Mathematics for Economists

Problem Set 7
Due date: Friday 25.11 at 12.15

Exercise 1

A consumer's utility function is $u(x, y) = x^2y^3$ and the budget constraint is $p_x x + p_y y \leq w$, where the parameters p_x, p_y and w are all strictly positive.

(a) Solve the consumer's utility maximization problem.

(b) Use the envelope theorem to estimate the change in the indirect utility function (i.e., the problem's value function) when the price p_y is changed to $p_y + \epsilon$, with $\epsilon > 0$.

Exercise 2

Consider the following constrained maximization problem:

$$\begin{aligned} \max_{x,y,z} \quad & f(x, y, z) = ax + by + cz \\ \text{s.t.} \quad & g(x, y, z) = \alpha x^2 + \beta y^2 + \gamma z^2 \leq G, \end{aligned}$$

where $a, b, c, \alpha, \beta, \gamma, G$ are all positive parameters.

(a) Solve the maximization problem.

(b) Let V be the value function of this maximization problem, $\mathcal{L}$ the corresponding Lagrangian function and λ the Lagrange multiplier. Verify that:

$$\frac{dV}{dG} = \lambda^*.$$

Exercise 3

Find the solutions of the following difference equations with the given initial values of x_0. In addition, check what happens when t goes to infinity.

(a) $x_{t+1} = 2x_t + 4$, $x_0 = 1$;

(b) $3x_{t+1} = x_t + 2$, $x_0 = 2$;

(c) $2x_{t+1} + 3x_t + 2 = 0$, $x_0 = -1$;

(d) $x_{t+1} - x_t + 3 = 0$, $x_0 = 3$.

Exercise 4

Solve the below difference equation system for initial values $x_0 = 25$ ja $y_0 = 0$.

$$\begin{aligned} x_{k+1} &= 1.5x_k + y_k, \\ y_{k+1} &= x_k. \end{aligned}$$

What happens to the ratio x_k/y_k, when k goes to infinity?

Exercise 5

Consider the following square matrix:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}.$$

(a) Show that $\boldsymbol{v}_1 = (2, 3)^T$ and $\boldsymbol{v}_2 = (1, -1)^T$ are the two eigenvectors of A.

(b) For each eigenvector, find the corresponding eigenvalue.

(c) Use the two eigenvectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ and the eigenvalues you obtained in question (b) to find a square matrix P and a diagonal matrix D such that

$$A = PDP^{-1}. \tag{1}$$

(d) Use (1) to calculate A^n, where $n \geq 1$.