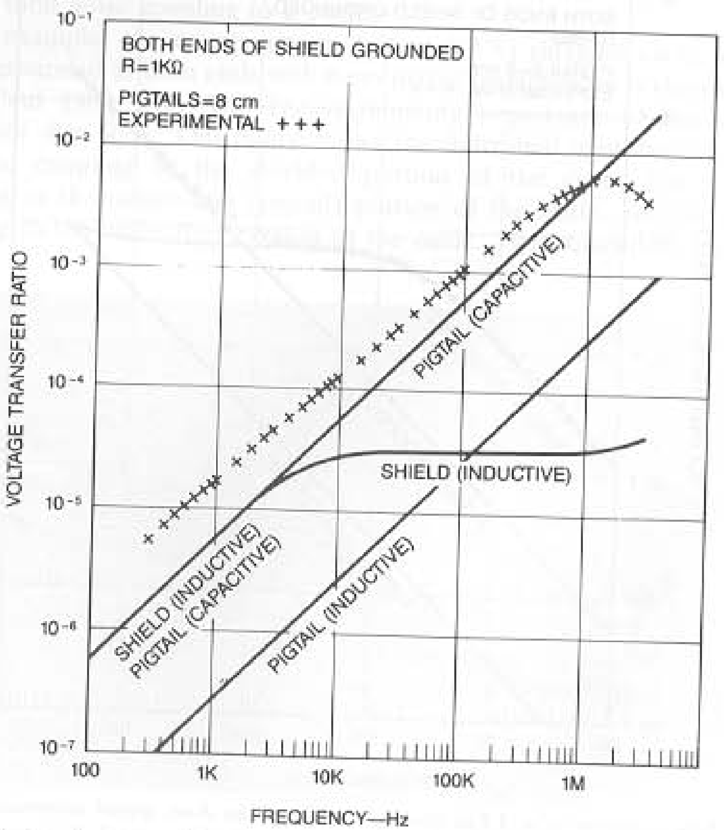

Figure 2-37. *Coupling to a 3.7-m shielded cable with an 8-cm pigtail termination. Circuit termination equals 1000 Ω (from Paul, 1980, © IEEE).*

RIBBON CABLES

A major cost associated with the use of cables is the expense related to the termination of the cable. The advantage of ribbon cables is that they allow low-cost multiple terminations. This is the primary reason for using ribbon cables.

Ribbon cables have a second advantage. They are "controlled cables" because the position and orientation of the wires within the cable is fixed, like the conductors on a printed wiring board. A normal wiring harness is a "random cable" because the position and orientation of the wires within the cable is random and varies from one harness to the next. Therefore the noise performance of a "random cable" can vary from one unit to the next.

The major problem associated with the use of ribbon cables relates to the way the individual conductors are assigned with respect to signal leads and grounds.

Figure 2-38A shows a ribbon cable where one conductor is a ground and all the remaining conductors are signal leads. This configuration is used because it minimizes the number of conductors required; however, it has three problems. First, it produces large loop areas between the signal conductors and their ground return, which results in radiation and susceptibility. The second problem is the common impedence coupling produced when all the signal conductors use the same ground return. The third problem is crosstalk between the individual conductors—both capacitive and magnetic; therefore this configuration should seldom be used. If it is used, the single ground should be assigned to one of the center conductors to minimize the loop areas.

Figure 2-38B shows a better configuration. In this arrangement the loop areas are small because each conductor has a separate ground return next to it. Since each conductor has a separate ground return, common impedance coupling is eliminated, and the crosstalk between leads is minimized. This is the preferred configuration for a ribbon cable, even though it does require twice as many conductors as Fig. 2-38A. In applications where crosstalk between cables is a problem, two grounds may be required between signal conductors.

A configuration that is only slightly inferior to Fig. 2-38B, and one that uses 25% fewer conductors is shown in Fig. 2-38C. This configuration also has a ground conductor next to every signal conductor and therefore has small loop areas. Two signal conductors share one ground, so there is some common impedance coupling, and the crosstalk is higher than in Fig. 2-38B because there is no ground between adjacent signal conductors. This configuration may provide adequate performance in some applications.

Ribbon cables are also available with a ground plane across the width of the cable as shown in Fig. 2-38D. In this case the loop areas are determined by the spacing between the signal conductor and the ground plane under it. Since this dimension is less than the lead-to-lead spacing in the cable, the loop areas are smaller than in the alternate ground configuration of Fig.

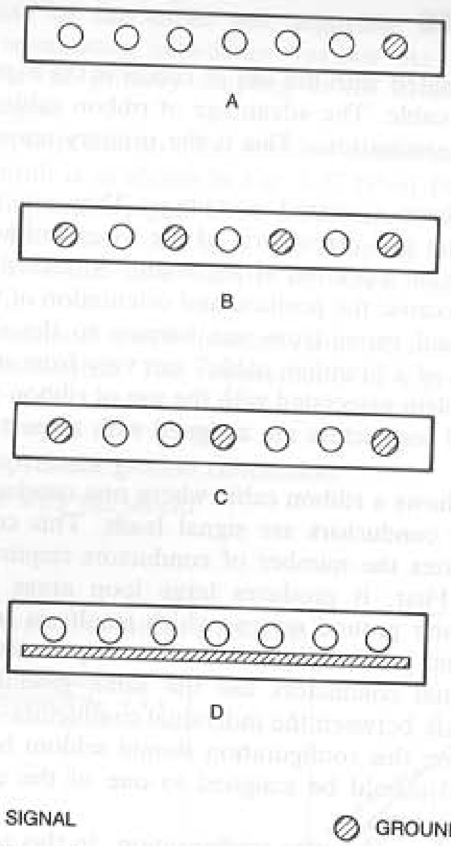

Figure 2-38. *Ribbon cable configurations: (A) single ground; (B) alternate ground; (C) ground/signal/signal/ground; (D) signal over ground plane.*

2-38B. If allowed to do so, the ground current will flow under the signal conductor, for the same reason that the current returned on the shield in Fig. 2-22. However, unless the cable is terminated with a full-width contact to the ground plane, the return currents will be forced out from under the signal leads, and the loop area will increase. Because it is difficult to terminate this kind of cable properly, it is not often recommended.

Shielded ribbon cables are also available; however, unless the shield is properly terminated with a 360° connection (a difficult thing to do), their effectivenesss is considerably reduced. The effect of shield termination on the radiation from ribbon cables was discussed by Palmgren (1981). Palmgren points out that the outside conductors in a shielded ribbon cable are not as well shielded as the conductors located closer to the center of the cable (typically 7 dB less shielding). This effect is due to the nonuniformity of the shield current at the outside edge of the shield. Therefore critical signals should not be placed on the outside conductors of shielded ribbon cables.

ELECTRICALLY LONG CABLES

The preceding analysis has assumed that the cables were short compared to a wavelength. What this really means is that all the current on the cable is in phase. Under these circumstances the theory predicts that both the electric and the magnetic field coupling increase with frequency indefinitely. In practice, however, the coupling levels off above some frequency.

As cables approach a quarter-wavelength in length, some of the current in the cable is out of phase. When the cable is a half-wavelength long the out-of-phase currents will cause the external coupling to be zero due to cancellation of effects. This does not alter the dependence of the coupling on the various other parameters of the problem; it only changes the numerical result. Therefore the parameters that determine the coupling remain the same, regardless of the length of the cables.

Figure 2-39 shows the coupling between two cables with and without the assumption that the cables are short. The results are similar up to the point where the phasing effects start to occur, about one-tenth of a wavelength. Above this point the actual coupling decreases because the current is not all in phase, whereas the short cable approximation predicts an increase in the

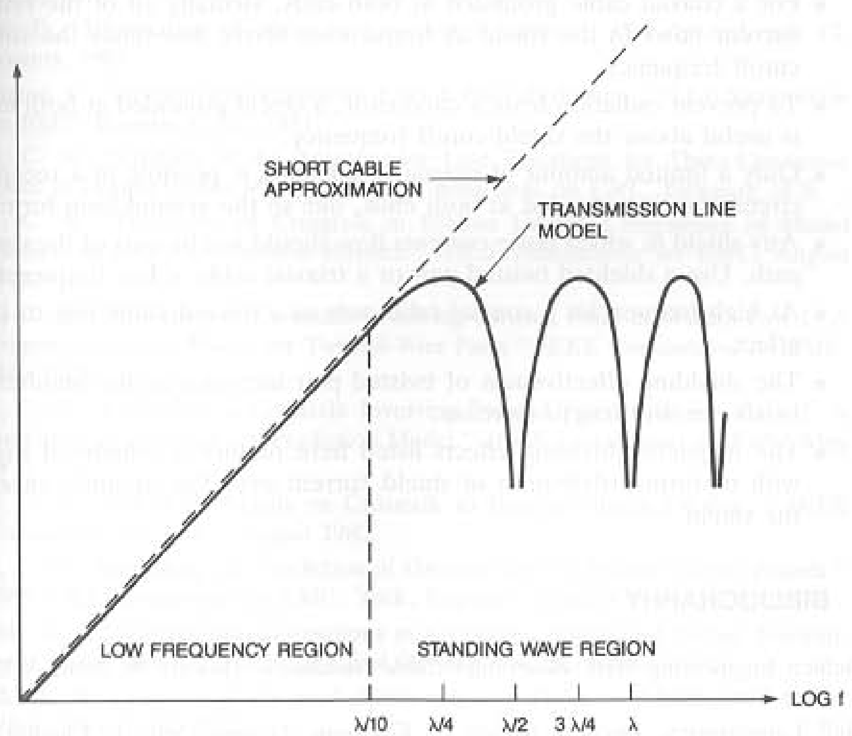

Figure 2-39. *Electric field coupling between cables using the short cable approximations and the transmission line model.*

coupling. If the rise in coupling predicted by the short cable approximation is truncated at a quarter-wavelength, it provides an approximation to the actual coupling. Note that the nulls and peaks produced by the phasing of the currents are not taken into account under these circumstances. However, unless one is planning to take advantage of these nulls and peaks in the design of equipment—a dangerous thing to do—their location is not important.

For more information on analyzing long cables see Paul (1979) and Smith (1977).

SUMMARY

- Electric fields are much easier to guard against than magnetic fields.
- The use of nonmagnetic shields around conductors provides no magnetic shielding.
- A shield grounded at one or more points shields against electric fields.
- The key to magnetic shielding is to decrease the area of the loop. To do that, use a twisted pair or a coaxial cable if the current return is through the shield instead of in the ground plane.
- For a coaxial cable grounded at both ends, virtually all of the return current flows in the shield at frequencies above five times the shield cutoff frequency.
- To prevent radiation from a conductor, a shield grounded at both ends is useful above the shield cutoff frequency.
- Only a limited amount of magnetic shielding is possible in a receptor circuit that is grounded at both ends, due to the ground loop formed.
- Any shield in which noise currents flow should not be part of the signal path. Use a shielded twisted pair or a triaxial cable at low frequencies.
- At high frequencies a coaxial cable acts as a triaxial cable due to skin effect.
- The shielding effectiveness of twisted pair increases as the number of twists per unit length increase.
- The magnetic shielding effects listed here require a cylindrical shield with uniform distribution of shield current over the circumference of the shield.

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3 GROUNDING

Grounding is one of the primary ways of minimizing unwanted noise and pickup. Proper use of grounding and cabling, in combination, can solve a large percentage of all noise problems. A good ground system must be designed; it is wishful thinking to expect a ground system to perform well if no thought has been given to its design. It is difficult to believe that expensive engineering time should be devoted to sorting out the minute details of circuit grounding, but in the end, not having to solve mysterious noise problems once the equipment is built and tested saves both money and time.

One advantage of a well-designed ground system is that it can provide protection against unwanted interference and emission, without any additional per-unit cost to the product. The only cost is the engineering time required to design the system. In comparison, an improperly designed ground system may be a primary source of interference and emission and therefore require considerable engineering time to eliminate the problem. Hence properly designed ground systems are truly cost-effective.

Grounds fall into two categories: (1) safety grounds and (2) signal grounds. If the ground is connected to the earth through a low impedance path, it may be called an earth ground. Safety grounds are usually at earth potential, whereas signal grounds may or may not be at earth potential. In many cases a safety ground is required at a point that is unsuitable for a signal ground, and this may complicate the noise problem.

SAFETY GROUNDS

Safety considerations require the chassis or enclosure for electric equipment to be grounded. Why this is so can be seen in Fig. 3-1. In the left-hand diagram Z_1 is the stray impedance between a point at potential V_1 and the chassis, and Z_2 is the stray impedance between the chassis and ground. The potential of the chassis is determined by impedances Z_1 and Z_2 acting as a voltage divider. The chassis potential is

$$V_{\text{chassis}} = \left(\frac{Z_2}{Z_1 + Z_2}\right) V_1 . \tag{3-1}$$

The chassis could be a relatively high potential and be a shock hazard, since its potential is determined by the relative values of the stray impedances

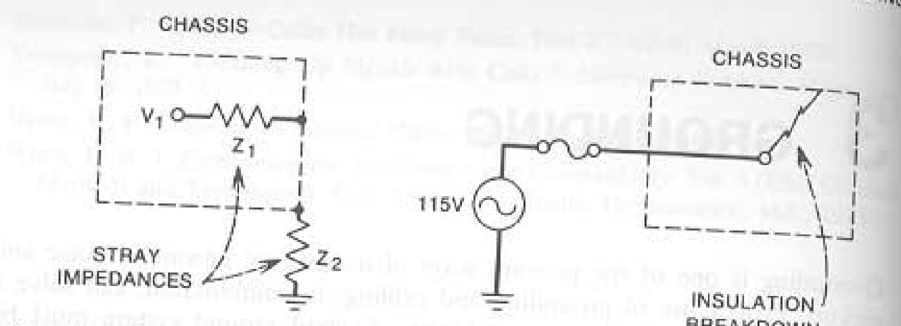

Figure 3-1. *Chassis should be grounded for safety. Otherwise, it may reach a dangerous voltage level through stray impedances (left) or insulation breakdown (right).*

over which there is very little control. If the chassis is grounded, however, its potential is zero since Z_2 becomes zero.

The right-hand diagram of Fig. 3-1 shows a second and far more dangerous situation: a fused ac line entering an enclosure. If there should be an insulation breakdown such that the ac line comes in contact with the chassis, the chassis would then be capable of delivering the full current capacity of the fused circuit. Anyone coming in contact with the chassis and ground would be connected directly across the ac power line. If the chassis is grounded, however, such an insulation breakdown will draw a large current from the ac line and cause the fuse to blow, thus removing the voltage from the chassis.

In the United States, ac power distribution and wiring standards are contained in the National Electrical Code. One requirement of this code specifies that 115-V ac power distribution in homes and buildings must be a three wire system, as shown in Fig. 3-2. Load current flows through the hot

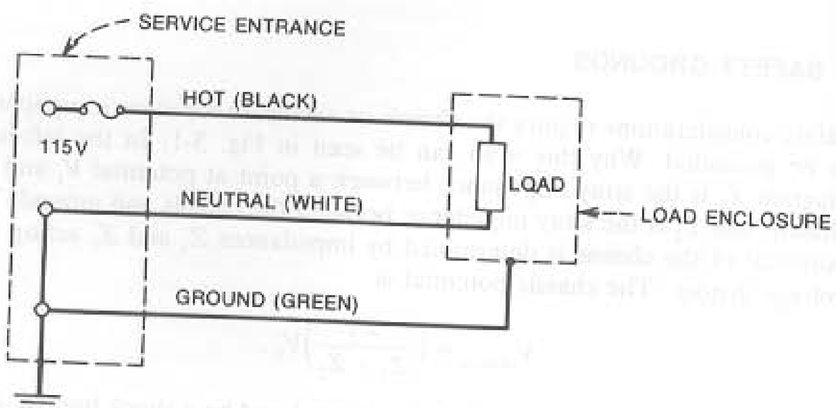

Figure 3-2. *Standard 115-V ac power distribution circuit has three leads.*

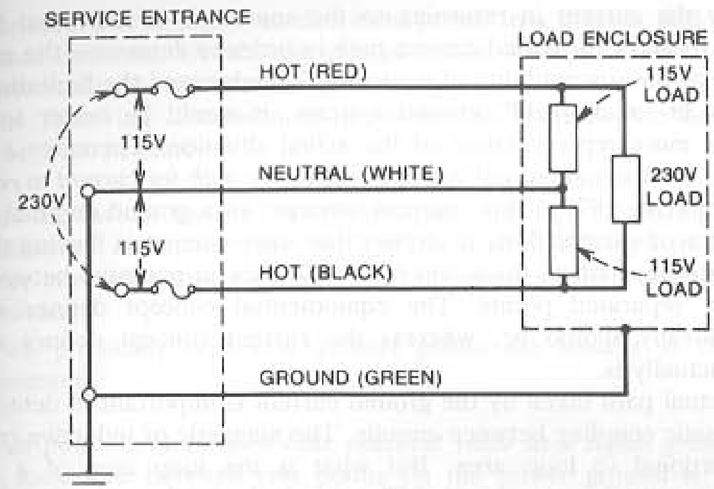

Figure 3-3. *Combination 115/230-V ac power distribution circuit has four leads.*

wire (black), which is fused, and returns through the neutral wire (white). In addition a safety ground wire (green) must be connected to all equipment enclosures and hardware. The only time the green wire carries current is during a fault, and then only momentarily until the fuse or breaker opens the circuit. Since no load current flows in the safety ground, it has no IR drop, and the enclosures connected to it are always at ground potential. The National Electrical Code specifies that the neutral and safety ground shall be connected together at only one point, and this point shall be at the main service entrance.* To do otherwise would allow some of the neutral current to return on the ground conductor. A combination 115/230-V system is similar, except an additional hot wire (red) is added, as shown in Fig. 3-3. If the load requires only 230 V the neutral (white) wire shown in Fig. 3-3 is not required.

SIGNAL GROUNDS

A ground is normally defined as an equipotential† point or plane that serves as a reference potential for a circuit or system. This definition, however, is not representative of practical ground systems because they are not equipotentials; also it does not emphasize the importance of the actual path

*Allowable exceptions are electric ranges, wall-mounted ovens, counter-mounted cooking units, and electric clothes dryers. For these appliances the case may be connected to ground via the neutral wire. See the National Electrical Code, paragraph 250-60, 1987.

†A point where the voltage does not change, regardless of the current applied to it or drawn from it.

taken by the current in returning to the source. It is important for the designer to know the actual current path in order to determine the radiated emission or the susceptibility of a circuit. To understand the limitations and problems of "real world" ground systems, it would be better to use a definition more representative of the actual situation. Therefore a better definition for a signal ground is *a low-impedance path for current to return to the source* (Ott, 1979). This "current concept" of a ground emphasizes the importance of current flow. It implies that since current is flowing through some finite impedance, there will be a difference in potential between two physically separated points. The equipotential concept defines what a ground ideally should be, whereas the current concept defines what a ground actually is.

The actual path taken by the ground current is important in determining the magnetic coupling between circuits. The magnetic or inductive coupling is proportional to loop area. But what is the loop area of a system containing multiple ground paths? The area is the total area enclosed by the actual current flow. An important consideration in determining this area is the ground path taken by the current in returning to the source. Often this is not the path intended by the designer.

In designing a ground it is important to ask: How does the current flow? The path taken by the ground current must be determined. Then, since any conductor-carrying current will have a voltage drop, the effect of this voltage drop on the performance of the other circuits connected to the ground must be considered.

The proper signal ground system is determined by the type of circuitry, the frequency of operation, the size of the system (self-contained or distributed), and other constraints, such as safety. No one ground system is appropriate for all applications.

Signal grounds usually fall into one of three categories: (1) single-point grounds, (2) multipoint grounds, and (3) hybrid grounds. Single-point and multipoint grounds are shown in Figs. 3-4 and 3-5, respectively. A hybrid ground is shown in Fig. 3-6. There are two subclasses of single-point grounds: those with series connections and those with parallel connections. The series connection is also called a common ground or daisy chain, and the parallel connection is called a separate ground system.

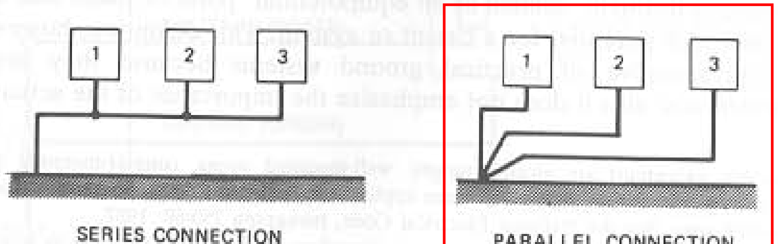

Figure 3-4. *Two types of single-point grounding connections.*

In general, it is desirable to distribute power in a manner that parallels the ground structure. Usually the ground system is designed first, and then the power is distributed in a similar manner.

In the following discussion of grounding techniques, two key points should be kept in mind:

1. All conductors have a finite impedance, generally consisting of both resistance and inductance. At 11 kHz, a straight length of 22-gauge wire one inch above a ground plane has more inductive reactance than resistance.
2. Two physically separated ground points are seldom at the same potential.

The ac power ground is of little practical value as a signal ground. The voltage measured between two points on the power ground is typically hundreds of millivolts, and in some cases many volts. This is excessive for low-level signal circuits. A single-point connection to the power ground is usually required for safety, however.

SINGLE-POINT GROUND SYSTEMS

With regard to noise, the most undesirable single-point ground system is the common ground system shown in Fig. 3-6. This is a series connection of all the individual circuit grounds. The resistances shown represent the impedance of the ground conductors, and I_1, I_2, and I_3 are the ground currents of

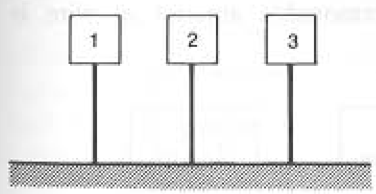

Figure 3-5. *Multipoint grounding connections.*

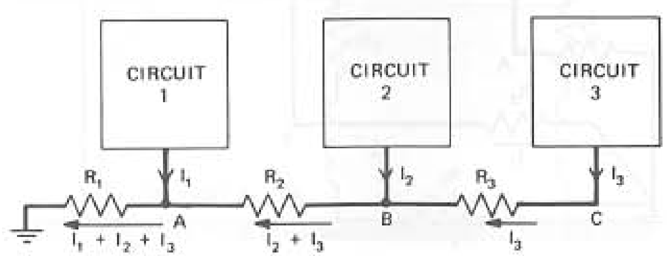

Figure 3-6. *Common ground system is a series ground connection and is undesirable from a noise standpoint but has the advantage of simple wiring.*

circuits 1, 2, and 3, respectively. Point A is not at zero potential but is at a potential of

$$V_A = (I_1 + I_2 + I_3)R_1 , \tag{3-2}$$

and point C is at a potential of

$$V_C = (I_1 + I_2 + I_3)R_1 + (I_2 + I_3)R_2 + I_3R_3 . \tag{3-3}$$

Although this circuit is the least desirable single-point grounding system, it is probably the most widely used because of its simplicity. For noncritical circuits it may be perfectly satisfactory. This system should not be used between circuits operating at widely different power levels, since the high-level stages produce large ground currents which in turn adversely affect the low-level stage. When this system is used, the most critical stage should be the one nearest the primary ground point. Note that point A in Fig. 3-6 is at a lower potential than point B or C.

The separate ground system (parallel connection) shown in Fig. 3-7 is the most desirable at low frequencies. That is because there is no cross coupling between ground currents from different circuits. The potentials at points A and C, for example, are as follows:

$$V_A = I_1R_1 , \tag{3-4}$$

$$V_C = I_3R_3 . \tag{3-5}$$

The ground potential of a circuit is now a function of the ground current and impedance of that circuit only. This system is mechanically cumbersome, however, since in a large system an unreasonable amount of wire is necessary.

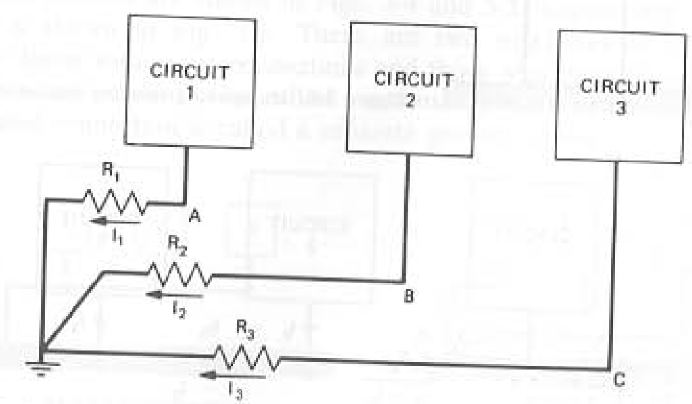

Figure 3-7. *Separate ground system is a parallel ground connection and provides good low-frequency grounding but is mechanically cumbersome.*

A limitation of the single-point ground system occurs at high frequencies, where the inductances of the ground conductors increase the ground impedance. At still higher frequencies the impedance of the ground wires can be very high if the length coincides with odd multiples of a quarter-wavelength. Not only will these grounds have large impedance, but they will also act as antennas and radiate noise. Ground leads should always be kept shorter than one-twentieth of a wavelength to prevent radiation and to maintain a low impedance. At high frequencies there is no such thing as a single-point ground.

MULTIPOINT GROUND SYSTEMS

The multipoint ground system is used at high frequencies and in digital circuitry to minimize the ground impedance. In this system, shown in Fig. 3-8, circuits are connected to the nearest available low-impedance ground plane, usually the chassis. The low ground impedance is due primarily to the lower inductance of the ground plane. The connections between each circuit and the ground plane should be kept as short as possible to minimize their impedance. In very high frequency circuits the length of these ground leads must be kept to a small fraction of an inch. Multipoint grounds should be avoided at low frequencies since ground currents from all circuits flow through a common ground impedance—the ground plane. At high frequencies, the common impedance of the ground plane can be reduced by silver plating the surface. Increasing the thickness of the ground plane has no effect on its high frequency impedance, since current flows only on the surface due to skin effect.

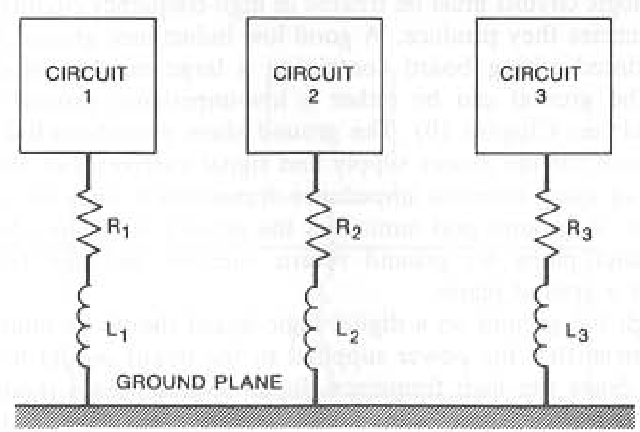

Figure 3-8. *Multipoint ground system is a good choice at frequencies above 10 MHz. Impedances R_1–R_3 and L_1–L_3 should be minimized.*

Normally at frequencies below one megahertz a single-point ground system is preferable; above 10 MHz, a multipoint ground system is best. Between 1 and 10 MHz a single point ground can usually be used, provided the length of the longest ground conductor is less than one-twentieth of a wavelength. If it is greater than one-twentieth of a wavelength, a multipoint ground system should be used.

Many ground system problems occur as the result of common impedance coupling. Common impedance coupling becomes a problem under one or more of the following conditions:

1. A high-impedance ground (usually too much inductance)
2. A large ground current (often due to power frequency currents or magnetic field pickup)
3. A very sensitive (low noise-margin) circuit connected to the ground

Single-point grounds overcome these problems by separating ground currents that are likely to interfere and forcing them to flow on different conductors. This is effective at low frequencies. However, the single current paths and long lead lengths increase the inductance which is detrimental at high frequencies. In addition at high frequencies single-point grounds are almost impossible to achieve because parasitic capacitance closes the ground loop.

Multipoint grounds overcome these problems by producing a very low ground impedance. The ground system is interconnected by a large number of parallel paths (a grid) or a solid metal plate (a plane). Multipoint grounds create ground loops that may be prone to magnetic field pickup. The obvious solution is to keep the area of the loops small by using a grid or plane and to avoid multipoint grounds with very low noise-margin (low millivolt or microvolts) circuits.

Digital logic circuits must be treated as high-frequency circuits, due to the high frequencies they produce. A good low inductance ground is necessary on any printed wiring board containing a large number of digital logic circuits. The ground can be either a low-impedance ground plane or a ground grid (see Chapter 10). The ground plane provides a low inductance ground return for the power supply and signal currents and allows for the possibility of using constant impedance transmission lines for signal interconnections. A ground grid minimizes the ground inductance by providing many parallel paths for ground return currents and can be almost as effective as a ground plane.

Although the ground on a digital logic board should be multipoint, that does not mean that the power supplied to the board should be multipoint grounded. Since the high frequency digital logic currents should not flow through the power supply conductors feeding the board, and since the power is low frequency (dc), it can be wired as a single point ground, even though the logic board ground is multipoint.

HYBRID GROUNDS

A hybrid ground is one in which the system-grounding configuration appears differently at different frequencies. Figure 3-9 shows a common type hybrid ground system that acts as a single-point ground at low frequencies and a multipoint ground at high frequencies. A practical application of this principle is the cable-grounding scheme shown in Fig. 3-38. At low frequencies the cable shield is single-point grounded, and at high frequencies it is multipoint grounded.

A different type of hybrid ground is shown in Fig. 3-10. This hybrid ground, although not as common as the one just mentioned, is used when a number of chassis must be grounded to the power system green wire ground for safety reasons, but it is desirable to have a single-point signal ground for the circuitry. The chokes provide a low-impedance safety ground at power line frequencies and ground isolation at higher frequencies.

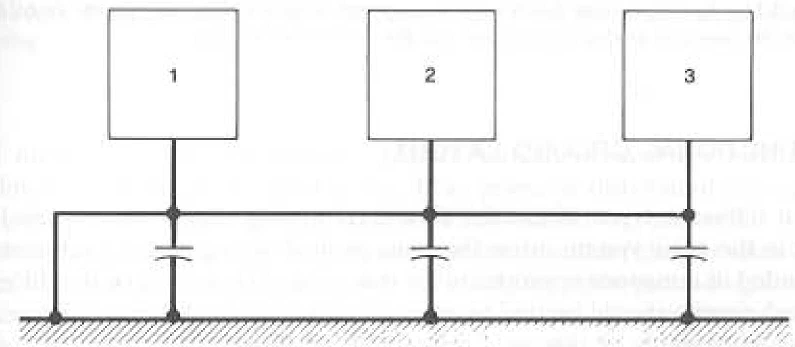

Figure 3-9. *A hybrid ground connection that acts as a single-point ground at low frequencies and a multipoint ground at high frequencies.*

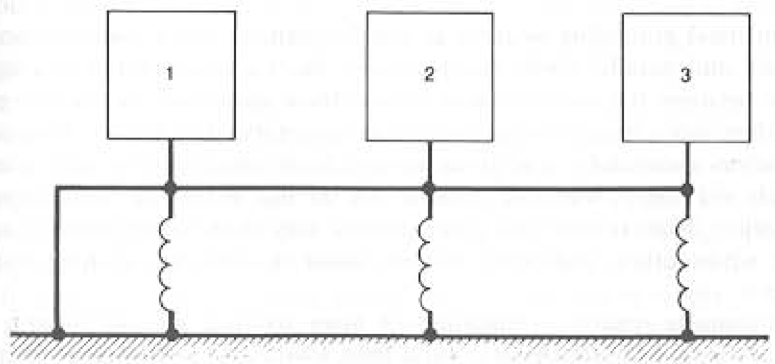

Figure 3-10. *A hybrid ground connection that acts as a multipoint ground at low frequencies and a single-point ground at high frequencies.*

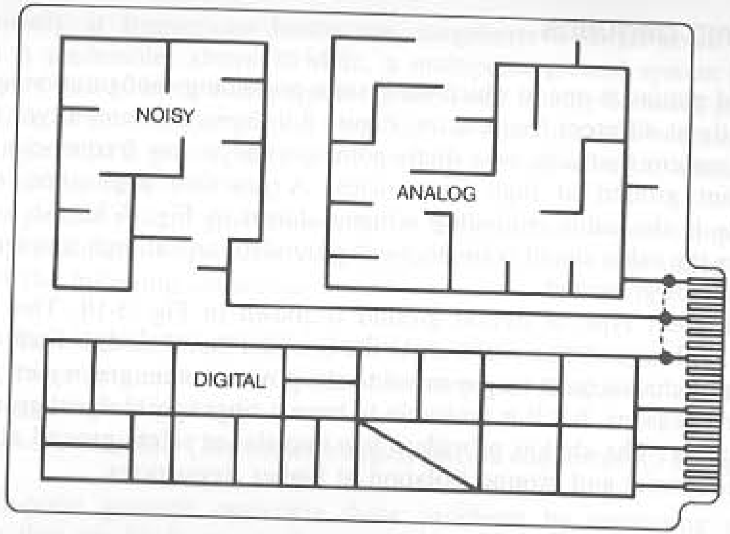

Figure 3-11. *A printed wiring board with three separate ground systems, one for the digital logic, one for the low-level analog circuits, and one for the "noisy" circuits.*

FUNCTIONAL GROUND LAYOUT

When different types of circuits (low-level analog, digital, noisy, etc.) are used in the same system or on the same printed wiring board, each must be grounded in a manner appropriate for that type of circuit. Then the different ground circuits should be tied together, usually at a single point. Figure 3-11 shows an example of this on a printed wiring board.

PRACTICAL LOW-FREQUENCY GROUNDING

Most practical grounding systems at low frequencies are a combination of the series and parallel single-point ground. Such a combination is a compromise between the need to meet the electrical noise criteria and the goal of avoiding more wiring complexity than necessary. The key to balancing these factors successfully is to group ground leads selectively, so that circuits of widely varying power and noise levels do not share the same ground return wire. Thus several low-level circuits may share a common ground return, while other high-level circuits share a different ground return conductor.

Most systems require a minimum of three separate ground returns, as shown in Fig. 3-12. The signal ground used for low-level electronic circuits should be separated from the "noisy" ground used for circuits such as relays

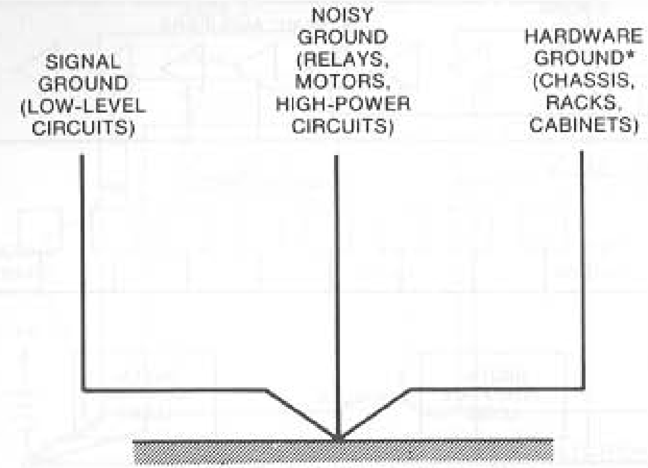

Figure 3-12. *These three classes of grounding connections should be kept separate to avoid noise coupling.*

and motors. A third "hardware" ground should be used for mechanical enclosures, chassis, racks, and so on. If ac power is distributed throughout the system, the power ground (green wire) should be connected to the hardware ground. The three separate ground return circuits should be connected together at only one point. Use of this basic grounding configuration in all equipment would greatly minimize grounding problems.

An illustration of how these grounding principles might be applied to a nine-track digital tape recorder is shown in Fig. 3-13. There are three signal grounds, one noisy ground, and one hardware ground. The most sensitive circuits, the nine read amplifiers, are grounded by using two separate ground returns. Five amplifiers are connected to one, and four are connected to the other. The nine write amplifiers, which operate at a much higher level than the read amplifiers, and the interface and control logic are connected to a third ground return. The three dc motors and their control circuits, the relays, and the solenoids are connected to the noisy ground. Of these elements, the capstan motor control circuit is the most sensitive; it is properly connected closest to the primary ground point. The hardware ground provides the ground for the enclosure and housing. The signal grounds, noisy ground, and hardware ground should be connected together only at the source of primary power, that is, the power supply.

When designing the grounding system for a piece of equipment, a block diagram similar to Fig. 3-13 can be very useful in determining the proper interconnection of the various circuit grounds.

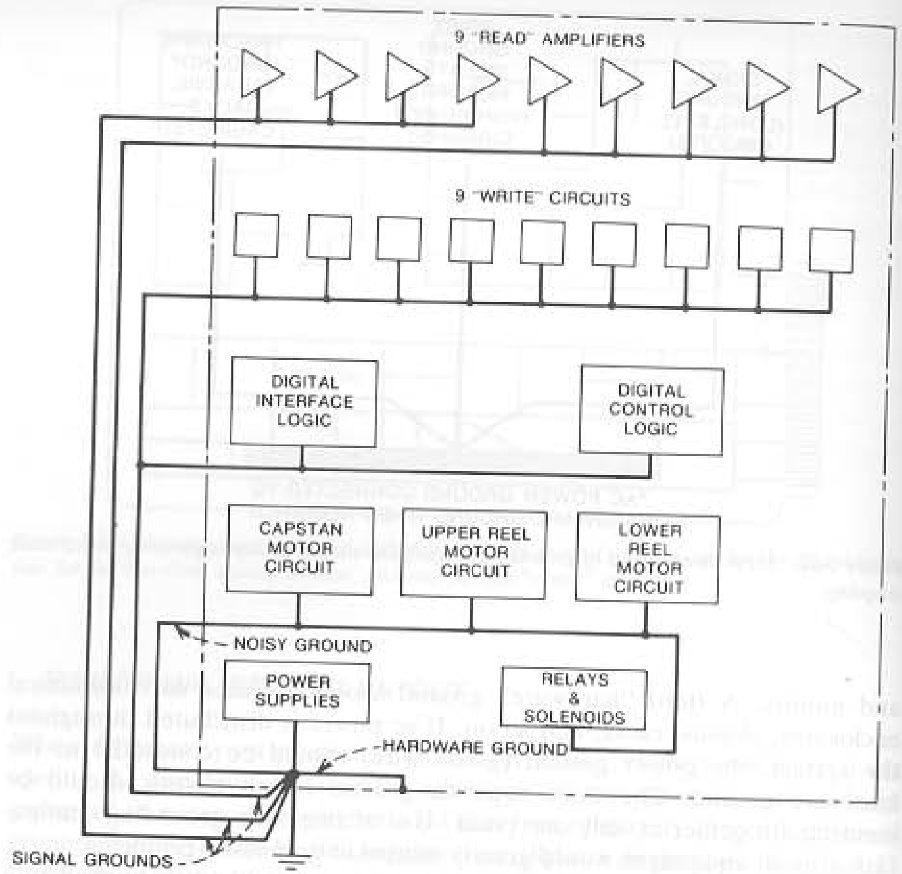

Figure 3-13. *Typical grounding system for nine-track digital tape recorder.*

HARDWARE GROUNDS

Electronic circuits for any large system are usually mounted in relay racks or cabinets. These racks and cabinets must be grounded for safety. In some systems such as electromechanical telephone offices, the racks serve as the return conductor for relay switching circuits. The rack ground is often very noisy, and it may have fairly high resistance due to joints and seams in the rack or in pull-out drawers.

Figure 3-14 shows a typical system consisting of sets of electronics mounted on panels which are then mounted to two relay racks. Rack number 1, on the left, shows correct grounding. The panel is strapped to the rack to provide a good ground, and the racks are strapped together and tied to ground at the primary power source. The electronics circuit ground does not make contact with the panel or rack. In this way noise currents on the

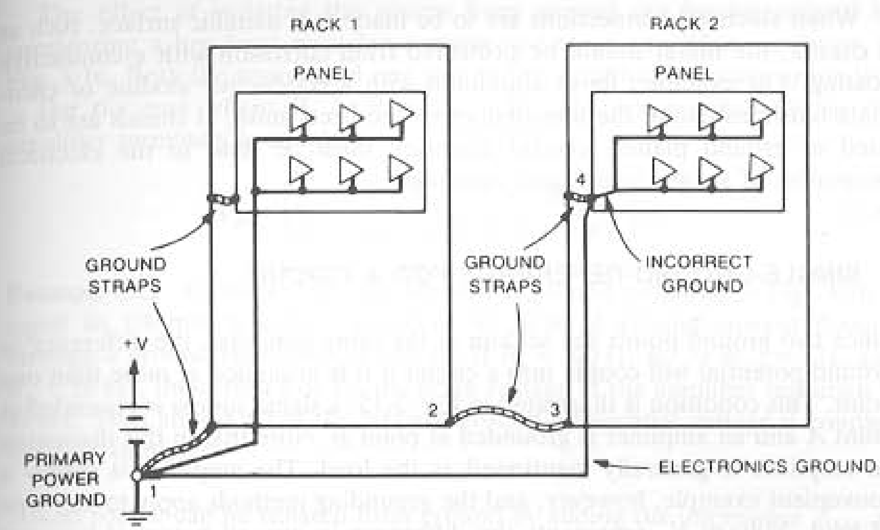

Figure 3-14. *Electronic circuits mounted in equipment racks should have separate ground connections. Rack 1 shows correct grounding; rack 2 shows incorrect grounding.*

rack cannot return to ground through the electronics ground. At high frequencies some of the rack noise current can return on the electronics ground due to capacitive coupling between the rack and electronics. This capacitance should therefore be kept as small as possible. Rack 2, on the right, shows an incorrect installation in which the circuit ground is connected to the rack ground.* Noise currents on the rack can now return on the electronics ground, and there is a ground loop between points 1, 2, 3, 4, and 1.

If the installation does not provide a good ground connection to the rack or panel, it is best to eliminate the questionable ground, and then provide a definite ground by some other means, or be sure that there is no ground at all. Do not depend on sliding drawers, hinges, and so on, to provide a reliable ground connection. When the ground is of a questionable nature, performance may vary from system to system or time to time, depending on whether or not the ground is made.

Hardware grounds produced by intimate contact, such as welding, brazing or soldering, are better than those made by screws and bolts. When joining dissimilar metals for grounding, care must be taken to prevent galvanic corrosion and to ensure that galvanic voltages are not troublesome. Improperly made ground connections may perform perfectly well on new equipment but may be the source of mysterious trouble later.

*A circuit ground to rack ground connection may be required for electrostatic discharge protection. See Chapter 12.

When electrical connections are to be made to a metallic surface, such as a chassis, the metal should be protected from corrosion with a conductive coating. For example, finish aluminum with a conductive alodine or chromate finish instead of the nonconductive anodized finish. If chassis are to be used as ground planes, careful attention must be paid to the electrical properties of seams, joints, and openings.

SINGLE-GROUND REFERENCE FOR A CIRCUIT

Since two ground points are seldom at the same potential, the difference in ground potential will couple into a circuit if it is grounded at more than one point. This condition is illustrated in Fig. 3-15; a signal source is grounded at point A and an amplifier is grounded at point B. Note that in this discussion an amplifier is generally mentioned as the load. The amplifier is simply a convenient example, however, and the grounding methods apply to any type of load. Voltage V_G represents the difference in ground potential between points A and B. In Fig. 3-15 and subsequent illustrations, two different ground symbols are used to emphasize that two physically separated grounds are not usually at the same potential. Resistors R_{C1} and R_{C2} represent the resistance of the conductors connecting the source to the amplifier.

In Fig. 3-15 the input voltage to the amplifier is equal to $V_s + V_G$. To eliminate the noise, one of the ground connections must be removed. Elimination of the ground connection at B means the amplifier must operate from an ungrounded power supply. A differential amplifier could also be used, as discussed later in this chapter. It is usually easier, however, to eliminate ground connection A at the source.

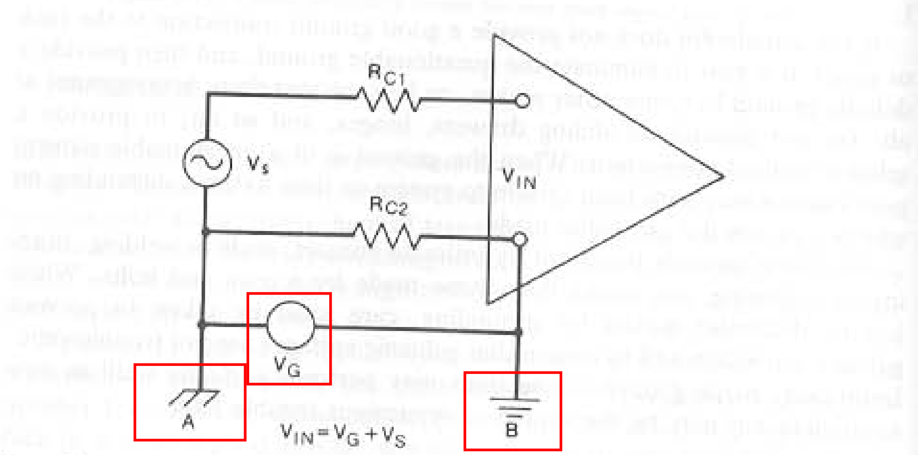

Figure 3-15. *Noise voltage V_G will couple into the amplifier if the circuit is grounded at more than one point.*

The effect of isolating the source from ground can be determined by considering a low-level transducer connected to an amplifier, as shown in Fig. 3-16. Both the source and one side of the amplifier input are grounded.

For the case where $R_{C2} \ll R_s + R_{C1} + R_L$, the noise voltage V_N at the amplifier terminals is equal to

$$V_N = \left[\frac{R_L}{R_L + R_{C1} + R_s}\right]\left[\frac{R_{C2}}{R_{C2} + R_G}\right] V_G . \tag{3-6}$$

Example 3-1. Consider the case where the ground potential in Fig. 3-16 is equal to 100 mV, a value equivalent to 10 A of ground current flowing through a ground resistance of 0.01 Ω. If $R_s = 500\ \Omega$, $R_{C1} = R_{C2} = 1\ \Omega$, and $R_L = 10\ \text{k}\Omega$, then from Eq. 3-6 the noise voltage at the amplifier terminals is 95 mV. Thus almost all of the 100-mV ground differential voltage is coupled into the amplifier.

The source can be isolated from ground by adding the impedance Z_{SG}, as shown in Fig. 3-17. Ideally, the impedance Z_{SG} would be infinite, but due to leakage resistance and capacitance it has some large finite value. For the case where $R_{C2} \ll R_s + R_{C1} + R_L$, and $Z_{SG} \gg R_{C2} + R_G$, the noise voltage V_N at the amplifier terminals is

$$V_N = \left[\frac{R_L}{R_L + R_{C1} + R_s}\right]\left[\frac{R_{C2}}{Z_{SG}}\right] V_G . \tag{3-7}$$

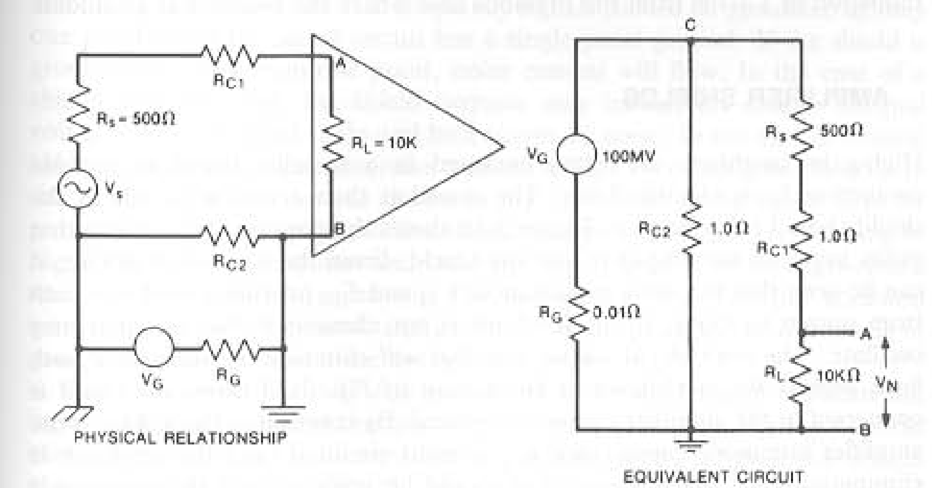

Figure 3-16. *With two ground connections, much of the ground-potential difference appears across the load as noise.*

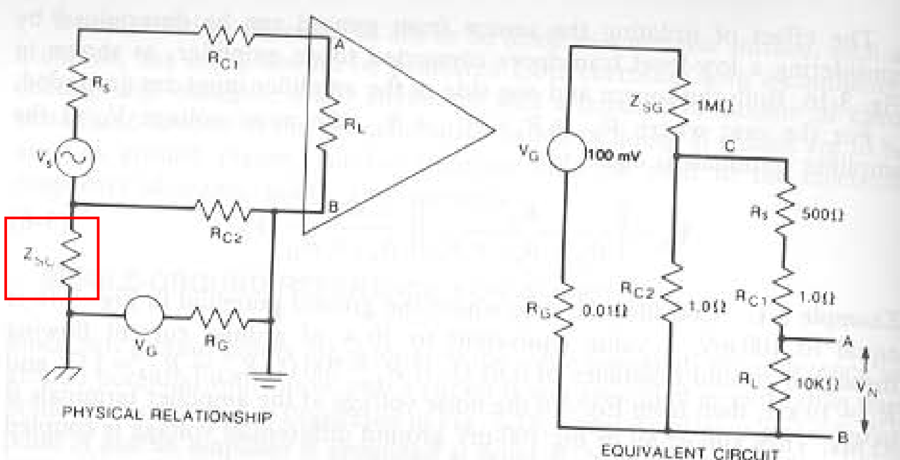

Figure 3-17. *A large impedance between the source and ground keeps most of the ground-potential difference away from the load and reduces noise.*

Most of the noise reduction obtained by isolating the source is due to the second term of Eq. 3-7. If Z_{SG} is infinite, there is no noise voltage coupled into the amplifier. If the impedance Z_{SG} from source to ground is 1 MΩ and all other values are the same as in the previous example, the noise voltage at the amplifier terminals is, from Eq. 3-7, now only 0.095 μV. This is a reduction of 120 dB from the previous case where the source was grounded.

AMPLIFIER SHIELDS

High-gain amplifiers are often enclosed in a metallic shield to provide protection from electric fields. The question then arises as to where the shield should be grounded. Figure 3-18 shows the parasitic capacitance that exists between the amplifier and the shield. From the equivalent circuit, it can be seen that the stray capacitances C_{3S} and C_{1S} provide a feedback path from output to input. If this feedback is not eliminated, the amplifier may oscillate. *The only shield connection that will eliminate the unwanted feedback path is the one shown at the bottom of Fig. 3-18 where the shield is connected to the amplifier common terminal.* By connecting the shield to the amplifier common, capacitance C_{2S} is short-circuited, and the feedback is eliminated. This shield connection should be made even if the common is not at earth ground.

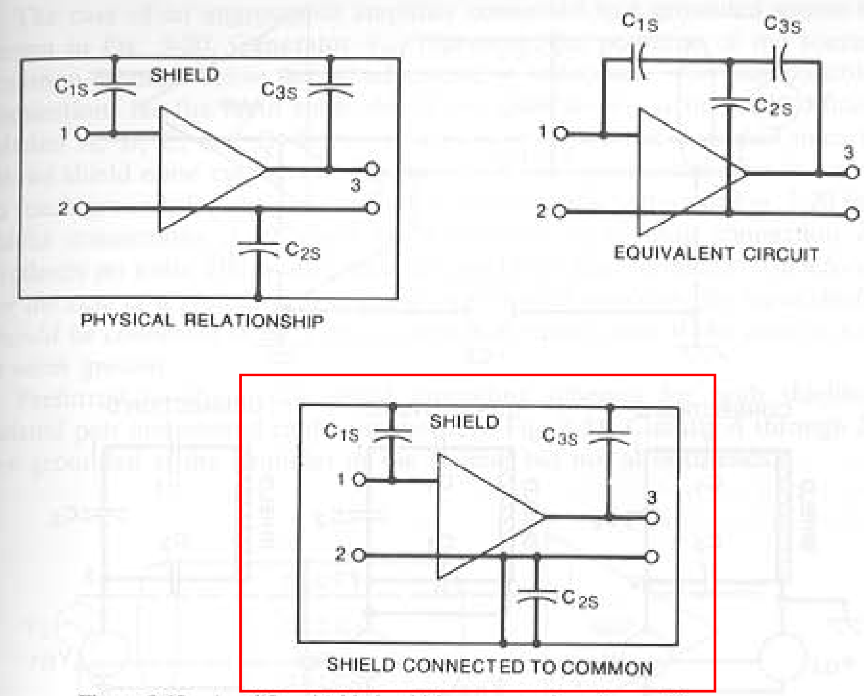

Figure 3-18. *Amplifier shield should be connected to the amplifier common.*

GROUNDING OF CABLE SHIELDS

Shields on cables used for low-frequency signals should be grounded at only one point when the signal circuit has a single-point ground. If the shield is grounded at more than one point, noise current will flow. In the case of a shielded twisted pair, the shield currents may inductively couple unequal voltages into the signal cable and be a source of noise. In the case of coaxial cable, the shield current generates a noise voltage by causing an IR drop in the shield resistance, as was shown in Fig. 2-25. But if the shield is to be grounded at only one point, where should that point be? The top drawing in Fig. 3-19 shows an amplifier and the input signal leads with an ungrounded source. Generator V_{G1} represents the potential of the amplifier common terminal above earth ground, and generator V_{G2} represents the difference in ground potential between the two ground points.

Since the shield has only one ground, it is the capacitance between the input leads and the shield that provides the noise coupling. The input shield may be grounded at any one of four possible points through the dotted connections labeled *A*, *B*, *C*, and *D*. Connection *A* is obviously not desirable, since it allows shield noise current to flow in one of the signal

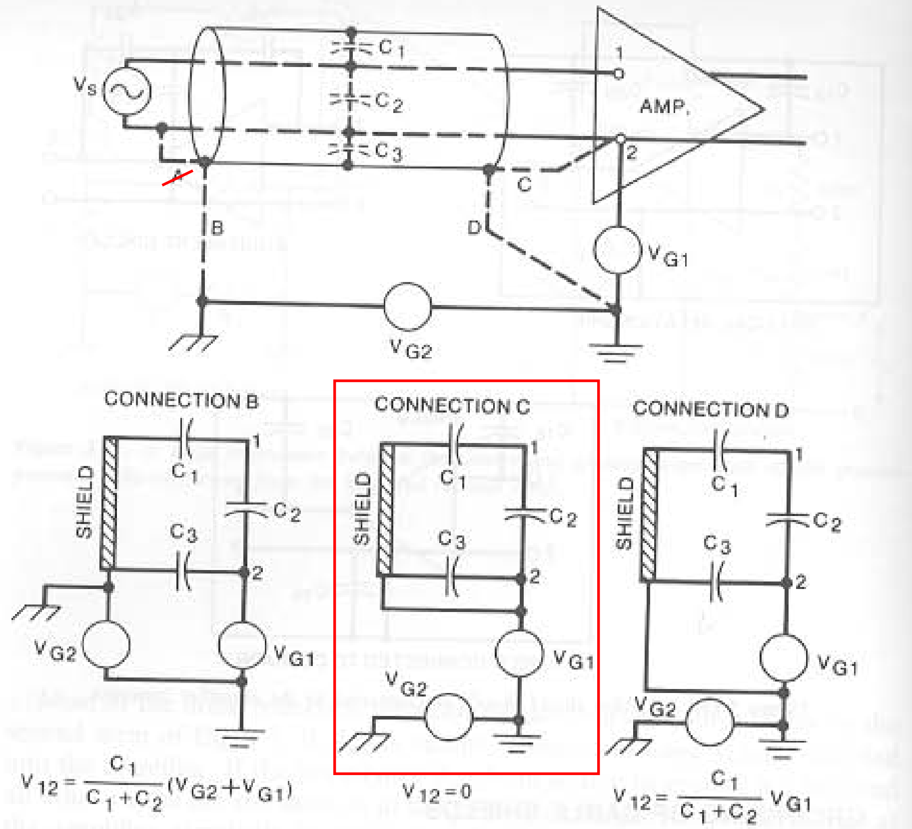

Figure 3-19. *When amplifier is grounded, the best shield connection is C, with shield connected to amplifier common.*

leads. This noise current flowing through the impedance of the signal lead produces a noise voltage in series with the signal.

The three lower drawings in Fig. 3-19 are equivalent circuits for grounding connections *B*, *C*, and *D*. Any extraneous voltage generated between the amplifier input terminals (points 1 and 2) is a noise voltage. With grounding arrangement *B*, a voltage is generated across the amplifier input terminals due to the generators V_{G2} and V_{G1} and the capacitive voltage divider formed by C_1 and C_2. This connection, too, is unsatisfactory. For ground connection *C*, there is no voltage V_{12}, regardless of the value of generators V_{G1} or V_{G2}. With ground connection *D*, a voltage is generated across the amplifier input terminals due to generator V_{G1} and the capacitive voltage divider C_1 and C_2. The only connection that precludes a noise voltage V_{12} is connection *C*. *Thus, for a circuit with an ungrounded source and a grounded amplifier, the input shield should always be connected to the amplifier common terminal, even if this point is not at earth ground.*

The case of an ungrounded amplifier connected to a grounded source is shown in Fig. 3-20. Generator V_{G1} represents the potential of the source common terminal above the actual ground at its location. The four possible connections for the input cable shield are again shown as the dashed lines labeled *A*, *B*, *C*, and *D*. Connection *C* is obviously not desirable since it allows shield noise currents to flow in one of the signal conductors in order to reach ground. Equivalent circuits are shown at the bottom of Fig. 3-20 for shield connections *A*, *B*, and *D*. As can be seen, only connection *A* produces no noise voltage between the amplifier input terminals. *Therefore, for the case of a grounded source and ungrounded amplifier, the input shield should be connected to the source common terminal, even if this point is not at earth ground.*

Preferred low-frequency shield grounding schemes for both shielded twisted pair and coaxial cable are shown in Fig. 3-21. Circuits *A* through *D* are grounded at the amplifier or the source, but not at both ends.

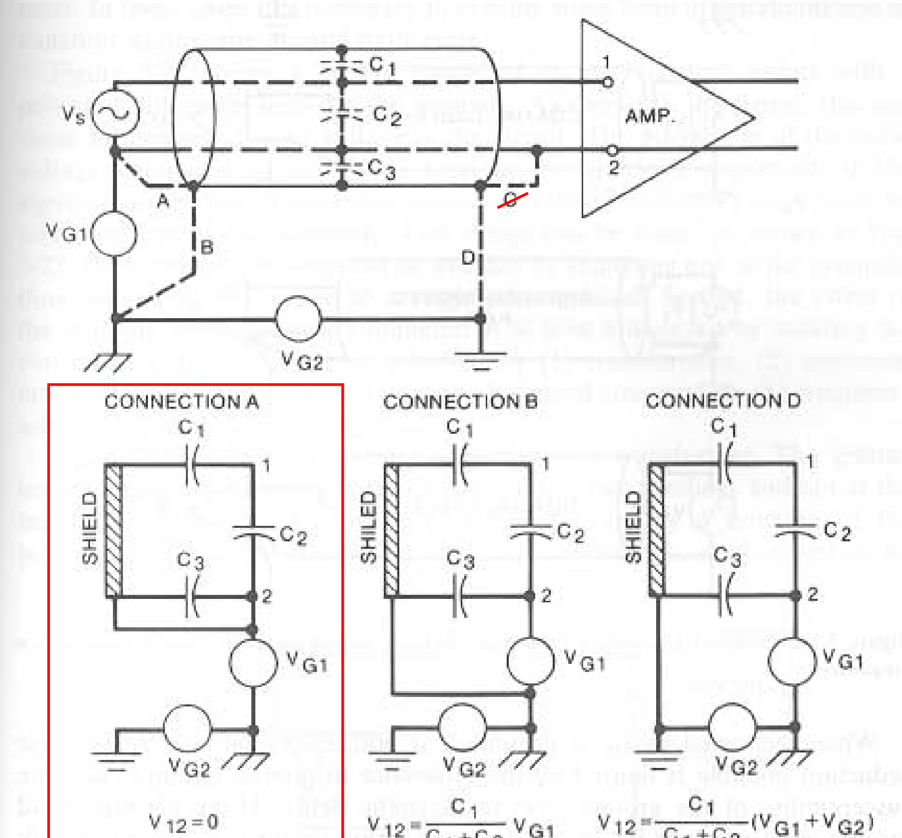

Figure 3-20. *When source is grounded, the best shield connection is A, with shield connected to the source common. The configuration can also be used with a differential amplifier.*

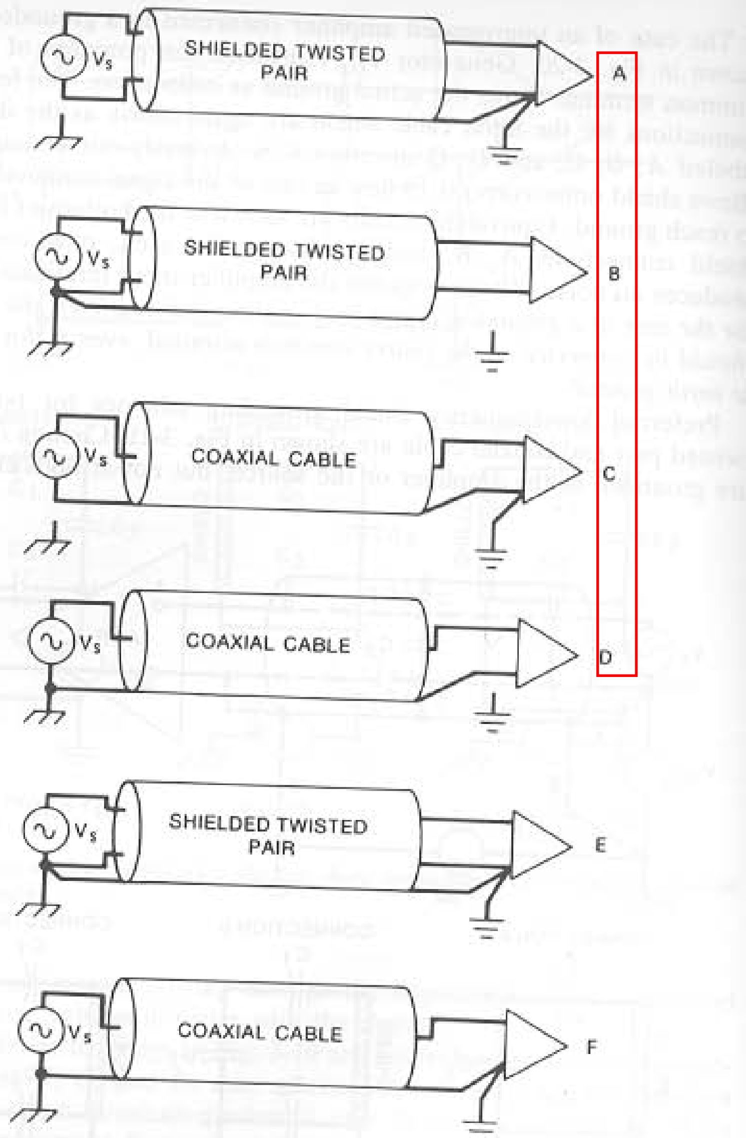

Figure 3-21. *Preferred grounded schemes for shielded, twisted pairs and coaxial cable at low frequency.*

When the signal circuit is grounded at both ends, the amount of noise reduction possible is limited by the difference in ground potential and the susceptibility of the ground loop to magnetic fields. The preferred shield ground configurations for cases where the signal circuit is grounded at both ends are shown in circuits *E* and *F* of Fig. 3-21. In circuit *F* the shield of the coaxial cable is grounded at both ends to force some ground-loop current to

flow through the lower-impedance shield, rather than the center conductor. In the case of circuit *E* the shielded twisted pair is also grounded at both ends to shunt some of the ground-loop current from the signal conductors. If additional noise immunity is required, the ground loop must be broken. This can be done by using transformers, optical couplers, or a differential amplifier.

An indication of the type of performance to be expected from the configurations shown in Fig. 3-21 can be obtained by referring to the results of the magnetic coupling experiment presented in Figs. 2-28 and 2-29.

GROUND LOOPS

Ground loops at times can be a source of noise. This is especially true when the multiple ground points are separated by a large distance and are connected to the ac power ground, or when low-level analog circuits are used. In these cases it is necessary to provide some form of discrimination or isolation against the ground-path noise.

Figure 3-22 shows a system grounded at two different points with a potential difference between the grounds. As shown in the figure, this can cause an unwanted noise voltage in the circuit. The magnitude of the noise voltage compared to the signal level in the circuit is important: if the signal-to-noise ratio is such that circuit operation is affected, steps must be taken to remedy the situation. Two things can be done, as shown in Fig. 3-22. First, the ground loop can be avoided by removing one of the grounds, thus converting the system to a single-point ground. Second, the effect of the multiple ground can be eliminated or at least minimized by isolating the two circuits. Isolation can be achieved by (1) transformers, (2) common-mode chokes, (3) optical couplers, (4) balanced circuitry, or (5) frequency selective grounding (hybrid grounds).

Figure 3-23 shows two circuits isolated with a transformer. The ground noise voltage now appears between the transformer windings and not at the input to the circuit. The noise coupling is primarily a function of the parasitic capacitance between the transformer windings, as discussed in the

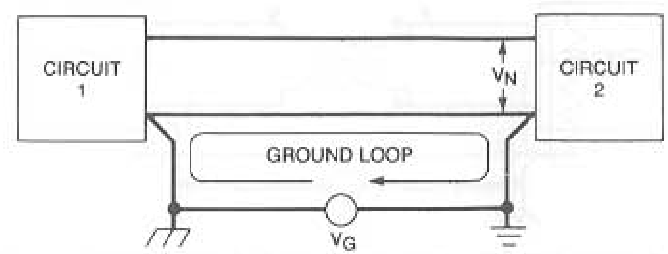

Figure 3-22. *A ground loop between two circuits.*

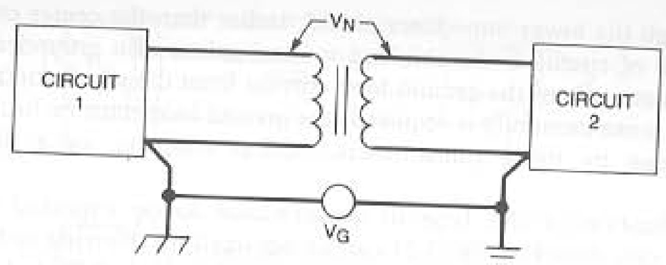

Figure 3-23. *A ground loop between two circuits can be broken by inserting a transformer.*

section on transformers in Chapter 5, and can be reduced by placing a shield between the windings. Although transformers can give excellent results, they do have disadvantages. They are large, have limited frequency response, provide no dc continuity, and are costly. In addition, if multiple signals are connected between the circuits, multiple transformers are required.

In Fig. 3-24 the two circuits are isolated with a transformer connected as a common-mode choke that will transmit dc and differential-mode signals while rejecting common-mode ac signals. The common-mode noise voltage now appears across the windings of the choke and not at the input to the circuit. Since the common-mode choke has no effect on the differential signals being transmitted, multiple signal leads can be wound on the same core without crosstalk. The operation of the common-mode choke is described in the next section.

Optical coupling (optical isolaters or fiber optics), as shown in Fig. 3-25, is a very effective method of eliminating common-mode noise since it completely breaks the metallic path between the two grounds. It is most useful when there are very large differences in voltage between the two grounds, even thousands of volts. The undesired common-mode noise voltage appears across the optical coupler and not across the input to the circuit.

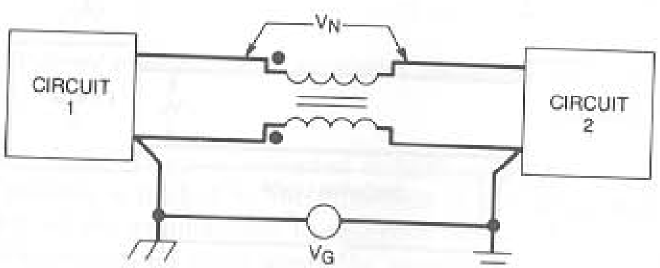

Figure 3-24. *A ground loop between two circuits can be broken by inserting a common-mode choke.*

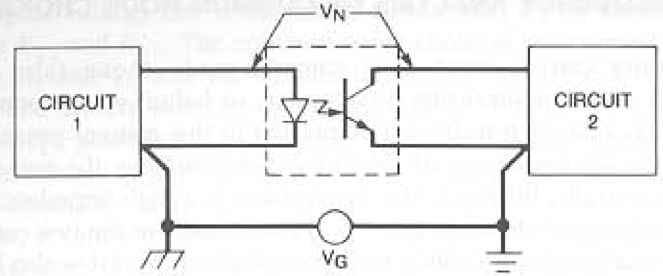

Figure 3-25. *An optical coupler can be used to break the ground loop between two circuits.*

Optical couplers are especially useful in digital circuits. They are less suitable for analog circuits because linearity through the coupler is not always satisfactory. Analog circuits have been designed, however, using optical feedback techniques to compensate for the inherent nonlinearity of the coupler (Waaben, 1975).

Balanced circuits, as shown in Fig. 3-26, provide another way to discriminate against common-mode ground noise voltages. In this case the common-mode voltages induce equal currents in both halves of the balanced circuit, and the balanced receiver responds only to the difference between the two inputs. The better the balance, the larger is the amount of common-mode rejection. As frequency increases, it becomes more and more difficult to achieve a high degree of balance. Balancing is discussed futher in Chapter 4.

When the common-mode noise voltages are at a frequency different from the desired signal, frequency-selective (hybrid) grounding can be used.

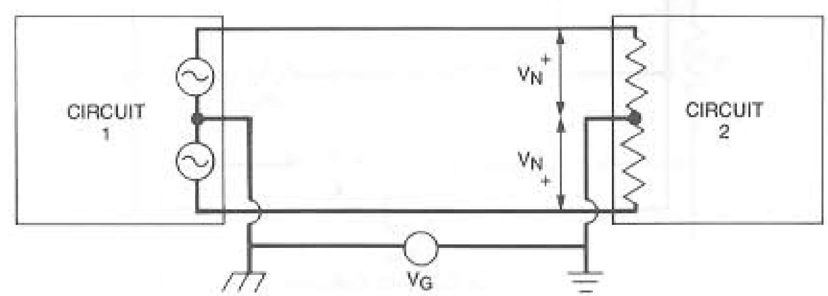

Figure 3-26. *A balanced circuit can be used to cancel out the effect of a ground loop between two circuits.*

LOW-FREQUENCY ANALYSIS OF COMMON-MODE CHOKE

A transformer can be used as a common-mode choke (also called a longitudinal choke, neutralizing transformer, or balun) when connected, as shown in Fig. 3-27. A transformer connected in this manner presents a low impedance to the signal current and allows dc coupling. To any common-mode noise current, however, the transformer is a high impedance.

The signal current shown in Fig. 3-27 flows equally in the two conductors, but in opposite directions. This is the desired current, and it is also known as the differential circuit current or metallic circuit current. The noise currents flow in the same direction along both conductors and are called common-mode currents.

Circuit performance for the common-mode choke of Fig. 3-27 may be analyzed by referring to the equivalent circuit. Voltage generator V_s represents a signal voltage that is connected to the load R_L by conductors with resistance R_{C1} and R_{C2}. The common-mode choke is represented by the two inductors L_1 and L_2 and the mutual inductance M. If both windings are identical and closely coupled on the same core, then L_1, L_2, and M are equal. Voltage generator V_G represents a common-mode voltage due either to magnetic coupling in the ground loop or to a ground differential voltage. Since the conductor resistance R_{C1} is in series with R_L and of much smaller magnitude, it can be neglected.

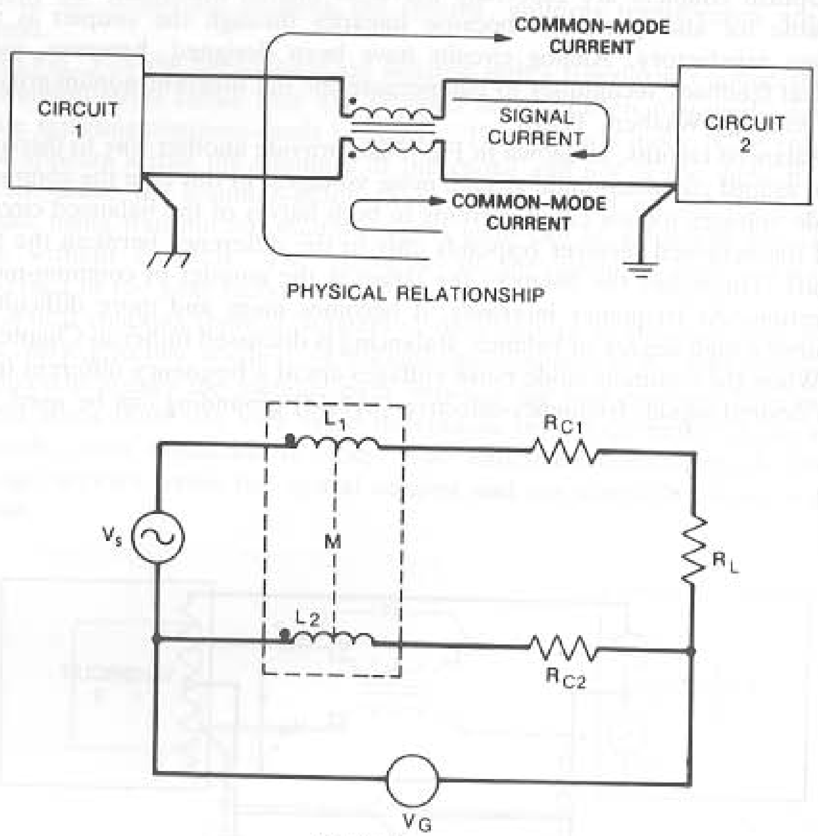

Figure 3-27. *When dc or low-frequency continuity is required, a common-mode choke can be used to break a ground loop.*

The first step is to determine the response of the circuit to the signal voltage V_s, neglecting the effect of V_G. The circuit of Fig. 3-27 can be redrawn, as shown in Fig. 3-28. This figure is similar to the circuit of Fig. 2-22. There it was shown that at frequencies greater than $\omega = 5R_{C2}/L_2$, virtually all the current I_s returned to the source through the second conductor and not through the ground plane. If L_2 is chosen such that the lowest signal frequency is greater than $5R_{C2}/L_2$ rad/s, then $I_G = 0$. Under these conditions the voltages around the top loop of Fig. 3-28 can be summed as follows:

$$V_s = j\omega(L_1 + L_2)I_s - 2j\omega M I_s + (R_L + R_{C2})I_s\,. \tag{3-8}$$

Remembering that $L_1 = L_2 = M$ and solving for I_s gives

$$I_s = \frac{V_s}{R_L + R_{C2}} = \frac{V_s}{R_L}, \tag{3-9}$$

provided R_L is much greater than R_{C2}. Equation 3-9 is the same that would have been obtained if the choke had not been present. It therefore has no effect on the signal transmission so long as the choke inductance is large enough that the signal frequency ω is greater than $5R_{C2}/L_2$.

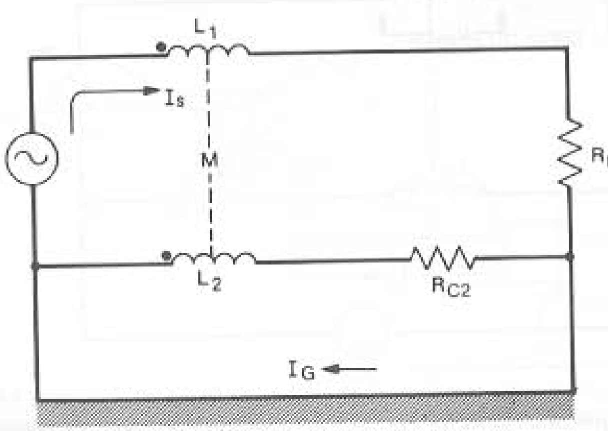

Figure 3-28. *Equivalent circuit for Fig. 3-27 for analysis of response to signal voltage V_s.*

The response of the circuit of Fig. 3-27 to the common-mode voltage V_G can be determined by considering the equivalent circuit shown in Fig. 3-29. If the choke were not present, the complete noise voltage V_G would appear across R_L.

When the choke is present, the noise voltage developed across R_L can be determined by writing equations around the two loops shown in the illustration. Summing voltages around the outside loop gives

$$V_G = j\omega L_1 I_1 + j\omega M I_2 + I_1 R_L \,. \tag{3-10}$$

The sum of the voltages around the lower loop is

$$V_G = j\omega L_2 I_2 + j\omega M I_1 + R_{C2} I_2 \,. \tag{3-11}$$

Equation 3-11 can be solved for I_2, giving the following result:

$$I_2 = \frac{V_G - j\omega M I_1}{j\omega L_2 + R_{C2}} \,. \tag{3-12}$$

Remembering that $L_1 = L_2 = M = L$, and sustituting Eq. 3-12 into Eq. 3-10, and solving for I_1, gives

$$I_1 = \frac{V_G R_{C2}}{j\omega L(R_{C2} + R_L) + R_{C2} R_L} \,. \tag{3-13}$$

The noise voltage V_N is equal to $I_1 R_L$, and since R_{C2} is normally much less than R_L, we can write

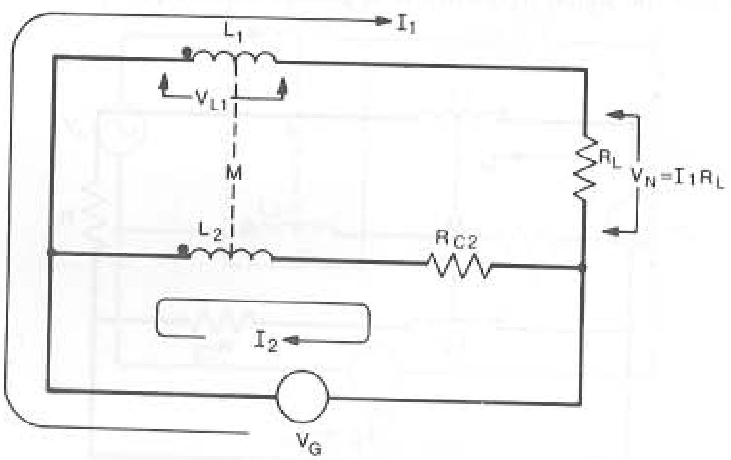

Figure 3-29. *Equivalent circuit for Fig. 3-27 for analysis of response to common-mode voltage V_G.*

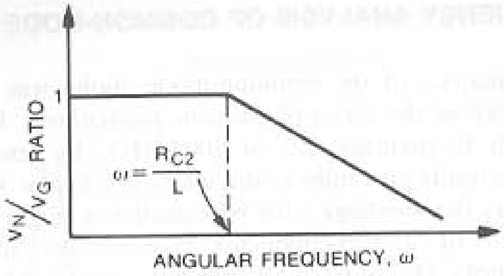

Figure 3-30. *Noise voltage may be significant if R_{C2} is large.*

$$V_N = \frac{V_G R_{C2}/L}{j\omega + R_{C2}/L} \,. \tag{3-14}$$

A plot of V_N/V_G is shown in Fig. 3-30. To minimize this noise voltage, R_{C2} should be kept as small as possible, and the choke inductance L should be such that

$$L \gg \frac{R_{C2}}{\omega} \tag{3-15}$$

where ω is the frequency of the noise. The choke also must be large enough that any unbalanced dc currents flowing in the circuit does not cause saturation.

The common-mode choke shown in Fig. 3-27 can be easily made; simply wind the conductors connecting the two circuits around a magnetic core, as shown in Fig. 3-31. The signal conductors from more than one circuit may be wound around the same core without the signal circuits interfering (crosstalking). In this way one core can be used to provide a common-mode choke for many circuits.

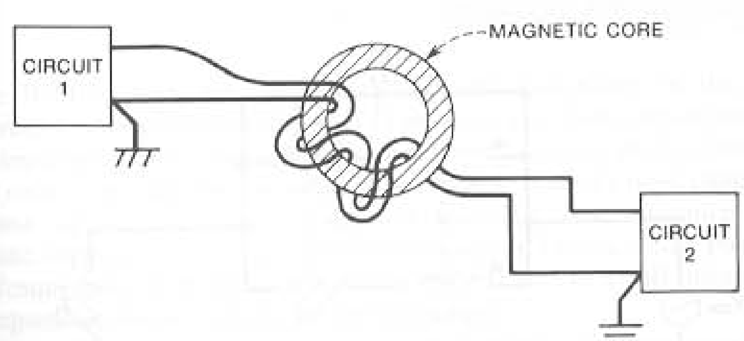

Figure 3-31. *An easy way to place a common-mode choke in the circuit is to wind both conductors around a toroidal magnetic core. A coaxial cable may also be used in place of the conductors shown.*

HIGH-FREQUENCY ANALYSIS OF COMMON-MODE CHOKE

The preceding analysis of the common-mode choke was a low-frequency analysis and neglected the effect of parasitic capacitance. If the choke is to be used at high frequencies (10 to 100 MHz), for example, to block common-mode currents on cables as discussed in Chapter 11, then the stray capacitance across the windings must be considered. Figure 3-32 shows the equivalent circuit of a two-conductor transmission line containing a common-mode choke (L_1 and L_2). R_{C1} and R_{C2} represent the resistance of the windings of the choke plus the cable conductors, and C_s is the stray capacitance across the windings of the choke. Z_L is the common-mode impedance of the cable and V_{CM} is the common-mode voltage driving the cable. In this analysis Z_L is not the differential-mode impedance, but the impedance of the cable acting as an antenna and may vary from about 50 to 350 Ω.

The insertion loss of the choke can be defined as the ratio of the common-mode current without the choke to the common-mode current with the choke. For $R_{C1} = R_{C2} = R$ and $L_1 = L_2 = L$, the insertion loss (IL) of the choke can be written as

$$\mathrm{IL} = Z_L \sqrt{\frac{[2R(1-\omega^2 LC_s)]^2 + R^4(\omega C_s)^2}{[R^2 + 2R(Z_L - \omega^2 LC_s Z_L)]^2 + [2R\omega L + \omega CR^2 Z_L]^2}}. \tag{3-16}$$

Figures 3-33 and 3-34 are plots of Eq. 3-16 for the case where $R_{C1} = R_{c2} = 5\,\Omega$, and $Z_L = 200\,\Omega$. Figure 3-33 shows the insertion loss for a 10-μH choke for various values of shunt capacitance, and Fig. 3-34 shows the insertion loss for a choke with 5 pF of shunt capacitance and various values of inductance. As can be seen from these two figures, the insertion loss

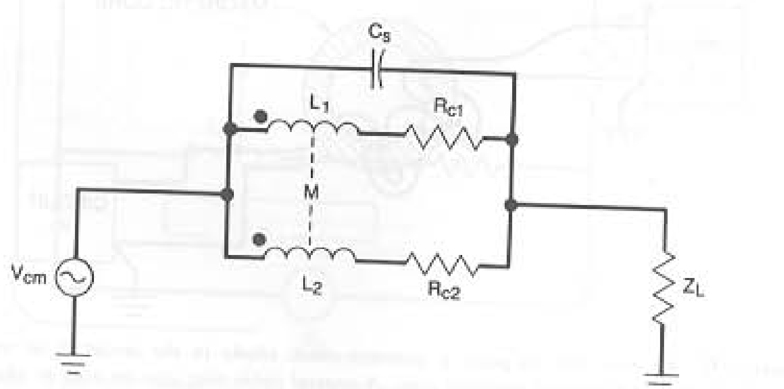

Figure 3-32. *Equivalent circuit of a common-mode choke with parasitic shunt capacitance C_S.*

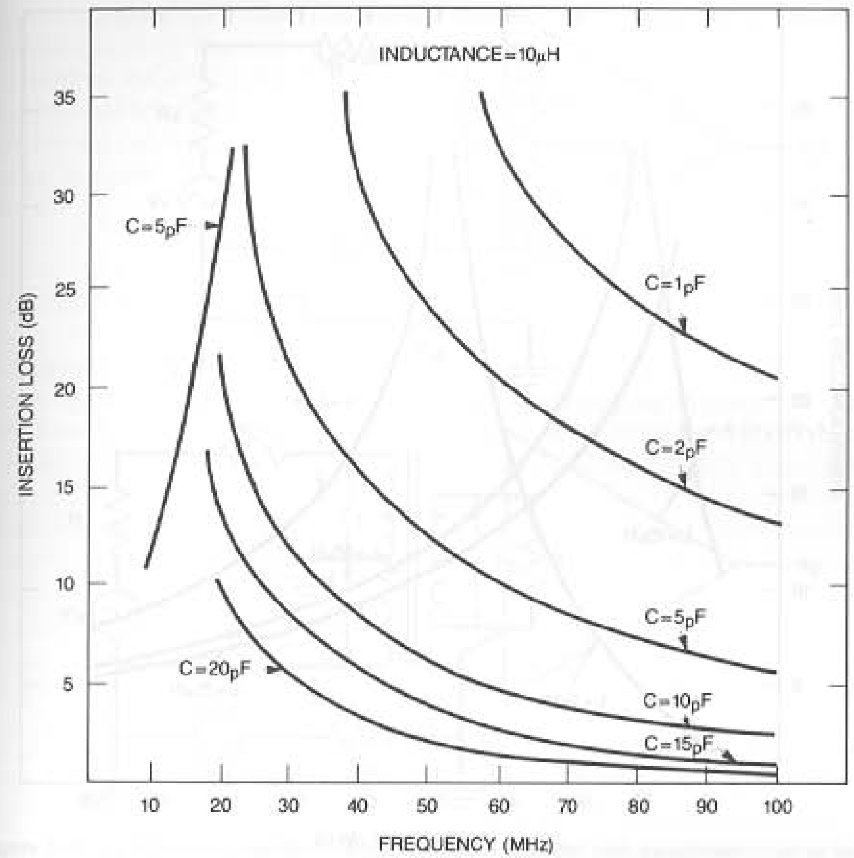

Figure 3-33. *Insertion loss of a 10-μH common-mode choke with various values of shunt capacitance.*

above 70 MHz does not vary much with the inductance of the choke, however, it varies considerably as a function of the shunt capacitance. *The most important parameter in determining the performance of the choke is the shunt capacitance and not the value of inductance.* Actually most chokes used in these applications are beyond self-resonance. The presence of the parasitic capacitance severely limits the maximum insertion loss possible at high frequencies. It is difficult to obtain more than 6 to 12 dB insertion loss at frequencies above 30 MHz by this technique.

At these frequencies the choke can be thought of as an open circuit to the common-mode noise currents. The total common-mode noise current on the cable is therefore determined by the parasitic capacitance, not the inductance of the choke.

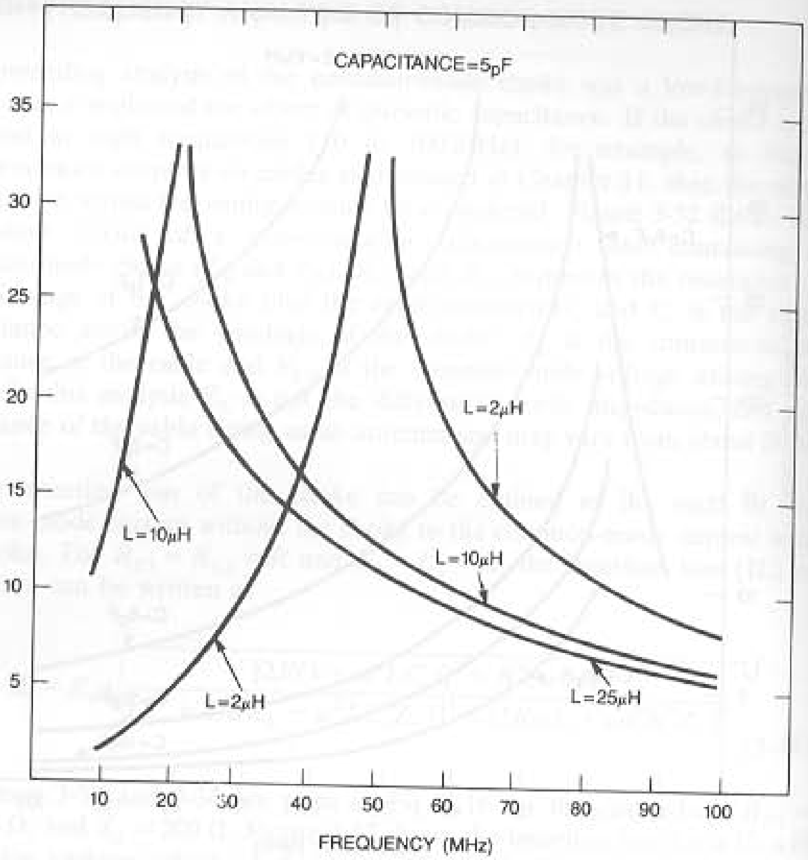

Figure 3-34. *Insertion loss of various inductance common-mode chokes with 5 pF of shunt capacitance.*

DIFFERENTIAL AMPLIFIERS

A differential (or balanced-input) amplifier may be used to decrease the effect of a common-mode noise voltage. This is shown in the upper drawing of Fig. 3-35, where V_G is the common-mode ground noise voltage. The differential amplifier has two input voltages V_1 and V_2, and the output voltage is equal to the amplifier gain (A) times the difference in the two input voltages, $V_0 = A(V_1 - V_2)$.

The lower drawing of Fig. 3-35 shows how a single-ended (or unbalanced) amplifier can be used to simulate the performance of a true balanced amplifier. The transformer primary has a grounded center tap, and the voltages across the two halves are V_1 and V_2. The secondary voltage (assuming a 1:1 turns ratio) is equal to $V_1 - V_2$. Amplifier output again is equal to the gain times this voltage difference, duplicating the balanced amplifier output.

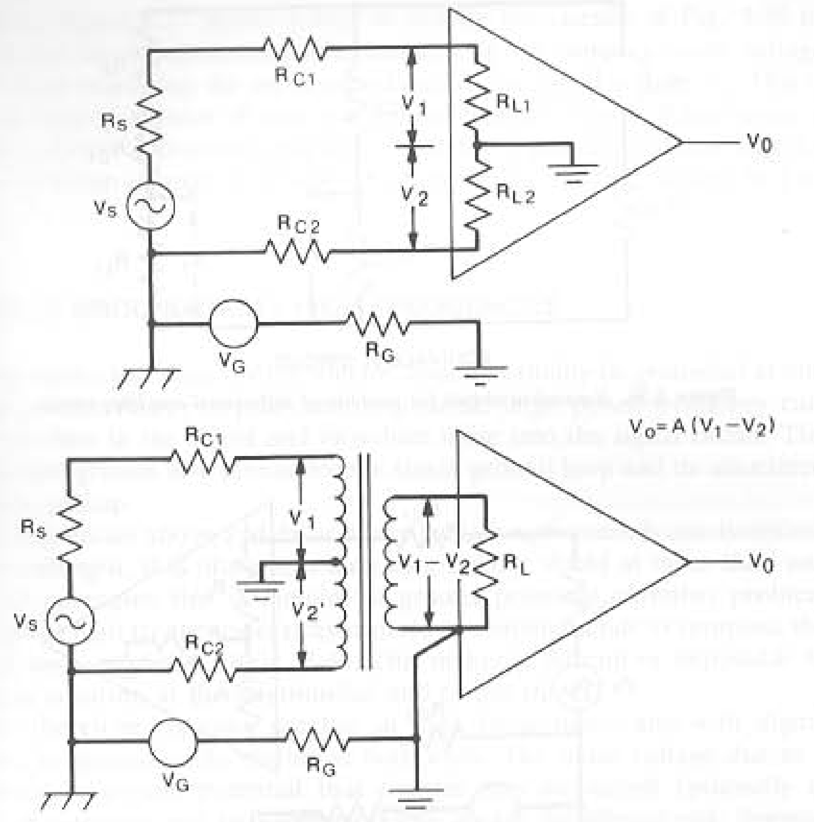

Figure 3-35. *A differential amplifier—or a single-ended amplifier with transformer—can be used to reduce the effects of a common-mode noise voltage.*

The response of either circuit in Fig. 3-35 to the noise voltage can be determined from the equivalent circuit shown in Fig. 3-36. For resistance R_{L2} much larger than R_G, the input voltage to the amplifier due to common-mode noise voltage V_G is as follows:

$$V_N = V_1 - V_2 = \left(\frac{R_{L1}}{R_{L1} + R_{C1} + R_s} - \frac{R_{L2}}{R_{L2} + R_{C2}} \right) V_G . \qquad (3\text{-}17)$$

Example 3-2. If in Fig. 3-36, $V_G = 100\,\text{mV}$, $R_G = 0.01\,\Omega$, $R_s = 500\,\Omega$, $R_{C1} = R_{C2} = 1\,\Omega$, and $R_{L1} = R_{L2} = 10\,\text{k}\Omega$, then from Eq. 3-17, $V_N = 4.6\,\text{mV}$. If, however, R_{L1} and R_{L2} were $100\,\text{k}\Omega$ instead of $10\,\text{k}\Omega$, then $V_N = 0.5\,\text{mV}$. This represents an almost 20 dB decrease in the input noise voltage.

From Example 3-2 it is obvious that increasing the input impedance (R_{L1} and R_{L2}) of the differential amplifier decreases the noise voltage coupled into the amplifier due to V_G. From Eq. 3-17 it can be seen that decreasing the source resistance R_s also decreases the noise voltage coupled into the

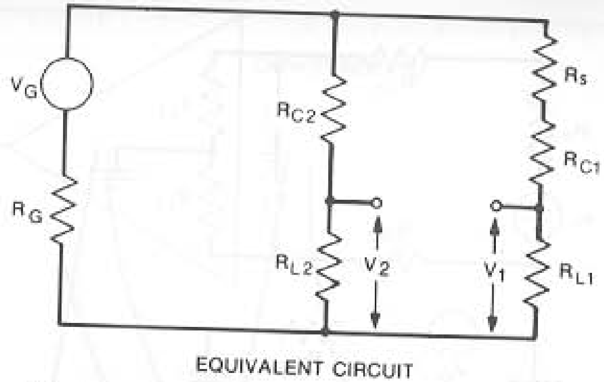

Figure 3-36. *Equivalent circuit for analysis of differential-amplifier circuit.*

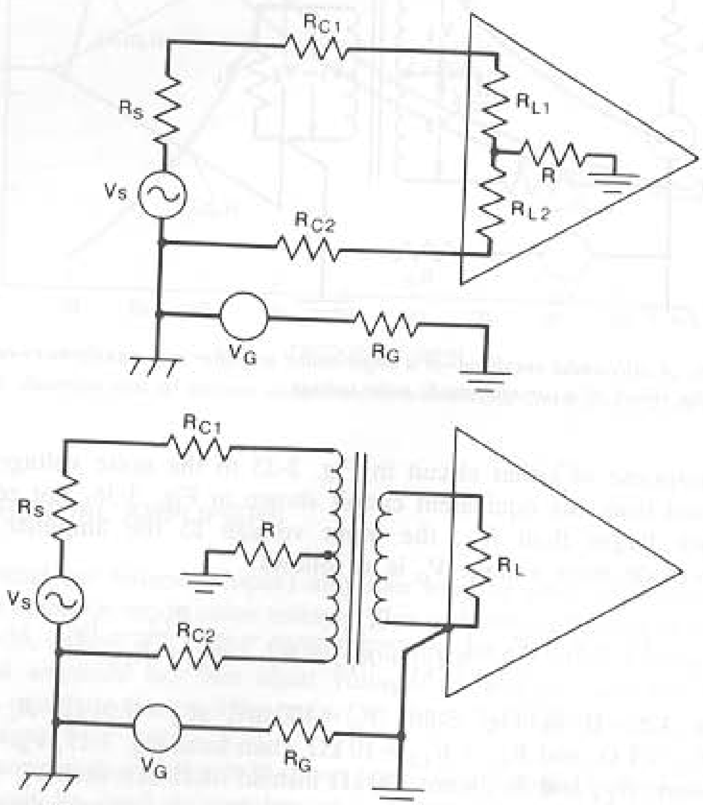

Figure 3-37. *Insertion of resistance R into ground lead decreases the noise voltage.*

amplifier. Figure 3-37 shows a way to modify the circuits of Fig. 3-35 to increase the input impedance of the amplifier to the common-mode voltage V_G, without increasing the input impedance to the signal voltage V_s. This is done by adding resistor R into the ground lead as shown. When using a high-impedance differential amplifier, both the input cable shield and the source common should be grounded at the source as was shown in Fig. 3-21B.

SHIELD GROUNDING AT HIGH FREQUENCIES

At frequencies less than 1 MHz, shields should normally be grounded at one end only. Otherwise, as previously explained, large power-frequency currents can flow in the shield and introduce noise into the signal circuit. The single-point ground also eliminates the shield ground loop and its associated magnetic pickup.

At frequencies above 1 MHz or where cable length exceeds one-twentieth of a wavelength, it is often necessary to ground a shield at more than one point to guarantee that it remains at ground potential. Another problem develops at high frequencies; stray capacitive coupling tends to complete the ground loop, as shown in Fig. 3-38. This makes it difficult or impossible to maintain isolation at the ungrounded end of the shield.

It is therefore common practice at high frequencies, and with digital circuits, to ground cable shields at both ends. The noise voltage due to a difference in ground potential that couples into the circuit (primarily at power frequencies and its harmonics) can usually be filtered out, because there is a large frequency difference between the noise and the signal frequency. At frequencies above 1 MHz the skin effect reduces the coupling due to signal and noise curent flowing on the shield. This skin effect causes the noise current to flow on the outside surface of the shield and the signal current to flow on the inside surface of the shield. The multiple ground also provides a degree of magnetic shielding at higher frequencies when coaxial cable is used.

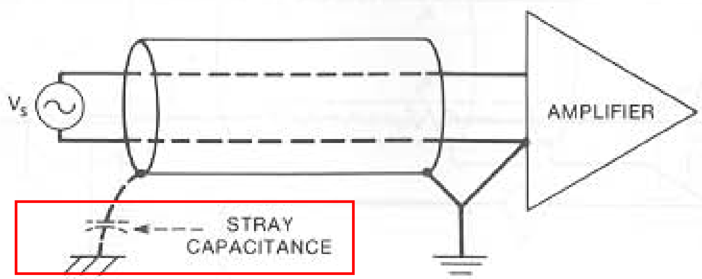

Figure 3-38. *At high frequencies stray capacitance tends to complete the ground loop.*

The characteristics of the circuit shown in Fig. 3-38 can be put to advantage by replacing the stray capacitance with an actual capacitor, thus forming a combination or hybrid ground. At low frequencies a single-point ground exists since the impedance of the capacitor is large. However, at high frequencies the capacitor becomes a low impedance, thus converting the circuit to one having a multiple ground. Such a ground configuration is often useful for circuits that must operate over a very wide frequency range.

GUARD SHIELDS

Noise reduction greater than that obtainable with a differential amplifier can be obtained by using an amplifier with a guard shield. A guard shield is placed around the amplifier and held at a potential which prevents current flow in the unbalanced source impedance. The effect of a guard shield can best be explained by considering an example in which a guard shield is used to cancel the effects of a difference in ground potential.

Figure 3-39 shows an amplifier connected by a shielded twisted pair to a grounded source. V_G is a common-mode voltage due to a difference in ground potentials. V_s and R_s are the differential signal voltage and source resistance, respectively. R_{IN} is the input impedance to the amplifier. C_{1G} and C_{2G} are stray capacitances between the amplifier input terminals and ground, including the cable capacitance. There are two undesirable currents flowing as a result of voltage V_G. Current I_1 flows through resistors R_s and R_1, and capacitance C_{1G}. Current I_2 flows through resistor R_2 and C_{2G}. If each current does not flow through the same total impedance, there will be a differential input voltage to the amplifier. If, however, a guard shield is

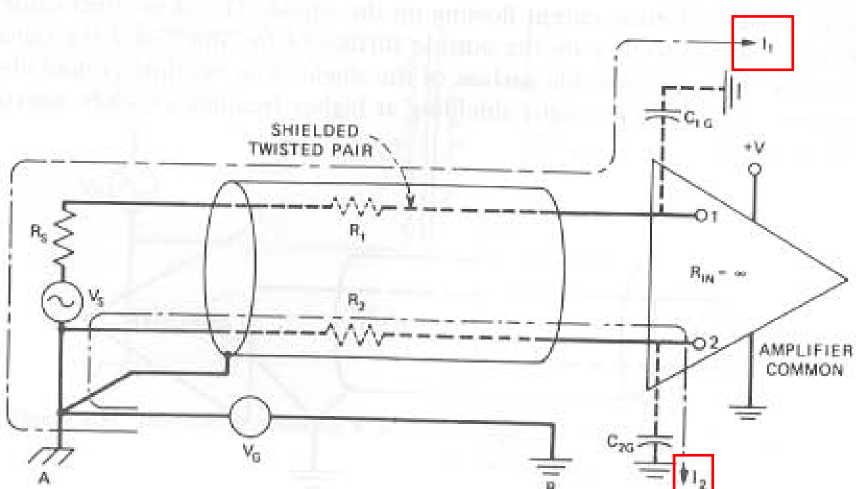

Figure 3-39. *Amplifier and a grounded source are connected by a shielded twisted pair.*

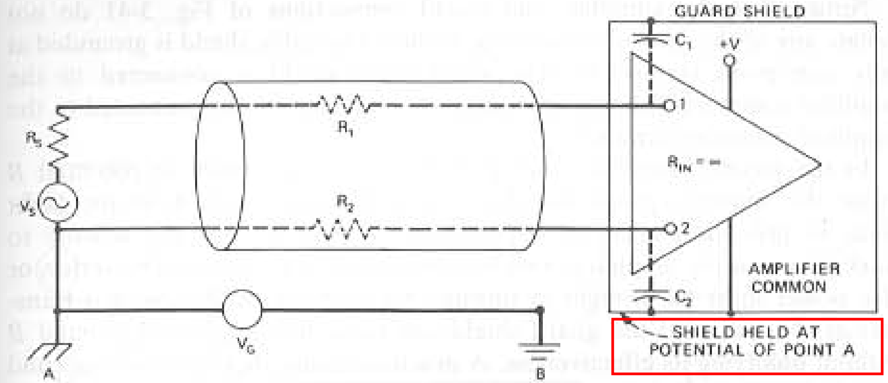

Figure 3-40. *Guard shield at potential of point A eliminates noise currents.*

placed around the amplifier, as shown in Fig. 3-40, and the shield is held at the same potential as point A, currents I_1 and I_2 both become zero because both ends of the path are at the same potential. Capacitances C_1 and C_2 now appear between the input terminals and the shield.

The shield accomplishes the objective of eliminating the differential input noise voltage. Unmentioned, however, has been the problem of how to hold the shield at the potential of point A. One way to do this is shown in Fig. 3-41, where the guard shield is held at the potential of point A by connecting it to the cable shield. The other end of the cable shield is then grounded at point A. This discussion assumes that the source common (lower) terminal is at the same potential as point A. That is, there is no noise voltage generated between point A and the source common. If there is any possibility of a noise voltage being generated between the common terminal of V_s and ground point A, the guard shield should be connected to the source common as illustrated, instead of directly to point A.

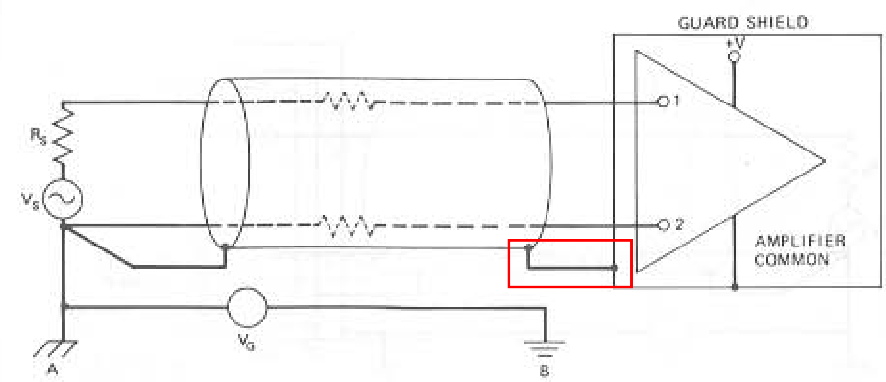

Figure 3-41. *Guard shield is connected to point A through the cable shield.*

Notice that the amplifier and shield connections of Fig. 3-41 do not violate any of the previously described rules. The cable shield is grounded at only one point (point A). The input cable shield is connected to the amplifier common. The shield around the amplifier is also connected to the amplifier common terminal.

In the guarded amplifier of Fig. 3-41, any ground point at potential B inside the amplifier guard shield increases the capacitance from the input leads to ground (unguarded capacitance). Therefore, for the scheme to work, it means the amplifier must be powered by self-contained batteries, or else power must be brought in through an electrostatically shielded transformer. No point of the guard shield can come in contact with ground B without nullifying its effectiveness. A practical circuit therefore has a second shield placed around the guard shield to guarantee the guard's integrity, as shown in Fig. 3-42. This second or external shield is grounded to the local ground, point B, and satisfies the safety requirements.

A guard shield is usually only required when extremely low-level signals are being measured, or when very large common-mode voltages are present and all other noise reduction techniques have also been applied to reduce the noise pickup to an absolute minimum. A guard shield may be used around a single-ended amplifier as well as a differential amplifier.

Example 3-3. Consider a numerical example, as illustrated in Fig. 3-43, where $R_1 = R_2 = 0$, $R_s = 2.6\,\text{k}\Omega$, $C_{1G} = C_{2G} = 100\,\text{pF}$, and $V_G = 100\,\text{mV}$ at 60 Hz. The reactance of 100 pF is 26 MΩ at 60 Hz. The differential input noise voltage across the amplifier input terminals without a guard shield can be written as

$$V_N = \left(\frac{R_s + R_1}{R_s + R_1 + Z_{1G}} - \frac{R_2}{R_2 + Z_{2G}}\right) V_G, \tag{3-18}$$

where Z_{1G} and Z_{2G} are the impedance of capacitance C_{1G} and C_{2G},

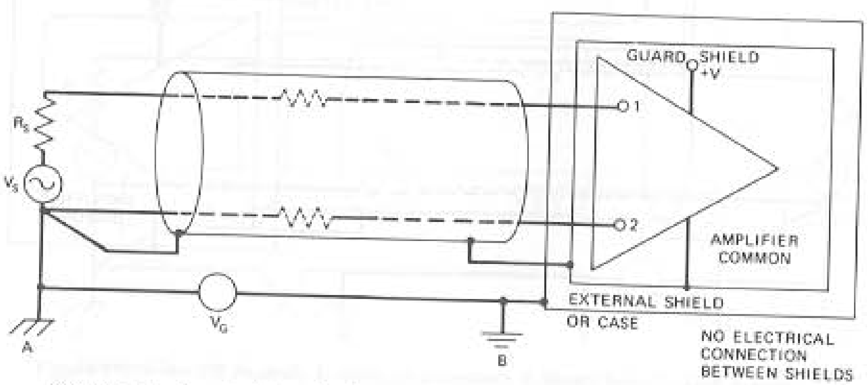

Figure 3-42. *Practical circuit often has a second shield around the guard shield.*

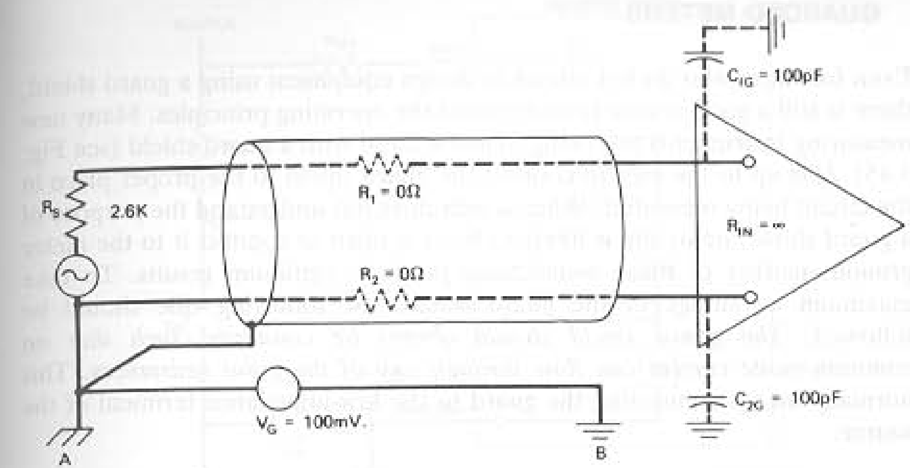

Figure 3-43. *Numerical example to illustrate need for guard shield.*

respectively. Substituting numerical values into Eq. 3-18, the input noise voltage without the guard shield is 10 μV. If the use of the guard shield reduces each line's capacitance to ground to 2 pF, as shown in Fig. 3-44, the differential input noise voltage across the amplifier input terminals with the guard shield in place can still be written as shown in Eq. 3-18, but the input noise voltage is now reduced to 0.2 μV, a 34 dB improvement. The 2-pF capacitance to ground is due to the fact that the guard shield is not perfect. If it were perfect, there would be no capacitance to ground and the noise voltage would be zero. It should be noted that the noise voltage coupled into the amplifier increases as the frequency of the noise source is increased, since the impedance of C_{1G} and C_{2G} decrease as the frequency is increased.

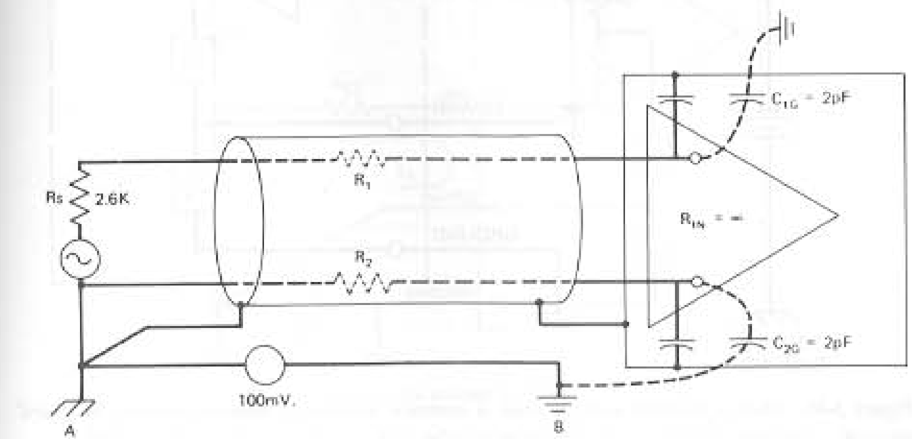

Figure 3-44. *Guard shield reduces line capacitance to ground and therefore noise voltage.*

GUARDED METERS

Even for those who do not intend to design equipment using a guard shield, there is still a good reason to understand the operating principles. Many new measuring instruments are being manufactured with a guard shield (see Fig. 3-45). It is up to the user to connect the guard shield to the proper place in the circuit being measured. When a user does not understand the purpose of a guard shield, he or she is likely to leave it open or connect it to the meter ground; neither of these connections produces optimum results. To take maximum advantage of the guard shield, the following rule should be followed: *The guard shield should always be connected such that no common-mode current can flow through any of the input resistances.* This normally means connecting the guard to the low-impedance terminal of the source.

Example 3-4. Refer to Fig. 3-45. The problem is to measure the voltage across resistor R_s, neither end of which is grounded, with a guarded digital voltmeter. What is the best connection for the guard shield? Five possible ways to connect the guard shield are shown in Figs. 3-46 through 3-50. Voltage V_G is the ground differential voltage, and V_N is the battery noise

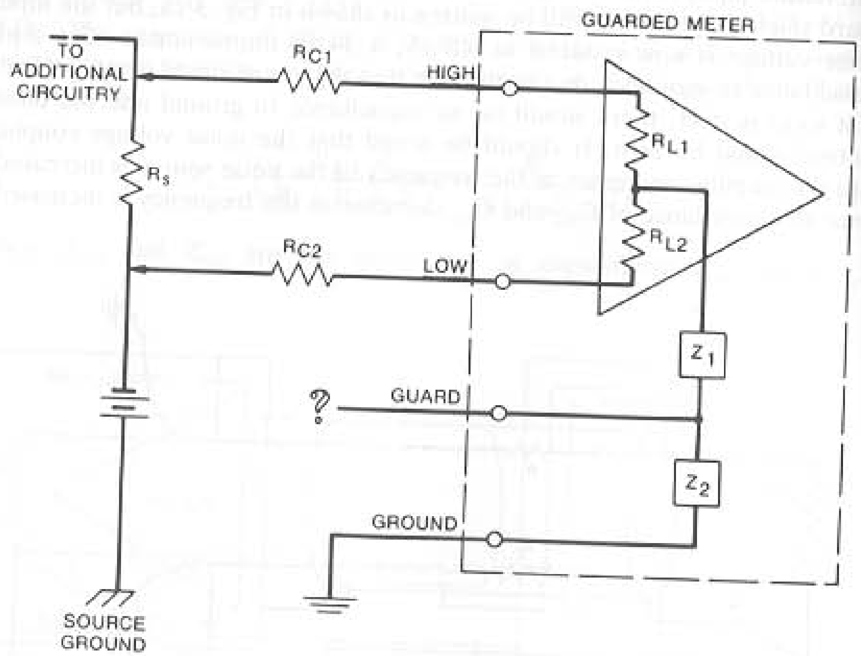

Figure 3-45. *When a guarded meter is used, a common problem is where to connect the guard terminal.*

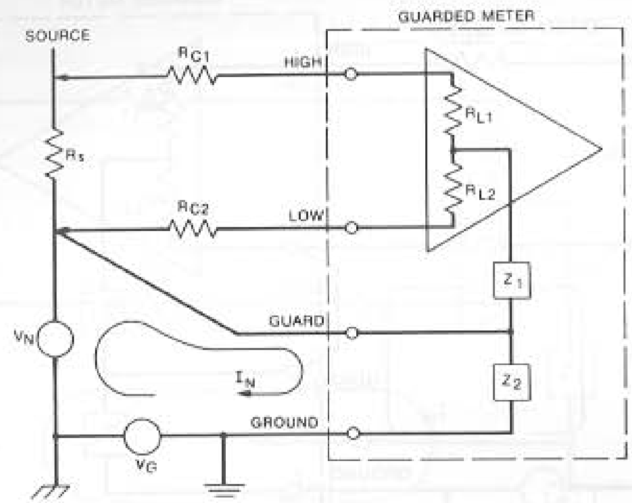

Figure 3-46. *When measuring voltage across R_s, best connection for the guard is to the low-impedance side of R_s; noise current does not affect amplifier.*

voltage. Figure 3-46 shows the best connection, with the guard connected to the low-impedance terminal of the source. Under this condition no noise current flows through the input circuit of the meter.

The connection shown in Fig. 3-47, where the guard is connected to the ground at the source is not as good as the previous connection. Here the noise current from the generator V_G is no problem, but noise current from

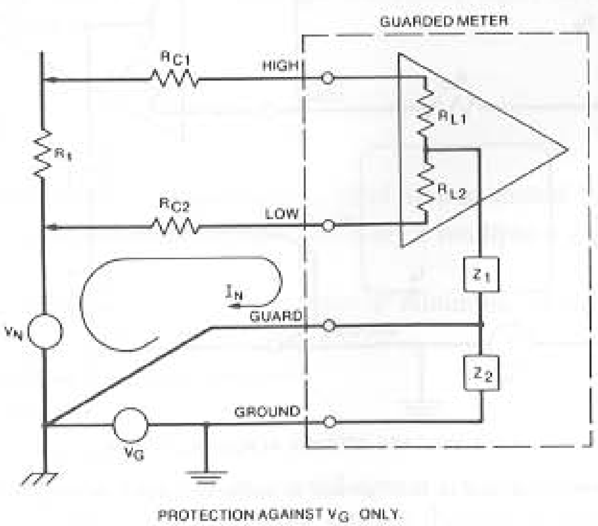

Figure 3-47. *Guard connected to source ground gives no protection against V_N.*

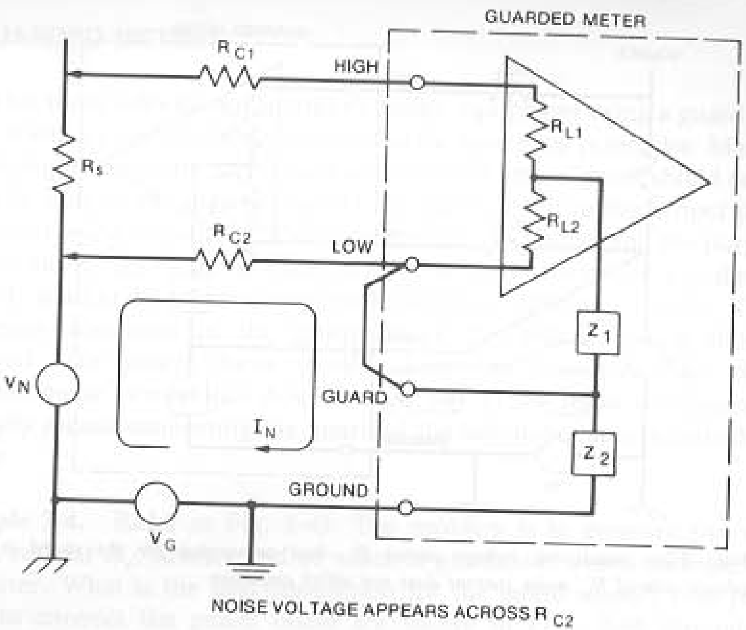

Figure 3-48. *Guard connected to low side of meter allows noise current to flow in line resistance* R_{C2}.

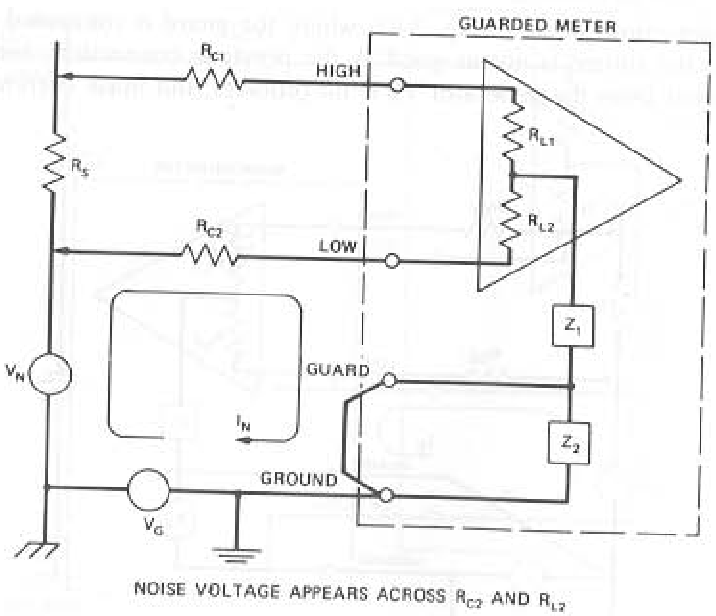

Figure 3-49. *Guard connected to local ground is ineffective; noise current flows through* R_{C2}, R_{L2}, *and* Z_1.

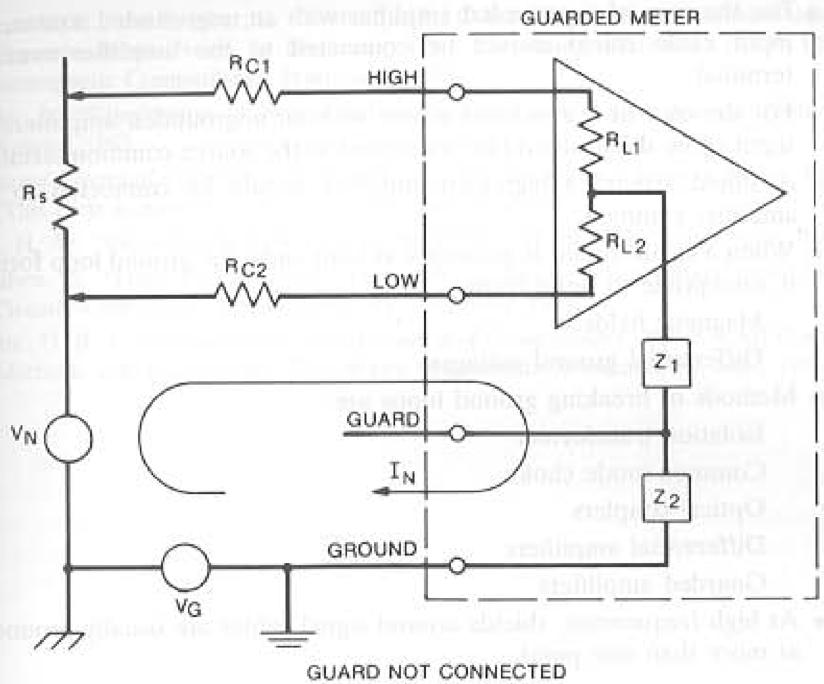

Figure 3-50. *Guard not connected; noise currents due to* V_N *and* V_G *flow through* R_{C2}, R_{L2}, Z_1, *and* Z_2.

V_N flows through impedances R_{C2}, R_{L2}, and Z_1 and causes a noise voltage to be coupled into the amplifier. The connections of Figs. 3-48, 3-49, and 3-50 all allow noise current to flow through the meter input circuit and are therefore undesirable.

SUMMARY

- At low frequencies a single-point ground system should be used.
- At high frequencies and in digital circuitry a multipoint ground system should be used.
- A low-frequency system should have a minimum of three separate ground returns. These should be:
  - Signal ground
  - Noisy ground
  - Hardware ground
- The basic objectives of a good ground system are to minimize the noise voltage from two ground currents flowing through a common impedance.

- For the case of a grounded amplifier with an ungrounded source, the input cable shield should be connected to the amplifier common terminal.
- For the case of a grounded source with an ungrounded amplifier, the input cable shield should be connected to the source common terminal.
- A shield around a high-gain amplifier should be connected to the amplifier common.
- When a signal circuit is grounded at both ends, the ground loop formed is susceptible to noise from:
  Magnetic fields
  Differential ground voltages
- Methods of breaking ground loops are:
  Isolation transformer
  Common-mode choke
  Optical couplers
  Differential amplifiers
  Guarded amplifiers
- At high frequencies, shields around signal cables are usually grounded at more than one point.

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9 ACTIVE DEVICE NOISE

Bipolar transistors, field effect transistors (FETs), and integrated circuit operational amplifiers (op-amps) have inherent noise generation mechanisms. This chapter discusses these internal noise sources and shows the conditions necessary to optimize noise performance.

Before covering active device noise, the general topics of how noise is specified and measured are presented. This general analysis provides a standard set of noise parameters that can then be used to analyze noise in various devices. The common methods of specifying device noise are (1) noise factor and (2) the use of a noise voltage and current model.

NOISE FACTOR

The concept of noise factor was developed in the 1940s as a method of evaluating noise in vacuum tubes. In spite of several serious limitations, the concept is still widely used today.

The noise factor (F) is a quantity that compares the noise performance of a device to that of an ideal (noiseless) device. It can be defined as

$$\boxed{F = \frac{\text{Noise power output of actual device } (P_{no})}{\text{Noise power output of ideal device}}} \tag{9-1}$$

The noise power output of an ideal device is due to the thermal noise power of the source resistance. The standard temperature for measuring the source noise power is 290°K. Therefore the noise factor can be written as

$$F = \frac{\text{Noise power output of actual device } (P_{no})}{\text{Power output due to source noise}}. \tag{9-2}$$

An equivalent definition of noise factor is the input signal-to-noise ratio divided by the output signal-to-noise ratio

$$\boxed{F = \frac{S_i/N_i}{S_o/N_o}}. \tag{9-3}$$

These signal-to-noise ratios must be power ratios unless the input impedance is equal to the load impedance, in which case they can be voltage squared, current squared, or power ratios.

All noise factor measurements must be taken with a resistive source, as shown in Fig. 9-1. The open circuit input noise voltage is therefore just the thermal noise of the source resistance R_s, or

$$V_t = \sqrt{4kTBR_s}. \tag{9-4}$$

At 290°K, this is

$$V_t = \sqrt{1.6 \times 10^{-20} BR_s}. \tag{9-5}$$

If the device has a voltage gain A, defined as the ratio of the output voltage measured across R_L to the open circuit source voltage, then the component of output voltage due to the thermal noise in R_s is AV_t. Using V_{no} for the total output noise voltage measured across R_L, the noise factor can be written as

$$F = \frac{(V_{no})^2/R_L}{(AV_t)^2/R_L}, \tag{9-6}$$

or

$$\boxed{F = \frac{(V_{no})^2}{(AV_t)^2}} \tag{9-7}$$

V_{no} includes the effects of both the source noise and the device noise. Substituting Eq. 9-4 into Eq. 9-7 gives

$$\boxed{F = \frac{(V_{no})^2}{4kTBR_sA^2}}. \tag{9-8}$$

The following three characteristics of noise factor can be seen by examining Eq. 9-8:

1. It is independent of load resistance R_L.
2. It does depend on source resistance R_s.
3. If a device were completely noiseless, the noise factor would equal one.

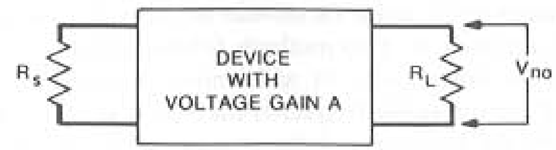


Figure 9-1. *Resistive source is used for noise factor measurements.*

Noise factor expressed in decibels is called noise figure (NF) and is equal to

$$NF = 10 \log F. \tag{9-9}$$

In a qualitative sense noise figure and noise factor are the same, and in casual conversation they are often interchanged.

Due to the bandwidth term in the denominator of Eq. 9-8, there are two ways to specify the noise factor: (1) a spot noise, measured at a specified frequency over a 1-Hz bandwidth, or (2) an integrated, or average noise, measured over a specified bandwidth. If the device noise is "white" and is generated prior to the bandwidth-limiting portion of the circuit both the spot and integrated noise factors are equal. This is because, as the bandwidth is increased, both the total noise and the source noise increase by the same factor.

The concept of noise factor has three major limitations:

1. Increasing the source resistance may decrease the noise factor while increasing the total noise in the circuit.
2. If a purely reactive source is used, noise factor is meaningless since the source noise is zero, making the noise factor infinite.
3. When the device noise is only a small percentage of the source thermal noise (as with some low noise FETs), the noise factor requires taking the ratio of two almost equal numbers. This can produce inaccurate results.

A direct comparison of two noise factors is only meaningful if both are measured at the same source resistance. Noise factor varies with the bias conditions, frequency, and temperature as well as source resistance, and all of these should be defined when specifying noise factor.

Knowing the noise factor for one value of source resistance does not allow the calculation of the noise factor at other values of resistance. This is because both the source noise and device noise vary as the source resistance is changed.

MEASUREMENT OF NOISE FACTOR

A better understanding of noise factor can be obtained by describing the methods used to measure it. Two methods follow: (1) the single-frequency method, and (2) the noise-diode, or white noise, method.

The test set up for the single-frequency method is shown in Fig. 9-2. V_s is an oscillator set to the frequency of the measurement, and R_s is the source resistance. With the source V_s turned off, the output rms noise voltage V_{no} is measured. This voltage consists of two parts: the first due to the thermal

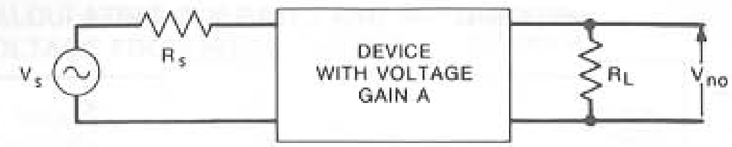


Figure 9-2. *Single-frequency method for measuring noise factor.*

noise voltage V_t of the source resistor, and the second due to the noise in the device.

$$V_{no} = \sqrt{(AV_t)^2 + (\text{Device noise})^2}. \tag{9-10}$$

Next, the generator V_s is turned on, and an input signal is applied until the output power doubles (output rms voltage increases by 3 dB over that previously measured). Under these conditions the following equation is satisfied

$$(AV_s)^2 + (V_{no})^2 = 2V_{no}^2; \tag{9-11}$$

therefore

$$AV_s = V_{no}. \tag{9-12}$$

Substituting Eq. 9-12 into Eq. 9-7 gives

$$F = \left(\frac{V_s}{V_t}\right)^2. \tag{9-13}$$

Substituting from Eq. 9-5 for V_t produces

$$F = \frac{V_s^2}{1.6 \times 10^{-20} BR_s}. \tag{9-14}$$

Since the noise factor is not a function of R_L, any value of load resistor can be used for the measurment.

The disadvantage of this method is that the noise bandwidth of the device* must be known.

A better method of measuring noise factor is to use a noise diode as a white noise source. The measuring circuit is shown in Fig. 9-3. I_{dc} is the direct current through the noise diode, and R_s is the source resistance. The shot noise in the diode is

$$I_{sh} = \sqrt{3.2 \times 10^{-19} I_{dc} B}. \tag{9-15}$$

*It should be remembered that the noise bandwidth is usually not equal to the 3-dB bandwidth (see Chapter 8).

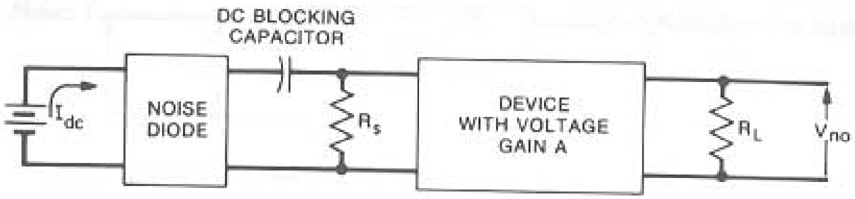


Figure 9-3. *Noise-diode method of measuring noise factor.*

Using Thevenin's theorem, the shot-noise current generator can be replaced by a voltage generator V_{sh} in series with R_s, where

$$V_{sh} = I_{sh}R_s . \tag{9-16}$$

The rms noise voltage output V_{no} is first measured with the diode current equal to zero. This voltage consists of two parts: that due to the thermal noise of the source resistor, and that due to the noise in the device. Therefore

$$V_{no} = \sqrt{(AV_t)^2 + (\text{Device noise})^2} . \tag{9-17}$$

Diode current is then allowed to flow and is increased until the output noise power doubles (output rms voltage increases by 3 dB). Under these conditions the following equation is satisfied:

$$(AV_{sh})^2 + (V_{no})^2 = 2(V_{no})^2 ; \tag{9-18}$$

therefore

$$V_{no} = AV_{sh} = AI_{sh}R_s . \tag{9-19}$$

Substituting V_{no} from Eq. 9-19 into Eq. 9-7, gives

$$F = \frac{(I_{sh}R_s)^2}{V_t^2} . \tag{9-20}$$

Substituting Eqs. 9-15 and 9-5 for I_{sh} and V_t, respectively, gives

$$F = 20 I_{dc} R_s . \tag{9-21}$$

The noise factor is now a function of only the direct current through the diode, and the value of the source resistance. Both of these quantities are easily measured. Neither the gain nor the noise bandwidth of the device need be known.

CALCULATING *S/N* RATIO AND INPUT NOISE VOLTAGE FROM NOISE FACTOR

Once noise factor is known, it can be used to calculate the signal-to-noise ratio and the input noise voltage. For these calculations it is important that the source resistance used in the circuit be the same as that used to make the noise factor measurement. Rearranging Eq. 9-8 gives

$$V_{no} = A\sqrt{4kTBR_sF} . \tag{9-22}$$

If the input signal is V_s, the output signal voltage is $V_o = AV_s$. Therefore the output signal-to-noise power ratio is

$$\frac{S_o}{N_o} = \frac{P_{\text{signal}}}{P_{\text{noise}}} , \tag{9-23}$$

or

$$\frac{S_o}{N_o} = \left(\frac{AV_s}{V_{no}}\right)^2 . \tag{9-24}$$

Using Eq. 9-22 to substitute for V_{no},

$$\frac{S_o}{N_o} = \frac{(V_s)^2}{4kTBR_sF} . \tag{9-25}$$

Signal-to-noise ratio, as used in Eqs. 9-23, 9-24, and 9-25, refers to a power ratio. However, signal-to-noise is sometimes expressed as a voltage ratio. Care should be taken as to whether a specified signal-to-noise ratio is a power or voltage ratio, since the two are not numerically equal. When expressed in decibels, the power signal-to-noise ratio is 10 log (S_o/N_o).

Another useful quantity is the total equivalent input noise voltage (V_{nt}) which is the output noise voltage (Eq. 9-22) divided by the gain

$$\boxed{V_{nt} = \frac{V_{no}}{A} = \sqrt{4kTBR_sF} .} \tag{9-26}$$

The total equivalent input noise voltage is a single noise source that represents the total noise in the circuit. *For optimum noise performance, V_{nt} should be minimized.* Minimizing V_{nt} is equivalent to maximizing the signal-to-noise ratio, provided the signal voltage remains constant. This is discussed further in the section on optimum source resistance.

The equivalent input noise voltage consists of two parts, one due to the thermal noise of the source and the other due to the device noise.

Representing the device noise by V_{nd}, we can write the total equivalent input noise voltage as

$$V_{nt} = \sqrt{(V_t)^2 + (V_{nd})^2}\,, \tag{9-27}$$

where V_t is the open circuit thermal noise voltage of the source resistance. Solving Eq. 9-27 for V_{nd} gives

$$V_{nd} = \sqrt{(V_{nt})^2 - (V_t)^2}\,. \tag{9-28}$$

Substituting Eqs. 9-4 and 9-26 into Eq. 9-28 gives

$$\boxed{V_{nd} = \sqrt{4kTBR_s(F-1)}\,.} \tag{9-29}$$

NOISE VOLTAGE AND CURRENT MODEL

A more recent approach, and one that overcomes the limitations of noise factor, is to model the noise in terms of an equivalent noise voltage and current. The actual network can be modeled as a noise-free device with two noise generators, V_n and I_n, connected to the input side of a network, as shown in Fig. 9-4. V_n represents the device noise that exists when R_s equals zero, and I_n represents the additional device noise that occurs when R_s does not equal zero. The use of these two noise generators plus a complex correlation coefficient (not shown) completely characterizes the noise performance of the device (Rothe and Dahlke 1956). Although V_n and I_n are normally correlated to some degree, values for the correlation coefficient are seldom given on manufacturers data sheets. In addition the typical spread of values of V_n and I_n for a device normally overshadows the effect of the correlation coefficient. Therefore it is common practice to assume the correlation coefficient is equal to zero. This will be done in the remainder of this chapter.

Figure 9-5 shows representative curves of noise voltage and noise current. As can be seen in Fig. 9-5, the data normally consist of a plot of $V_n/\sqrt{B}$ and

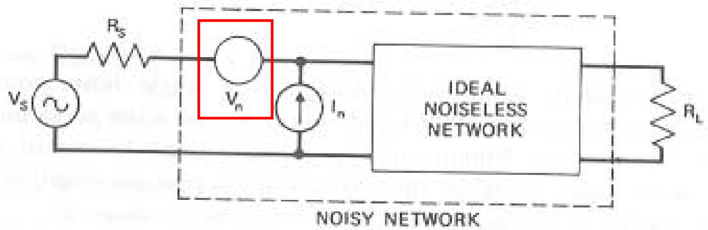


Figure 9-4. *A noisy network modeled by the addition of an input noise voltage and current source.*

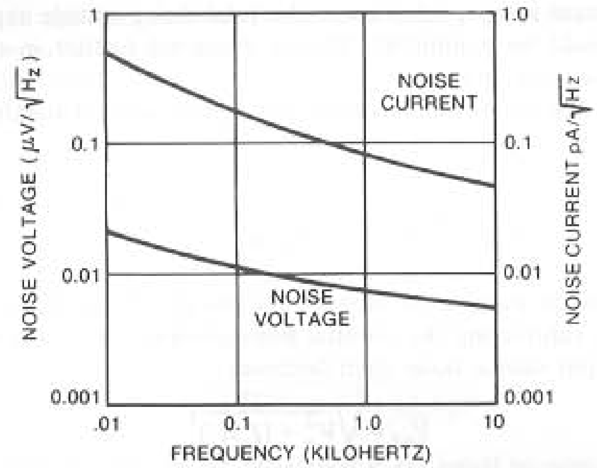


Figure 9-5. *Typical noise voltage $V_n/\sqrt{B}$ and noise current $I_n/\sqrt{B}$ curves.*

$I_n/\sqrt{B}$ versus frequency. The noise voltage or current over a band of frequencies can be found by integrating $[V_n/\sqrt{B}]^2$ or $[I_n/\sqrt{B}]^2$ versus frequency and then taking the square root of the result. In the case when $V_n/\sqrt{B}$ or $I_n/\sqrt{B}$ is constant over the desired bandwidth, the total noise voltage or current can be found simply by multiplying $V_n/\sqrt{B}$ or $I_n/\sqrt{B}$ by the square root of the bandwidth.

Using these curves and the equivalent circuit of Fig. 9-4, the total equivalent input noise voltage, signal-to-noise ratio, or noise factor for any circuit can be determined. This can be done for any source impedance, resistive or reactive, and across any frequency spectrum. The device must, however, be operated at or near the bias conditions for which the curves are specified. Quite often, additional curves are given showing the variation of these noise generators with bias points. With a set of these curves the noise performance of the device is completely specified under all operating conditions.

The representation of noise data in terms of the equivalent parameters V_n and I_n can be used for any device. Field effect transistors and op-amps are usually specified in this manner. Some bipolar transistor manufacturers are also beginning to use the V_n–I_n parameters instead of noise factor.

The total equivalent input noise voltage of a device is an important parameter. Assuming no correlation between noise sources, this voltage, which combines the affect of V_n, I_n, and the thermal noise of the source, can be written as

$$\boxed{V_{nt} = \sqrt{4kTBR_s + V_n^2 + (I_nR_s)^2}\,,} \tag{9-30}$$

where V_n and I_n are the noise voltage and noise current over the bandwidth

B. For optimum noise performance the total noise voltage represented by Eq. 9-30 should be minimized. This is discussed further in the optimum source resistance section.

The total equivalent input voltage per square root of bandwidth can be written as

$$\frac{V_{nt}}{\sqrt{B}} = \sqrt{4kTR_s + \left(\frac{V_n}{\sqrt{B}}\right)^2 + \left(\frac{I_n R_s}{\sqrt{B}}\right)^2}\,. \tag{9-31}$$

The equivalent input noise voltage due to the device noise only can be calculated by subtracting the thermal noise component from Eq. 9-30. The equivalent input device noise then becomes

$$V_{nd} = \sqrt{V_n^2 + (I_n R_s)^2}\,. \tag{9-32}$$

Figure 9-6 is a plot of the total equivalent noise voltage per square root of the bandwidth for a typical low-noise bipolar transistor, junction field effect transistor, and op-amp. The thermal noise voltage generated by the source resistance is also shown. The thermal noise curve places a lower limit on the total input noise voltage. As can be seen from this figure, when the source resistance is between 10,000 Ω and 1 MΩ, this FET has a total noise voltage only slightly greater than the thermal noise in the source resistance. On the basis of noise this FET approaches an ideal device when the source

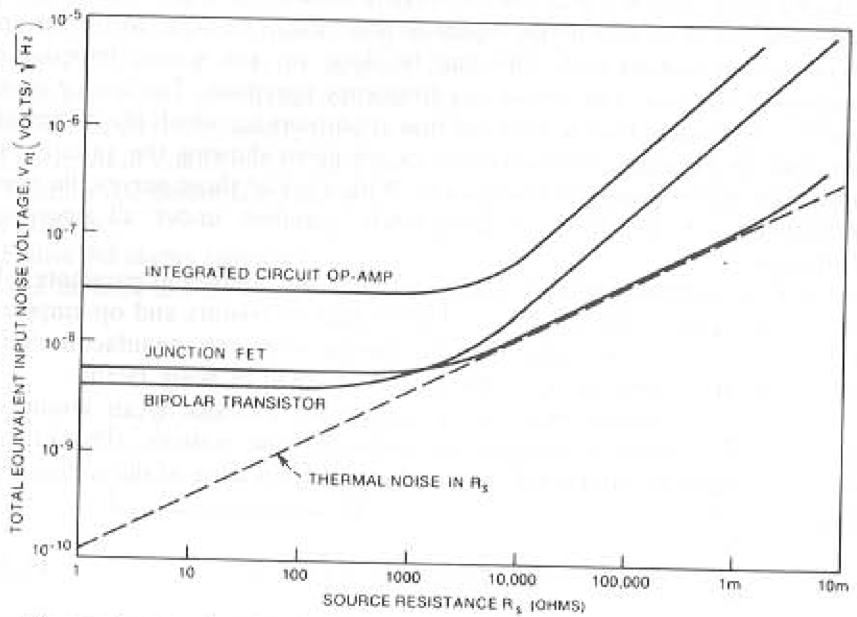


Figure 9-6. *Typical total equivalent noise voltage curves for three types of devices.*

resistance is in this range. With low source resistance, however, a bipolar transistor generally has less noise than an FET. In most cases the op-amp has more noise than either of the other devices. The reasons for this are discussed in the section on op-amp noise.

MEASURMENT OF V_n AND I_n

It is a relatively simple matter to measure the parameters V_n and I_n for a device. The method can best be described by referring to Fig. 9-4 and recalling from Eq. 9-30 that the total equivalent noise voltage V_{nt} is

$$V_{nt} = \sqrt{4kTBR_s + V_n^2 + (I_n R_s)^2}\,. \tag{9-33}$$

To determine V_n, the source resistance is set equal to zero, causing two terms in Eq. 9-33 to equal zero, and the output noise voltage V_{no} is measured. If the voltage gain of the circuit is A,

$$\boxed{V_{no} = AV_{nt} = AV_n\,, \qquad \text{for } R_s = 0} \tag{9-34}$$

The equivalent input noise voltage is

$$V_n = \frac{V_{no}}{A}\,, \qquad \text{for } R_s = 0\,. \tag{9-35}$$

To measure I_n, a second measurement is made with a very large source resistance. The source resistance should be large enough so that the first two terms in Eq. 9-33 are negligible. This will be true if the measured output noise voltage V_{no} is

$$V_{no} \gg A\sqrt{4kTBR_s + V_n^2}\,.$$

Under these conditions the equivalent input noise current is

$$\boxed{I_n = \frac{V_{no}}{AR_s}\,, \qquad \text{for } R_s \text{ large}} \tag{9-36}$$

CALCULATING NOISE FACTOR AND *S/N* RATIO FROM V_n–I_n

Knowing the equivalent input noise voltage V_n, the current I_n, and the source resistance R_s, the noise factor can be calculated by referring to Fig. 9-4. This derivation is left as a problem in Appendix D. The result is

$$\boxed{F = 1 + \frac{1}{4kTB}\left(\frac{V_n^2}{R_s} + I_n^2 R_s\right)}\,. \tag{9-37}$$

where V_n and I_n are the equivalent input noise voltage and current over the bandwidth B of interest.

The value of R_s producing the minimum noise factor can be determined from Eq. 9-37 by differentiating it with respect to R_s. The resulting R_s for minimum noise factor is

$$R_{so} = \frac{V_n}{I_n}. \tag{9-38}$$

If Eq. 9-38 is substituted back into Eq. 9-37, the minimum noise factor can be determined and is

$$\boxed{F_{\min} = 1 + \frac{V_n I_n}{2kTB}}. \tag{9-39}$$

The output power signal-to-noise ratio can also be calculated from the circuit of Fig. 9-4. This derivation is left as a problem in Appendix D. The result is

$$\frac{S_o}{N_o} = \frac{(V_s)^2}{(V_n)^2 + (I_n R_s)^2 + 4kTBR_s}, \tag{9-40}$$

where V_s is the input signal voltage.

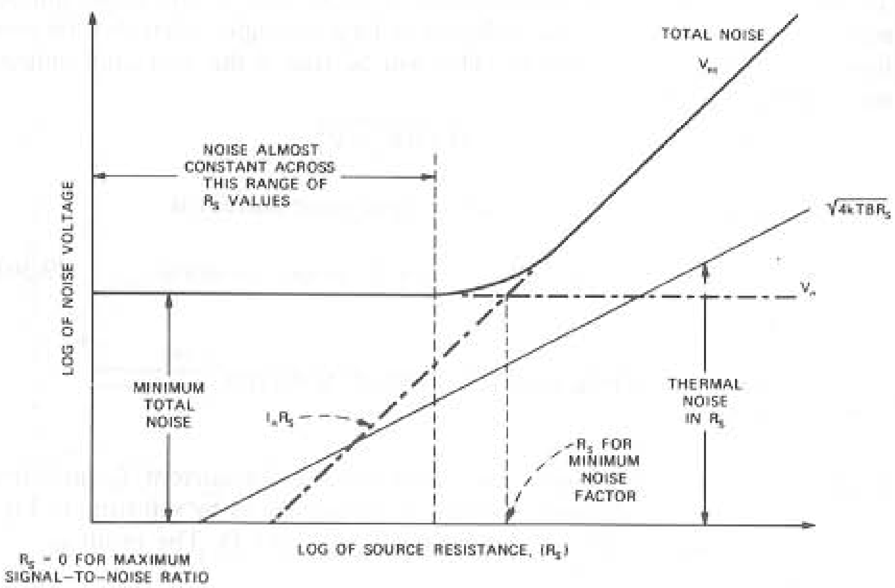


Figure 9-7. *Total equivalent input noise voltage V_{nt} for a typical device. The total noise voltage is made up of three components (thermal noise, V_n, and I_nR_s) as was shown in Eq. 9-30.*

For constant V_s maximum signal-to-noise ratio occurs when $R_s = 0$, and is

$$\left.\frac{S_o}{N_o}\right|_{\max} = \left(\frac{V_s}{V_n}\right)^2. \tag{9-41}$$

It should be noted that when V_s is constant and R_s is variable minimum noise factor occurs when $R_s = V_n/I_n$, but maximum signal-to-noise ratio occurs at $R_s = 0$. Minimum noise factor therefore does not necessarily represent maximum signal-to-noise ratio or minimum noise. This can best be understood by referring to Fig. 9-7, which is a plot of the total equivalent input noise voltage V_{nt} for a typical device. When $R_s = V_n/I_n$, the ratio of the device noise to the thermal noise is a minimum. However, the device noise and the thermal noise are both minimum when $R_s = 0$. Although minimum equivalent input noise voltage (and maximum signal-to-noise ratio) occurs mathematically at $R_s = 0$, there is actually a range of values of R_s over which it is almost constant, as shown in Fig. 9-7. In this range, V_n of the device is the predominant noise source. For large values of source resistance, I_n is the predominant noise source.

OPTIMUM SOURCE RESISTANCE

Since the maximum signal-to-noise ratio occurs at $R_s = 0$ and minimum noise factor occurs at $R_s = V_n/I_n$, the question of what is the optimum source resistance for the best noise performance arises. The requirement of a zero resistance source is impractical since all actual sources have a finite source resistance. However, as was shown in Fig. 9-7, as long as R_s is small there is a range of values over which the total noise voltage is almost constant.

In practice, the circuit designer does not always have control over the source resistance. A source of fixed resistance is used for one reason or another. The question then arises as to whether this source resistance should be transformed to the value that produces minimum noise factor. The answer to this question depends on how the transformation is made.

If the actual source resistance is less than $R_s = V_n/I_n$, a physical resistor should not be inserted in series with R_s to increase the resistance. To do this would produce three detrimental effects:

1. It increases the thermal noise due to the larger source resistance. (This increase is proportional to $\sqrt{R}$.)
2. It increases the noise due to the current from the input noise current generator flowing through the larger resistor. (This increase is proportional to R.)
3. It decreases the amount of the signal getting to the amplifier.

The noise performance can, however, be improved by using a transformer to effectively raise the value of R_s to a value closer to $R_s = V_n/I_n$, thus minimizing the noise produced by the device. At the same time the signal voltage is stepped up by the turns ratio of the transformer. This effect is cancelled by the fact that the thermal noise voltage of the source resistance is also stepped up by the same factor. There is, however, a net increase in signal-to-noise ratio when this is done.

If the actual source resistance is greater than that required for minimum noise factor, noise performance can still be improved by transforming the higher value of R_s to a value closer to $R_s = V_n/I_n$. The noise will, however, be greater than if a lower-impedance source were used.

For optimum noise peformance, the lowest possible source impedance should be used. Once this is decided, noise performance can be further improved by transformer coupling this source to match the impedance $R_s = V_n/I_n$.

The improvement in signal-to-noise ratio that is possible by using a transformer can best be seen by rewriting Eq. 9-3 as

$$\frac{S_o}{N_o} = \frac{1}{F}\left(\frac{S_i}{N_i}\right). \tag{9-42}$$

Assuming a fixed source resistance, adding an ideal transformer of any turns ratio does not change the input signal-to-noise ratio. With the input signal-to-noise ratio fixed, the output signal-to-noise ratio will be maximized when the noise factor F is a minimum. F is a minimum when the device sees a source resistance $R_s = V_n/I_n$. Therefore transformer coupling the actual source resistance minimizes F and maximizes the output signal-to-noise ratio. If the value of the source resistance is not fixed, choosing R_s to minimize F does not necessarily produce optimum noise performance. However, for a given source resistance R_s, the least noisy circuit is the one with the smallest F.

When using transformer coupling, thermal noise in the transformer winding must be accounted for. This can be done by adding to the source resistance the primary winding resistance, plus the secondary winding resistance divided by the square of turns ratio. The turns ratio is defined as the number of turns of the secondary divided by the number of turns of the primary. Despite this additional noise introduced by the transformer, the signal-to-noise ratio is normally increased sufficiently to justify using the transformer if the actual source resistance is more than an order of magnitude different than the optimum source resistance.

Another source of noise to consider when using a transformer is its sensitivity to pickup from magnetic fields. Shielding the transformer is often necessary to reduce this pickup to an acceptable level.

The improvement in signal-to-noise ratio due to transformer coupling can be expressed in terms of the signal-to-noise improvement (SNI) factor

defined as

$$\text{SNI} = \frac{(S/N)\text{ using transformer}}{(S/N)\text{ without transformer}}. \tag{9-43}$$

It can be shown that the signal-to-noise improvement factor can also be expressed in a more useful form as

$$\text{SNI} = \frac{(F)\text{ without tranformer}}{(F)\text{ with transformer}}. \tag{9-44}$$

NOISE FACTOR OF CASCADED STAGES

Signal-to-noise ratio and total equivalent input noise voltage should be used in designing the components of a system for optimum noise performance. Once the components of a system have been designed, it is usually advantageous to express the noise performance of the individual components in terms of noise factor. The noise factor of the various components can then be combined as follows.

The overall noise factor of a series of networks connected in cascade (see Fig. 9-8) was shown by Friis (1944) to be

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots + \frac{F_m - 1}{G_1G_2\cdots G_{m-1}}, \tag{9-45}$$

where F_1 and G_1 are the noise factor and available power gain* of the first stage, F_2, G_2 are those of the second stage.

Equation 9-45 clearly shows the important fact that *with sufficient gain G_1 in the first stage of a system, the total noise factor is primarily determined by the noise factor F_1 of the first stage.*

Example 9-1. Figure 9-9 shows a number of identical amplifiers operating in cascade on a transmission line. Each amplifier has an available power gain

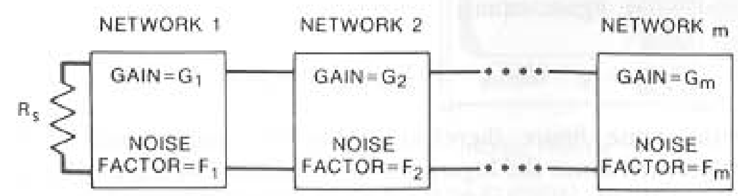


Figure 9-8. *Networks in cascade.*

*$G = A^2R_s/R_o$, where A is the open-circuit voltage gain (open-circuit output voltage divided by source voltage). R_s is the source resistance, and R_o is the network output impedance.

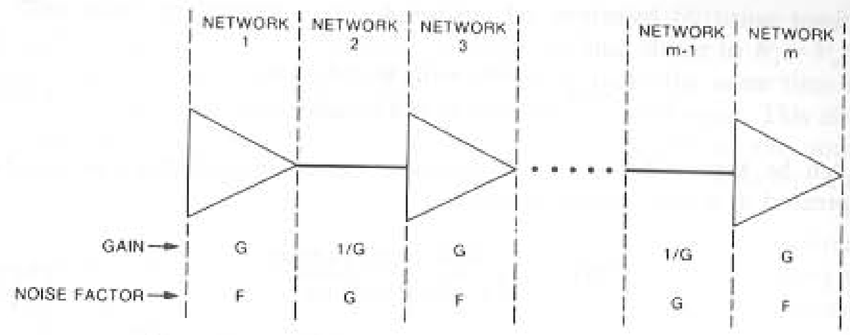


Figure 9-9. *Amplifiers equally spaced on a transmission line.*

G, and the amplifiers are spaced so the loss in the section of cable between amplifiers is also G. This type of arrangement can be used in a telephone trunk circuit or a CATV distribution system. The amplifiers have an available power gain equal to G and a noise factor F. The cable sections have an insertion gain $1/G$ and a noise factor G.* Equation 9-45 then becomes

$$F_t = F + \frac{G-1}{G} + \frac{F-1}{1} + \frac{G-1}{G} + \frac{F-1}{1} + \cdots + \frac{F-1}{1}, \quad (9\text{-}46)$$

$$F_t = F + 1 - \frac{1}{G} + F - 1 + 1 - \frac{1}{G} + F - 1 + \cdots + F - 1. \quad (9\text{-}47)$$

For K amplifiers and $K-1$ cable sections,

$$F_t = KF - \frac{K-1}{G}. \quad (9\text{-}48)$$

If $FG \gg 1$,

$$F_t = KF. \quad (9\text{-}49)$$

The overall noise figure equals

$$(NF)_t = 10 \log F + 10 \log K. \quad (9\text{-}50)$$

The overall noise figure therefore equals the noise figure of the first amplifier plus ten times the logarithm of the number of stages. Another way of looking at this is that every time the number of stages is doubled, the

*This can be derived by applying the basic noise factor definition (Eq. 9-1) to the cable section. The cable is considered a matched transmission line operating at its characteristic impedance.

noise figure increases by 3 dB. This limits the maximum number of amplifiers that can be cascaded.

Example 9-2. Figure 9-10 shows an antenna connected to a TV set by a section of 300-Ω matched transmission line. If the transmission line has 6 dB of insertion loss and the TV set has a noise figure of 14 dB, what signal voltage is required at the antenna terminal for a 40-dB signal-to-noise ratio at the terminals of the TV set? To solve this problem, all the noise sources in the system are converted to equivalent noise voltages at one point, in this case the input to the TV set. The noise voltages can then be combined, and the appropriate signal level needed to produce the required signal-to-noise ratio can be calculated.

The thermal noise at the input of the TV set due to a 300-Ω input impedance with a 4-MHz bandwidth is -53.2 dBmV (2.2 μV).* Since the TV set adds 14 dB of noise to the input thermal noise, the total input noise level is -39.2 dBmV (thermal noise voltage in dB + noise figure). Since a signal-to-noise ratio of 40 dB is required, the signal voltage at the amplifier input must be $+0.5$ dBmV (total input noise in dB + signal-to-noise ratio in

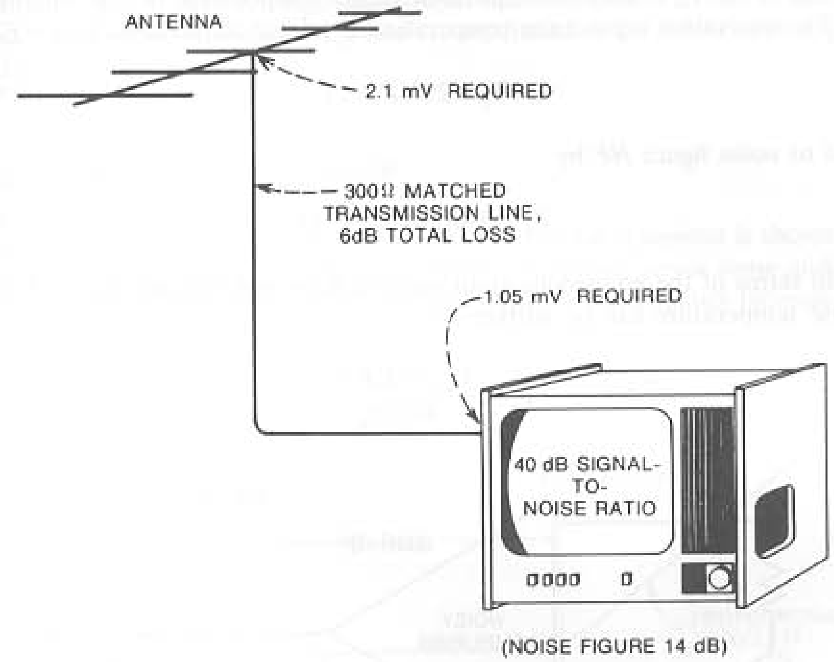


Figure 9-10. *TV set connected to antenna.*

*The open circuit noise voltage at room temperature (290°K) for a 300-Ω resistor and a 4-MHz bandwidth is 4.4 μV. When this source is connected to a 300-Ω load, it delivers one-half of this voltage, or 2.2 μV, to the load.

dB). The transmission line has 6 dB of loss, so the signal voltage at the antenna terminal must be +6.5 dBmV or 2.1 mV. To be able to add terms directly, as in this example, all the quantities must be referenced to the same impedance level, in this case 300 Ω.

NOISE TEMPERATURE

Another method of specifying noise performance of a circuit or device is by the concept of equivalent input noise temperature (T_e).

The equivalent input noise temperature of a circuit can be defined as the increase in source resistance temperature necessary to produce the observed noise power at the output of the circuit. The standard reference temperature T_0 for noise temperature measurements is 290°K.

Figure 9-11 shows a noisy amplifier with a source resistance R_s at temperature T_0. The total measured output noise is V_{no}. Figure 9-12 shows an ideal noiseless amplifier having the same gain as the amplifier in Fig. 9-11 and also a source resistance R_s. The temperature of the source resistance is now increased by T_e, so the total measured output noise V_{no} is the same as in Fig. 9-11. T_e is then the equivalent noise temperature of the amplifier. The equivalent input noise temperature is related to the noise factor F by

$$T_e = 290(F - 1) \tag{9-51}$$

and to noise figure NF by

$$T_e = 290(10^{NF/10} - 1)\,. \tag{9-52}$$

In terms of the equivalent input noise voltage and current ($V_n - I_n$), the noise temperature can be written as

$$\boxed{T_e = \frac{V_n^2 + (I_n R_s)^2}{4kBR_s}}\,. \tag{9-53}$$

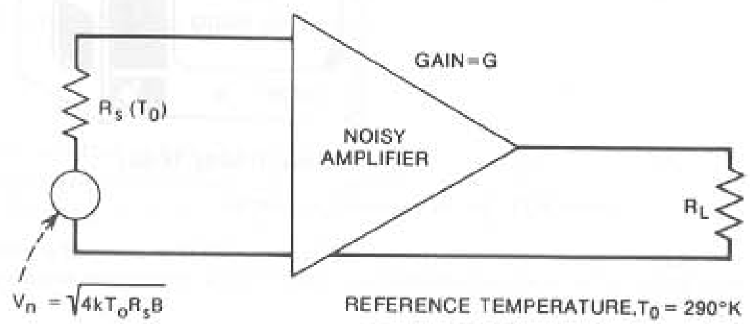


Figure 9-11. *Amplifier with noise.*

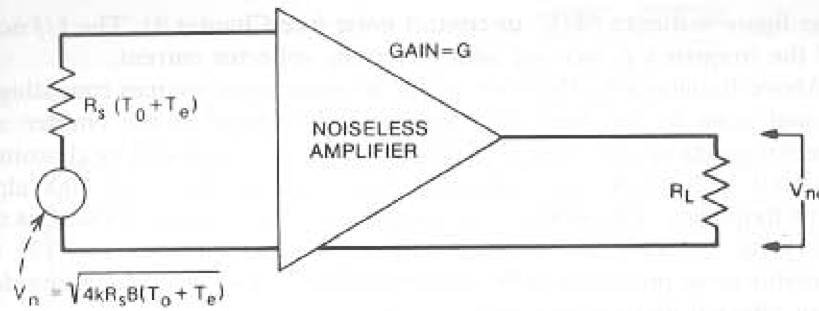


Figure 9-12. *Source resistance temperature increased to account for amplifier noise.*

The equivalent input noise temperature of a number of amplifiers in cascade can be shown to be

$$T_{e(\text{total})} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdots, \tag{9-54}$$

where T_{e1} and G_1 are the equivalent input noise temperature and available power gain of the first stage, T_{e2} and G_2 are the same for the second stage, and so on.

BIPOLAR TRANSISTOR NOISE

The noise figure versus frequency for a typical bipolar transistor is shown in Fig. 9-13. It can be seen that the noise figure is constant across some middle range of frequencies and rises on both sides. The low-frequency increase in

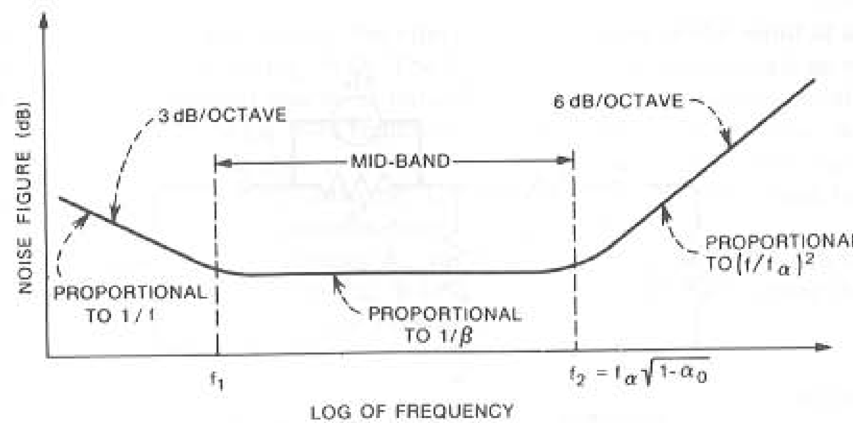


Figure 9-13. *Noise figure versus frequency for bipolar transistor.*

noise figure is due to "$1/f$" or contact noise (see Chapter 8). The $1/f$ noise and the frequency f_1 increase with increasing collector current.

Above frequency f_1, the noise is due to white noise sources consisting of thermal noise in the base resistance and shot noise in the emitter and collector junctions. The white noise sources can be minimized by choosing a transistor with small base resistance, large current gain, and high alpha cutoff frequency. The increase in noise figure at frequencies above f_2 is due to (1) the decrease in transistor gain at these frequencies and (2) the transistor noise produced in the output (collector) junction, which therefore is not affected by transistor gain.

For a typical audio transistor the frequency f_1, below which the noise begins to increase, may be between 1 and 50 kHz. The frequency f_2, above which the noise increases, is usually greater than 10 MHz. In transistors designed for rf use, f_2 may be much higher.

Transistor Noise Factor

The theoretical expression for bipolar transistor noise factor can be derived by starting with the T-equivalent circuit of a transistor, as shown in Fig. 9-14, neglecting the leakage term I_{CBO}. By neglecting $r_c (r_c \gg R_L)$ and adding the following noise sources—(1) thermal noise of the base resistance, (2) shot noise in emitter diode, (3) shot noise in collector, and (4) thermal noise in the source resistance—the circuit can be revised to form the equivalent circuit shown in Fig. 9-15.

The noise factor can be obtained from the circuit in Fig. 9-15 and the relationships

$$I_c = \alpha_o I_e, \tag{9-55}$$

$$r_e = \frac{kT}{qI_e} \approx \frac{26}{I_e(\text{ma})}, \tag{9-56}$$

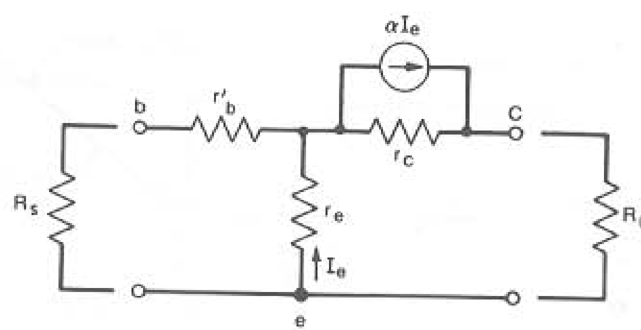


Figure 9-14. *T-equivalent circuit for a transistor.*

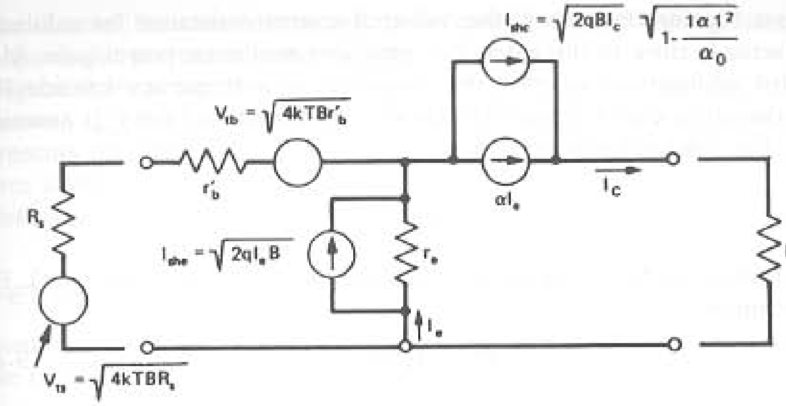


Figure 9-15. *Equivalent circuit of a transistor including noise sources.*

$$|\alpha| = \frac{|\alpha_o|}{\sqrt{1 + (f/f_\alpha)^2}}, \tag{9-57}$$

where α_o is the dc value of the transistor common-base current gain alpha, k is Boltzmann's constant, q is the electron charge, f_α is the transistor alpha cutoff frequency, and f is the frequency variable. Using this equivalent circuit, Nielsen (1957) has shown the noise factor of the transistor to be

$$F = 1 + \frac{r'_b}{R_s} + \frac{r_e}{2R_s} + \frac{(r_e + r'_b + R_s)^2}{2r_eR_s\beta_o}\left[1 + \left(\frac{f}{f_\alpha}\right)^2(1 + \beta_o)\right], \tag{9-58}$$

where β_o is the dc value of the common-emitter current gain beta,

$$\beta_o = \frac{\alpha_o}{1 - \alpha_o}. \tag{9-59}$$

This equation does not include the effect of the $1/f$ noise and is valid at all frequencies above f_1 in Fig. 9-13. The $1/f$ noise can be represented as an additional noise current source in parallel with αI_e in the collector circuit.

The second term in Eq. 9-58 represents the thermal noise in the base, the third term represents shot noise in the emitter, and the fourth term represents shot noise in the collector. This equation is applicable to both the common-emitter and the common-base configurations.

The value of source resistance R_{so} for the minimum noise factor can be determined by differentiating Eq. 9-58 with respect to R_s and setting the result equal to zero. This source resistance is found to be

$$R_{so} = \left[(r'_b + r_e)^2 + \frac{(2r'_b + r_e)\beta_o r_e}{1 + (f/f_\alpha)^2(1 + \beta_o)}\right]^{1/2} \tag{9-60}$$

For most bipolar transistors, the value of source resistance for minimum noise factor is close to the value that produces maximum power gain. Most transistor applications operate the transistor at a frequency considerably below the alpha cutoff frequency. Under this condition ($f \ll f_\alpha$), assuming $\beta_o \gg 1$, Eq. 9-60 reduces to

$$R_{so} = \sqrt{(2r_b' + r_e)\beta_o r_e}\,. \tag{9-61}$$

If in addition the base resistance r_b' is negligible (not always the case), Eq. 9-61 becomes

$$R_{so} \approx r_e\sqrt{\beta_o}\,. \tag{9-62}$$

This equation is also useful for making quick approximations of the source resistance that produces minimum noise factor. Equation 9-62 shows that the higher the common-emitter current gain β_o of the transistor, the higher will be the value of R_{so}.

V_n–I_n for Transistor

To determine the parameters for the equivalent input noise voltage and current model, we must first determine the total equivalent input noise voltage V_{nt}. Substituting Eq. 9-58 into Eq. 9-26, and squaring the result, gives

$$V_{nt}^2 = 2kTB(r_e + 2r_b' + 2R_s) + \frac{2kTB(r_e + r_b' + R_s)^2}{r_e\beta_o}\left[1 + \left(\frac{f}{f_\alpha}\right)^2(1+\beta_o)\right]. \tag{9-63}$$

The equivalent input noise voltage squared V_n^2 is obtained by making $R_s = 0$ in Eq. 9-63 (see Eqs. 9-34 and 9-35), giving

$$V_n^2 = 2kTB(r_e + 2r_b') + \frac{2kTB(r_e + r_b')^2}{r_e\beta_o}\left[1 + \left(\frac{f}{f_\alpha}\right)^2(1+\beta_o)\right]. \tag{9-64}$$

To determine I_n^2, we must divide Eq. 9-63 by R_s^2 and then make R_s large (see Eqs. 9-34 and 9-36), giving

$$I_n^2 = \frac{2kTB}{r_e\beta_o}\left[1 + \left(\frac{f}{f_\alpha}\right)^2(1+\beta_o)\right]. \tag{9-65}$$

JUNCTION FIELD EFFECT TRANSISTOR NOISE

There are three important noise mechanisms in a junction FET: (1) the shot noise produced in the reverse biased gate, (2) the thermal noise generated

in the channel between source and drain, and (3) the $1/f$ noise generated in the space charge region between gate and channel.

Figure 9-16 is the noise equivalent circuit for a junction FET. Noise generator I_{sh} represents the shot noise in the gate circuit, and generator I_{tc} represents the thermal noise in the channel. I_{ts} is the thermal noise of the source admittance G_s. The FET has an input admittance g_{11}, and a forward transconductance g_{fs}.

FET Noise Factor

Assuming no correlation between I_{sh} and I_{tc}* in Fig. 9-16, the total output noise current can be written as

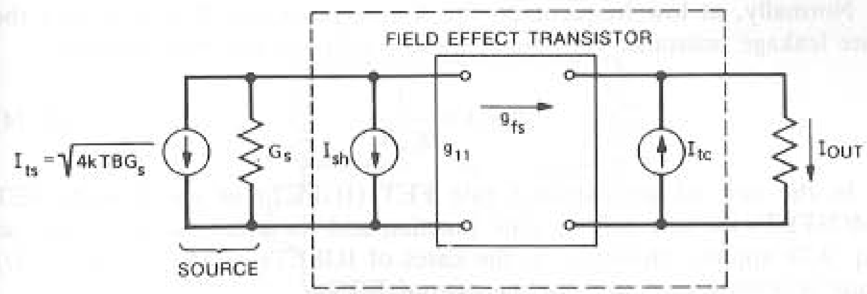
$$I_{out} = \left[\frac{4kTBG_s g_{fs}^2}{(G_s + g_{11})^2} + \frac{I_{sh}^2 g_{fs}^2}{(G_s + g_{11})^2} + I_{tc}^2\right]^{1/2}. \tag{9-66}$$

The output noise current due to the thermal noise of the source only is

$$I_{out}(\text{source}) = \left(\frac{\sqrt{4kTBG_s}}{G_s + g_{11}}\right)g_{fs}\,. \tag{9-67}$$

The noise factor F is Eq. 9-66 squared, divided by Eq. 9-67 squared, or

$$F = 1 + \frac{I_{sh}^2}{4kTBG_s} + \frac{I_{tc}^2}{4kTBG_s(g_{fs})^2}(G_s + g_{11})^2. \tag{9-68}$$



g_{11} = INPUT ADMITTANCE
g_{fs} = FORWARD TRANSCONDUCTANCE (AMPERES PER VOLT)
G_s = SOURCE ADMITTANCE

Figure 9-16. *Noise equivalent of junction field effect transistor.*

*At high frequencies, noise generators I_{sh} and I_{tc} show some correlation. As a practical matter, however, this is normally neglected.

I_{sh} is the input shot noise and equals

$$I_{sh} = \sqrt{2qI_{gss}B}\ , \tag{9-69}$$

where I_{gss} is the total gate leakage current. I_{tc} is the thermal noise of the channel and equals

$$I_{tc} = \sqrt{4kTBg_{fs}}\ . \tag{9-70}$$

Substituting Eqs. 9-69 and 9-70 into 9-68, and recognizing that

$$\frac{2q}{4kT} I_{gss} = g_{11}\ , \tag{9-71}$$

gives for the noise factor

$$F = 1 + \frac{g_{11}}{G_s} + \frac{1}{G_s g_{fs}} (G_s + g_{11})^2\ . \tag{9-72}$$

Rewriting Eq. 9-72 in terms of the resistances instead of admittances gives

$$F = 1 + \frac{R_s}{r_{11}} + \frac{R_s}{g_{fs}}\left(\frac{1}{R_s} + \frac{1}{r_{11}}\right)^2 . \tag{9-73}$$

Neither Eq. 9-72 nor Eq. 9-73 include the effect of the $1/f$ noise. The second term in the equations represent the contribution from the shot noise in the gate junction. The third term represents the contribution of the thermal noise in the channel.

For low noise operation, an FET should have high gain (large g_{fs}) and a high input resistance r_{11} (small gate leakage).

Normally, at low frequencies, the source resistance R_s is less than the gate leakage resistance r_{11}. Under these conditions Eq. 9-73 becomes

$$\boxed{F \approx 1 + \frac{1}{g_{fs}R_s}}\ . \tag{9-74}$$

In the case of an insulated gate FET (IGFET) or metal oxide FET (MOSFET) there is no p–n gate junction and therefore no shot noise, so Eq. 9-74 applies. However, in the cases of IGFETs or MOSFETs the $1/f$ noise is often greater than in the case of JFETs.

$V_n - I_n$ Representation of FET

The total equivalent input noise voltage can be obtained by substituting Eq. 9-73 into Eq. 9-26, giving

$$V_{nt}^2 = 4kTBR_s\left[1 + \frac{R_s}{r_{11}} + \frac{R_s}{g_{fs}}\left(\frac{1}{R_s} + \frac{1}{r_{11}}\right)^2\right] . \tag{9-75}$$

Making $R_s = 0$ in Eq. 9-75 gives the equivalent input noise voltage squared (see Eqs. 9-34 and 9-35) as

$$V_n^2 = \frac{4kTB}{g_{fs}}\ . \tag{9-76}$$

To determine I_n^2, we must divide Eq. 9-75 by R_s^2 and then make R_s large (see Eqs. 9-34 and 9-36), giving

$$I_n^2 = \frac{4kTB(1 + g_{fs}r_{11})}{g_{fs}r_{11}^2}\ . \tag{9-77}$$

For the case when $g_{fs}r_{11} \gg 1$, Eq. 9-77 becomes

$$I_n^2 = \frac{4kTB}{r_{11}}\ . \tag{9-78}$$

NOISE IN IC OPERATIONAL AMPLIFIERS

The input stage of an operational amplifier is of primary concern in determining the noise performance of the device. Most monolithic op-amps use a differential input configuration that uses two and sometimes four input transistors. Figure 9-17 shows a simplified schematic of a typical two-transistor input circuit used in an operational amplifier. Since two input transistors are used, the noise voltage is approximately $\sqrt{2}$ times that for a single-transistor input stage. In addition some monolithic transistors have

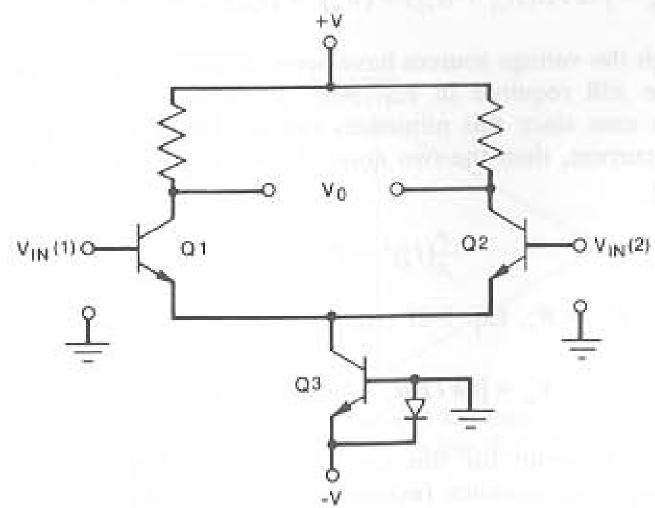


Figure 9-17. *Typical input circuit schematic of an IC operational amplifier. Transistor Q_3 acts as a constant current source to provide dc bias for the input transistors Q_1 and Q_2.*

lower current gains (β) than discrete transistors, and that also increases the noise.

Therefore, in general, operational amplifiers are inherently higher noise devices than discrete transistor amplifiers. This can be seen in the typical equivalent input noise voltage curves shown in Fig. 9-6. A discrete bipolar transistor stage preceding an op-amp can often provide lower noise performance along with the other advantages of the operational amplifier. Op-amps do have the advantage of a balanced input with low temperature drift and low-input offset currents.

The noise characteristics of an operational amplifier can best be modeled by using the equivalent input noise voltage and current $V_n - I_n$. Figure 9-18A shows a typical operational amplifier circuit. Figure 9-18B shows this same cicuit with the equivalent noise voltage and current sources included.

The equivalent circuit in Fig. 9-18B can be used to calculate the total equivalent input noise voltage, which is

$$V_{nt} = [4kTB(R_{s1} + R_{s2}) + V_{n1}^2 + V_{n2}^2 + (I_{n1}R_{s1})^2 + (I_{n2}R_{s2})^2]^{1/2} . \tag{9-79}$$

It should be noted that V_{n1}, V_{n2}, I_{n1}, and I_{n2} are also functions of the bandwidth B.

The two noise voltage sources of Eq. 9-79 can be combined by defining

$$(V_n')^2 = V_{n1}^2 + V_{n2}^2 . \tag{9-80}$$

Equation 9-79 can then be rewritten as

$$V_{nt} = [4kTB(R_{s1} + R_{s2}) + (V_n')^2 + (I_{n1}R_{s1})^2 + (I_{n2}R_{s2})^2]^{1/2} . \tag{9-81}$$

Although the voltage sources have been combined, the two noise current sources are still required in Eq. 9-81. If, however, $R_{s1} = R_{s2}$, which is usually the case since this minimizes the dc output offset voltage due to input bias current, then the two noise current generators can be combined by defining

$$(I_n')^2 = I_{n1}^2 + I_{n2}^2 . \tag{9-82}$$

For $R_{s1} = R_{s2} = R_s$, Eq. 9-81 reduces to

$$V_{nt} = [8kTBR_s + (V_n')^2 + (I_n'R_s)^2]^{1/2} . \tag{9-83}$$

The equivalent circuit for this case is shown in Fig. 9-18C. To obtain optimum noise performance (maximum signal-to-noise ratio) from an op-amp, the total equivalent input noise voltage V_{nt} should be minimized.

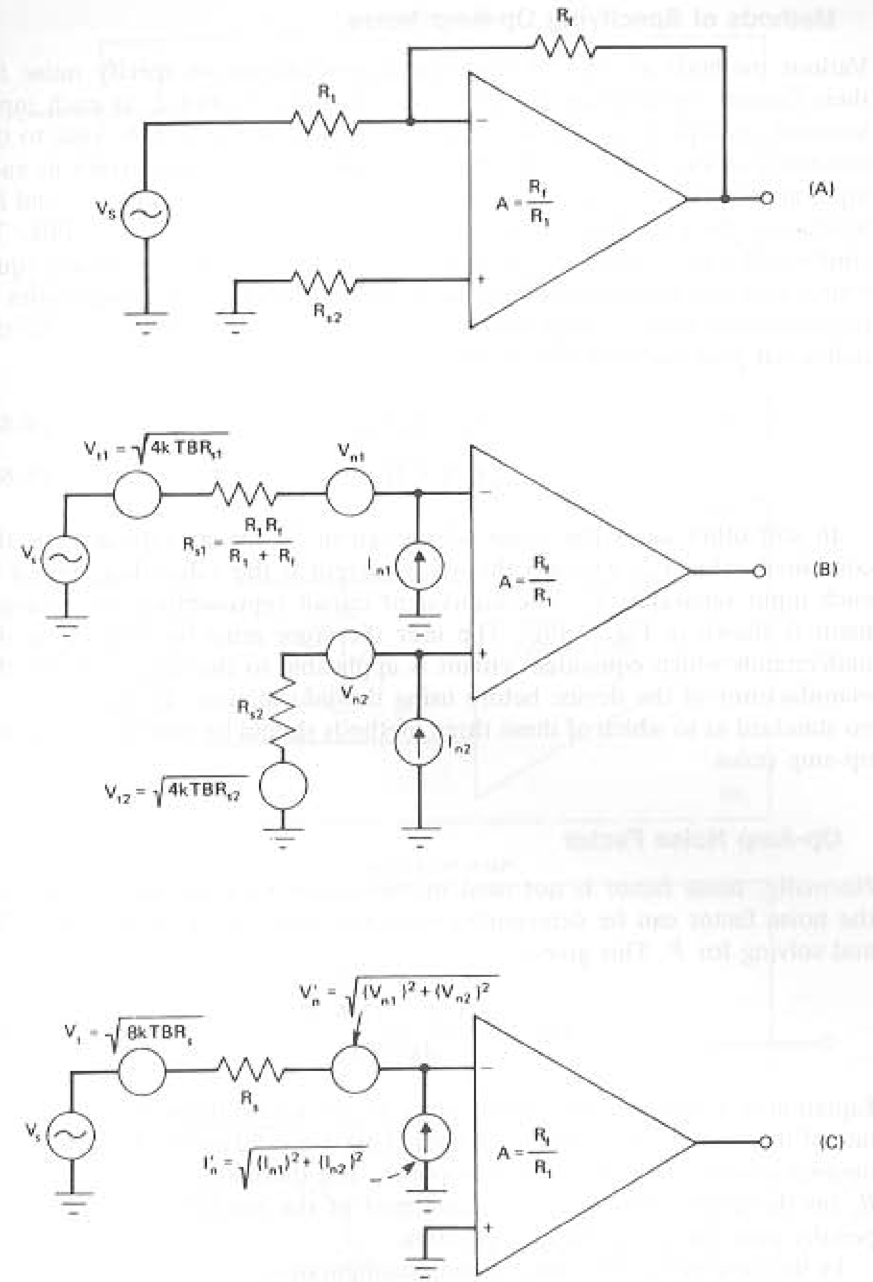


Figure 9-18. *(A) Typical op-amp circuit; (B) Circuit of A with noise sources added; (C) Circuit of B with noise sources combined at one terminal for the case $R_{s1} = R_{s2} = R_s$.*

Methods of Specifying Op-Amp Noise

Various methods are used by op-amp manufacturers to specify noise for their devices. Sometimes they provide values for V_n and I_n at each input terminal, as represented by the equivalent circuit in Fig. 9-19A. Due to the symmetry of the input circuit, the noise voltage and noise current at each input are equal. A second method is to provide combined values, V'_n and I'_n, which are then applied to one input only, as shown in Fig. 9-19B. To combine the two noise current generators, it must be assumed that equal source resistors are connected to the two input terminals. The magnitudes of the combined noise voltage generators in Fig. 9-19B, with respect to the individual generators in Fig. 9-19A, are

$$V'_n = \sqrt{2}V_n\,, \tag{9-84}$$

$$I'_n = \sqrt{2}I_n\,. \tag{9-85}$$

In still other cases the noise voltage given by the manufacturer is the combined value V'_n, whereas the noise current is the value that applies to each input separately I_n. The equivalent circuit representing this arrangement is shown in Fig. 9-19C. The user therefore must be sure he or she understands which equivalent circuit is applicable to the data given by the manufacturer of the device before using the information. To date, there is no standard as to which of these three methods should be used for specifying op-amp noise.

Op-Amp Noise Factor

Normally, noise factor is not used in connection with op-amps. However, the noise factor can be determined by substituting Eq. 9-83 into Eq. 9-26, and solving for F. This gives

$$F = 2 + \frac{(V'_n)^2 + (I'_n R_s)^2}{4kTBR_s}\,. \tag{9-86}$$

Equation 9-86 assumes the source noise is due to the thermal noise in just one of the source resistors R_s, not both. This is a valid assumption when the op-amp is used as a single-ended amplifier. The thermal noise in the resistor R_s on the unused input is considered part of the amplifier noise and is a penalty paid for using this configuration.

In the case of the inverting op-amp configuration, the noise due to R_s at the unused input may be bypassed with a capacitor. This is not possible, however, in the noninverting configuration, since the feedback is connected to this point.

A second method of defining the noise factor for an op-amp is to assume the source noise is due to the thermal noise of both source resistors ($2R_s$ in

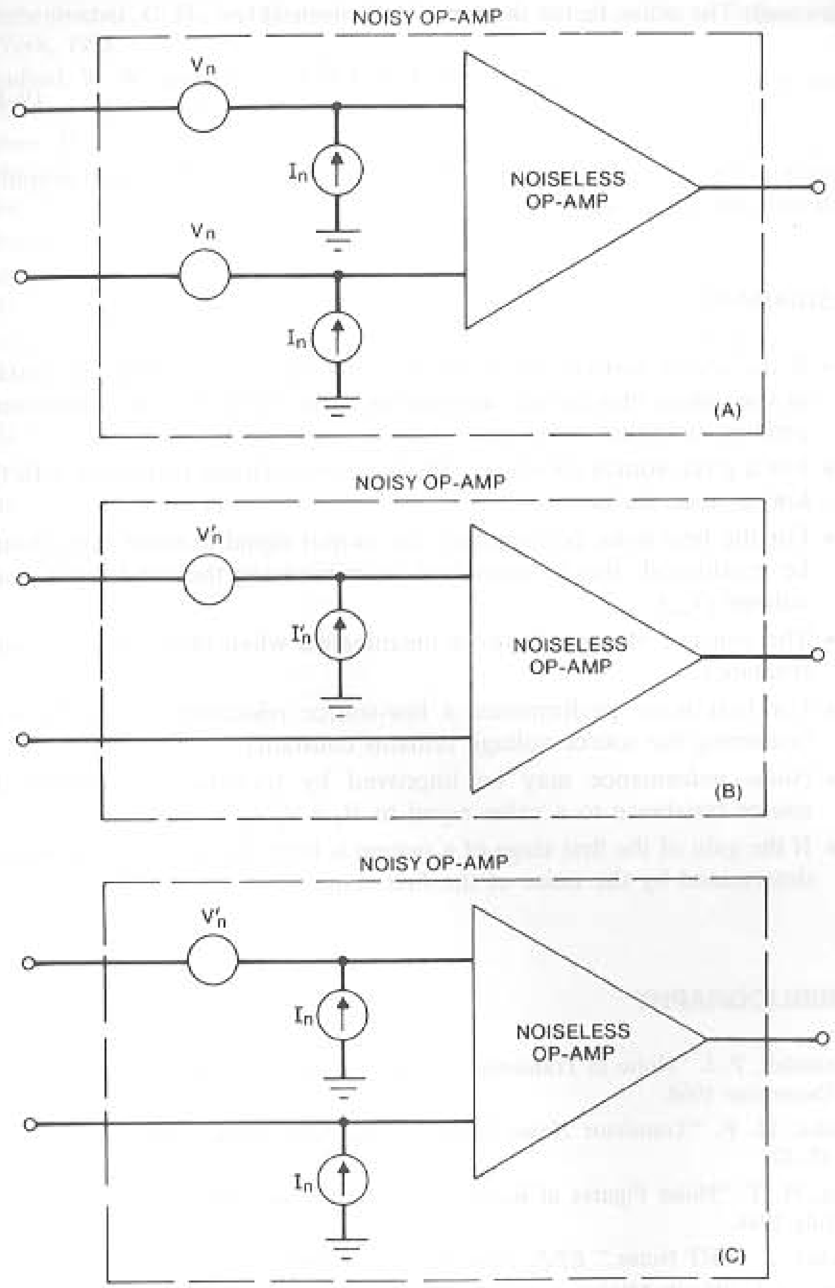


Figure 9-19. *Methods of modeling op-amp noise: (A) separate noise generators at each input; (B) noise generators combined at one input; (C) separate noise current generators with combined noise voltage generator.*

this case). The noise factor then can be written as

$$F = 1 + \frac{(V_n')^2 + (I_n'R_s)^2}{8kTBR_s} . \tag{9-87}$$

Equation 9-87 is applicable if the op-amp is used as a differential amplifier with both inputs driven.

SUMMARY

- If the source resistance is a variable and the source voltage a constant in the design of a circuit, minimizing noise factor does not necessarily produce optimum noise performance.
- For a given source resistance, the least noisy circuit is the one with the lowest noise factor.
- For the best noise performance the output signal-to-noise ratio should be maximized, this is equivalent to minimizing the total input noise voltage (V_{nt}).
- The concept of noise factor is meaningless when the source is a pure reactance.
- For best noise performance a low-source resistance should be used (assuming the source voltage remains constant).
- Noise peformance may be improved by transformer coupling the source resistance to a value equal to $R_s = V_n/I_n$.
- If the gain of the first stage of a system is high, the total system noise is determined by the noise of the first stage.

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