Matrix Algebra
MS-A0001
Hakula
Mock Exam, 2022

I'll be available on Zoom on Dec 12, if you have any questions. I'll also write up solutions or at least ideas of solutions...

PROBLEM 1 Given

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix},$$

find matrices $B$ such that $AB = BA$.

PROBLEM 2 Show that, if $A$ and $B$ are orthogonal, then both $AB$ and $A^{-1}$ are orthogonal.

PROBLEM 3 (a) Are the vectors $\begin{pmatrix} 0 & 2 & -4 & 8 \end{pmatrix}^{\mathrm{T}}$, $\begin{pmatrix} 6 & 12 & 3 & 3 \end{pmatrix}^{\mathrm{T}}$, $\begin{pmatrix} 2 & 5 & -1 & 5 \end{pmatrix}^{\mathrm{T}}$ linearly independent? (b) Is the vector $\begin{pmatrix} -2 & 0 & -9 & 15 \end{pmatrix}^{\mathrm{T}}$ a linear combination of the first three?

PROBLEM 4 Let

$$A = (a_1\ a_2\ a_3) = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 1 & -1 & 1 \end{pmatrix}.$$

(a) Compute $PA = LU$.

(b) What is the volume spanned by $a_i$, $i = 1, 2, 3$?

PROBLEM 5 Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

Find the eigenvalues and orthonormal eigenvectors.

PROBLEM 6 Let the matrix $A$ have exactly two eigenvalues $\lambda_1 = 1$, $\lambda_2 = 1/2$, and the corresponding eigenvectors $v_1 = (1, 1)^T$, $v_2 = (-1, 1)^T$. Find the limit $\lim_{k\to\infty} A^k$.