Mathematics for Economists

Problem Set 8
Due date: Friday 2.12 at 12.15

Exercise 1

a) Assume that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and $a, b, c, d > 0$ satisfy $a + b = 1$ and $c + d = 1$. Verify that $r = 1$ is and eigenvalue of A with the corresponging eigenvetor $\mathbf{x} = (1, 1)$.
b) Use the property of the determinant $\det(B) = \det(B^T)$ to show that the eigenvalues of A and A^T are the same.

Exercise 2

a) Assume that $A = \begin{pmatrix} 0.3 & 0.6 \\ 0.7 & 0.4 \end{pmatrix}$. Utilize the results of exercise 1 and the formula that $r_1 + r_2 = \text{tr}(A)$ to find eigenvalues of A.
b) Find the eigenvector corresponding to the largest eigenvalue of A (.i.e., so called principal eigenvector). How is this eigenvector related to the asymptotic behavior of $\mathbf{z}^{k+1} = A\mathbf{z}^k$?

Exercise 3

Find the general solution of the following differential equations:

(a) $3\ddot{y} + 8\dot{y} = 0$;

(b) $4\ddot{y} + 4\dot{y} + y = 0$.

Exercise 4

a) Find the general solution of $\dot{x}_1 = 2x_1 + x_2$, $\dot{x}_2 = -12x_1 - 5x_2$ by using eigenvalues. Is origin a stable steady-state?

b) Find the steady state of

$$\begin{cases} \dot{x} &= 5x - \frac{1}{2}y - 12 \\ \dot{y} &= -2x + 5y - 24 \end{cases}.$$

Is the steady state globally asymptotically stable? Why or why not?

Exercise 5

Consider the growth model $\dot{k} = sf(k) - (1+\delta)k$, where $k \geq 0$ is per capita level of capital, $f(k) = \sqrt{k}$ is the production function, $s \in (0,1]$ and $\delta > 0$ are exogenous. The other steady-state is $k^* = 0$. What is the other one? Is the steady-state stable? (Tip: stability may depend on s and δ)