Model Solutions 8

1. (a) The profit maximizing pricing scheme is to either sell to the high-type (seller is in a hurry) or to sell both low and high-type customers.

We can calculate first the profit if the firm sells only to high-types. The profit maximizing price in this case is to price overnight delivery, denoted as p_H, at 40€.Denote profit for this strategy as π_H. There are 1 million potential deliveries of which 50 percent are in a hurry. The profit in €m is:

$$\pi_H = (40 - 5) \cdot 0.5 = 17.5$$

The company can also sell to both types of customers. The profit maximizing strategy is to price the low-type package so that the surplus for customers that are not in a hurry is zero. Price of the regular service, p_L, should be 12€. Because customers can choose which service to choose, the company needs to price its services so that high-type customers want to choose the high-type service. In other words, the incentive compatibility constraint needs to hold:

$$B_H(H) - P_H \geq B_H(L) - P_L$$

We know that $P_L = 12$, $B_H(H) = 40$, $B_H(L) = 15$. Note that when selling to both customer types, the higher quality service cannot be priced at 40 when lower type is priced at 12 because then customers in a hurry will choose the regular service. The IC-constraint needs to hold as an equality because profit is increasing in p_H:

$$40 - P_H = 15 - 12$$
$$P_H = 40 - 3 = 37$$

Profit when selling to both customers, π_B, is:

$$\pi_B = 37 \cdot 0.5 + 12 \cdot 0.5 - 1 \cdot 5 = 19.5$$

The optimal strategy is to sell to both types and profit is 19.5 €m.

(b) We can calculate profits of different pricing schemes for the company. One alternative is to sell only to high-types at 40€, which generates 17.5 €m profits. Another alternative is to sell to both types at one price. The optimal price would be to sell only overnight service at 20€. Profit for this service would be:

$$\pi_B = (20 - 5) \cdot 1 = 15$$

So the optimal is to sell only to high-types at 40. The consumer surplus is zero in this case. In a.) the low-types received no surplus and the high-types received as much as the IC-constraint allowed them to have. The surplus in 1a is:

$$CS_T = CS_L + CS_H = 0 + (B_H(H) - P_H) \cdot 0.5 = (40 - 37) \cdot 0.5 = 1.5$$

So the loss in CS_T in 1a compared to 1b is -1.5€m.

(c) Denote marginal cost for regular service as $MC_r = x$ and $MC_o = 5$. We can rewrite profit when selling to both types as a function of regular service's marginal cost. The optimal in a.) was to sell to both types. Then we can calculate the point where the optimal strategy just holds:

$$\begin{aligned} \pi_B &\geq \pi_H \\ (37-5)\cdot 0.5 + (12-x)\cdot 0.5 &\geq 17.5 \\ 22 - 0.5x &\geq 17.5 \\ 0.5x &\leq 4.5 \\ x &\leq 9 \end{aligned}$$

So the marginal cost for regular service needs to be lower or equal to 9 for the optimal strategy in 1a to hold. So if $MC_r = 9$, the strategy just holds.

2. (a) This is a problem of quantity discounts, where we want to solve for the optimal package sizes and prices for chocoholics (High type) and ordinaries (Low type). Let's first express the total benefit from consuming Q units for both types by using the formula $B(Q) = \alpha Q - (\beta/2)Q^2$ (area of a trapezoid):

$$Q_H(p) = 50 - 10p \Leftrightarrow B_H(q) = 5q - \frac{1}{20}q^2$$
$$Q_L(p) = 30 - 6p \Leftrightarrow B_L(q) = 5q - \frac{1}{12}q^2$$

i). The large package is of the efficient size for H-types ("no distortion at the top"):

$$\begin{aligned} P_H(q_H) = \text{MC} \Leftrightarrow 5 - \frac{1}{10}q_H &= 0.5 \\ q_H^* &= 45 \end{aligned}$$

ii). The price of the small package will extract all surplus from the low types ("no surplus at the bottom"):

$$P_L(q_L) = B_L(q_L) \Rightarrow P_L(q_L) = 5q_L - \frac{1}{12}q_L^2$$

iii). Price of large package is such that high types will choose that and not the low-type package ("self-selection constraint"):

$$\begin{aligned} P_H(q_L) &= B_L(q_L) + (B_H(q_H^*) - B_H(q_L)) \\ &= 5q_L - \frac{1}{12}q_L^2 + (5\times 45 - \frac{1}{20}\times 45^2 - (5q_L - \frac{1}{20}q_L^2)) \\ &= -\frac{1}{30}q_L^2 + \frac{495}{4} \end{aligned}$$

iv). Using these results, let's formulate the profit function and maximize:

$$\begin{aligned}\Pi(q_L) &= N_L P_L(q_L) + N_H P_H(q_L) - (N_L q_L + N_H q_H^*) \times \text{MC}\\ &= 100(5q_L - \frac{1}{12}q_L^2) + 200(-\frac{1}{30}q_L^2 + \frac{495}{4}) - (100q_L + 200 \times 45) \times 0.5\\ &= 500q_L - \frac{25}{3}q_L^2 - \frac{20}{3}q_L^2 + 24\ 750 - 50q_L - 4\ 500\\ &= -15q_L^2 + 450q_L + 20\ 250\end{aligned}$$

Maximization:

$$\begin{aligned}\frac{\partial \Pi(q_L)}{\partial q_L} &= -30q_L + 450 = 0\\ \implies q_L^* &= 15\end{aligned}$$

The optimal {small, large} packages $\{q_L^* = 15, q_H^* = 45\}$ priced at:

$$\begin{aligned}P_L^* &= B_L(45) = 5 \times 15 - \frac{1}{12}15^2 = 56.25\ €\\ P_H^* &= P_H(45) = -\frac{1}{30}15^2 + \frac{495}{4} = 116.25\ €\end{aligned}$$

v). Comparison of profits. Selling to both types with optimal quantity discount:

$$\Pi(15) = -15 \times 15^2 + 450 \times 15 + 20\ 250 = 23\ 625\ €$$

Selling only to high types (at high type reservation price):

$$\begin{aligned}\Pi_H(q_H^*) &= N_H(B_H(q_H^*) - \text{MC} \times q_H^*)\\ &= 200 \times (5 \times 45 - \frac{1}{20}45^2 - 0.5 \times 45)\\ &= 20\ 250\ €\end{aligned}$$

Selling to both types is more profitable than selling only to high types.

(b) *i*). The new optimal large package size:

$$\begin{aligned}P_H(q_H) = \text{MC} \Leftrightarrow 5 - \frac{1}{10}q_H &= 1.4\\ q_H^* &= 36\end{aligned}$$

ii). The price of the large package:

$$\begin{aligned}P_H(q_L) &= B_L(q_L) + (B_H(q_H^*) - B_H(q_L))\\ &= 5q_L - \frac{1}{12}q_L^2 + (5 \times 36 - \frac{1}{20} \times 36^2 - (5q_L - \frac{1}{20}q_L^2))\\ &= -\frac{1}{30}q_L^2 + \frac{576}{5}\end{aligned}$$

iii). The profit function:

$$\begin{aligned}
\Pi(q_L) &= N_L P_L(q_L) + N_H P_H(q_L) - (N_L q_L + N_H q_H^*) \times \text{MC} \\
&= 100(5q_L - \frac{1}{12}q_L^2) + 200(-\frac{1}{30}q_L^2 + \frac{576}{5}) - (100q_L + 200 \times 36) \times 1.4 \\
&= 500q_L - \frac{25}{3}q_L^2 - \frac{20}{3}q_L^2 + 23\ 040 - 140q_L - 10\ 080 \\
&= -15q_L^2 + 360q_L + 12\ 960
\end{aligned}$$

And its maximization:

$$\begin{aligned}
&\frac{\partial \Pi(q_L)}{\partial q_L} = -30q_L + 360 = 0 \\
&\implies q_L^* = 12
\end{aligned}$$

The optimal {small, large} packages $\{q_L^* = 12, q_H^* = 36\}$ priced at:

$$\begin{aligned}
P_L^* &= B_L(12) = 5 \times 12 - \frac{1}{12}12^2 = 48 \text{ €} \\
P_H^* &= P_H(12) = -\frac{1}{30}12^2 + \frac{576}{5} = 110.4 \text{ €}
\end{aligned}$$

Let's then calculate the inflation in price per chocolate piece for both customer types by comparing prices per piece before and after the supply crunch (Price inflation = $\frac{P_{new}}{Q_{new}} / \frac{P_{old}}{Q_{old}}$):

$$\begin{aligned}
\text{P.infl}_L &= \frac{48}{12} / \frac{56.25}{15} \approx 6.7\% \\
\text{P.infl}_H &= \frac{110.4}{36} / \frac{116.25}{45} \approx 18.7\%
\end{aligned}$$

The high-type customers end up with higher price inflation. Lastly, let's verify that this still is the profit-maximizing strategy by comparing the profits of selling to both types to the profits of selling only to high types:

$$\begin{aligned}
\text{Both types: } \Pi(12) &= -15 \times 12^2 + 360 \times 12 + 12\ 960 = 15\ 120 \text{ €} \\
\text{Only high types: } \Pi_H(q_H^*) &= 200 \times (5 \times 36 - \frac{1}{20}36^2 - 1.4 \times 36) = 12\ 960 \text{ €}
\end{aligned}$$

Selling to both types is more profitable, which means the price inflation calculations are valid.

3. All valuations in this exercise are, unless otherwise stated, **net of marginal cost**, to make comparison of profits easier. Profits are expressed in euros per every three customers. The net valuations are:

Valuations net of MC

€	V_P	V_S	V_B
Away fans	4	16	20
Home fans	18	6	24
Tourists	12	14	26

Marginal costs are $MC_P = 2$, $MC_S = 4$ and $MC_{P+S} = 6$.

(a) Since the only available bundling strategy is to sell all the goods in a bundle, the profit-maximizing bundling strategy is to sell to all customers at $P_B = 20 + MC_{P+S} = 26$, earning $3 \times V_B^A = 60$ euros in profits. The optimal basic pricing strategy would be to sell pennants to Tourists and Home fans, and scarves to Away Fans and Tourists, which would lead to $2 \times V_P^T + 2 \times V_S^T = 2 \times 12 + 2 \times 14 = 52$ euros in profits. Pure bundling is thus optimal.

(b) Now it is possible to use mixed bundling. Since Away fans have the highest valuation for the scarf and the lowest valuation for the bundle, while Home fans and Tourists have high valuations for the pennant, the profit-maximizing mixed bundling strategy is to price the bundle so that Home fans and Tourists buy it and the scarf so that Away fans buy it and not the bundle. The same is shown graphically in figure 1.

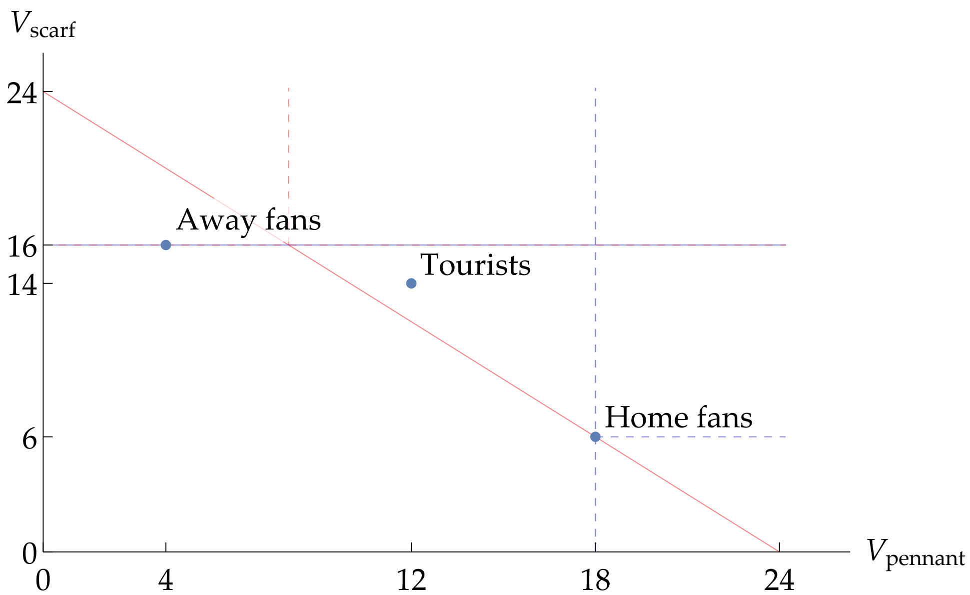

Figure 1: Valuations of different customer types

The prices are $P_S = 16 + \text{MC}_S = 20$ and $P_B = 24 + \text{MC}_{P+S} = 30$. Profits are $(P_S - \text{MC}_S) + 2 \times (P_B - \text{MC}_{P+S}) =$ €64. This is higher than the profits from pure bundling, so it is the optimal pricing strategy.

(c) Now the products are substitutes and the valuation table becomes:

Valuations net of MC

€	V_P	V_S	$V_\text{B} = ((V_P + \text{MC}_P) + (V_S + \text{MC}_S))/2 - (\text{MC}_P + \text{MC}_S)$
Away fans	4	16	$((4+2)+(16+4))/2 - 6 = 7$
Home fans	18	6	$((18+2)+(6+4))/2 - 6 = 9$
Tourists	12	14	$((12+2)+(14+4))/2 - 6 = 10$

The valuations for the bundle are now significantly lower than before. The optimal bundling strategy would be the same as in part 3b, with prices $P_S = 16 + \text{MC}_S = 20$ and $P_B = 9 + \text{MC}_B = 15$ and profits $V_S^A + 2 \times V_B^H = 16 + 2 \times 9 = 34$ euros.

This strategy does not maximize profits. The firm could improve by using basic pricing and selling scarves to Away fans and Tourists and Pennants Home fans. The optimal prices are $P_S = 14 + \text{MC}_S = 18$ and $P_P = 18 + \text{MC}_P = 20$. Profits are $V_P^H + 2 \times V_S^T = 18 + 2 \times 14 = 46$. This is the profit-maximizing pricing strategy.

4. (a) The expected net benefit of a Zorgian who is considering sending a satellite to LZO is, as a function of existing satellites in LZO:

$$\begin{aligned} E(\Pi(n)) &= 10(1 - p(n)) - 10p(n) \\ &= 10 - 10 \times 10^{-6} n^2 \end{aligned}$$

The expected benefit declines as the number of satellites increases. Zorgians will send satellites to the orbit up to the point where the expected private net benefit drops to zero. Let's solve for this point:

$$E(\Pi(n)) = 10 - 10 \times 10^{-6} n^2 = 0$$
$$\Leftrightarrow$$
$$n = 1000$$

There will be 1000 satellites in LZO. Expected value generated is zero.

(b) Total welfare (TW) of LZO satellites equals to the number of satellites times expected private net benefit per satellite. Let's express this as a function of n:

$$\begin{aligned} E(\text{TW}(n)) &= n \times E(\Pi(n)) \\ &= n \times \Big\{ 10(1 - p(n)) - 10p(n) \Big\} \\ &= n(10 - 10 \times 10^{-6} n^2) \\ &= 10n - 10^{-5} n^3 \end{aligned}$$

Let's differentiate this wrt. n to get the optimal number of satellites

$$\frac{\partial E(\text{TW}(n))}{\partial n} = 10 - 3 \times 10^{-5} n^2 = 0$$
$$\implies n^2 = \frac{10}{3 \times 10^{-5}}$$
$$n \approx 577$$

577 satellites in LZO maximizes expected total welfare.

(c) A satellite sender considers only her private benefit but causes a negative externality to all other satellites in LZO. An optimal tax balances the expected private benefit of an additional satellite with the negative externality. Since we know that the negative externality exceeds the expected marginal net benefit if the number of satellites is higher than 577, we need to solve for a tax per satellite sent that makes it unprofitable to send more than 577 satellites to LZO. Thus, the tax needs to be equal to the expected private net benefit evaluated at $n = 577$:

$$E(\Pi(577)) = 10 - 10 \times 10^{-6} \times 577^2 \approx 6.67 \text{ \$Alt}$$

The optimal tax is 6.67 $Alt.

5. To be released later