ELEC-E8101 Digital and Optimal Control

Exercise 11

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1. A constant variable x is measured through two different sensors. However, the measurements are noisy and have different accuracy. Assume the system is described by

$$\begin{aligned} x[k+1] &= x[k] \\ y[k] &= Cx[k] + e[k] \end{aligned}$$

where $C^T = [1 \quad 1]$ and $e[k]$ is zero-mean, white-noise vector with the covariance matrix

$$R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$

Estimate x as

$$\hat{x}[k] = a_1 y_1[k] + a_2 y_2[k]$$

Determine constant a_1 and a_2 such that the mean value of the prediction error is zero and such that the variance of the prediction error is as low as possible. Compare the minimum variance with the cases when only one of the measurements is used.

2. A stochastic process is generated as

$$\begin{aligned} x[k+1] &= 0.5x[k] + v[k] \\ y[k] &= x[k] + e[k] \end{aligned}$$

where v and e are uncorrelated white-noise processes with covariances r_1 and r_2, respectively. Further, $x[0]$ is normally distributed with zero mean and standard deviation σ.

   a) Determine the Kalman filter for the system.

   b) What is the gain in steady state?