Welcome to Greifswald

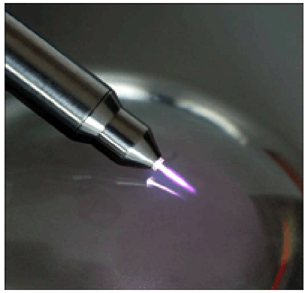
Plasma physics in Greifswald

Max-Planck-Institut für Plasmaphysik

- fusion research

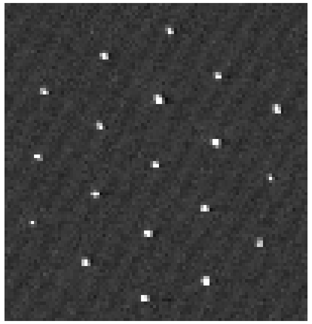
Leibniz-Institut für Plasmaforschung

- plasma technology



Institut für Physik Universität Greifswald

- academic research



Basic plasma physics and the motion of charged particles in magnetic fields

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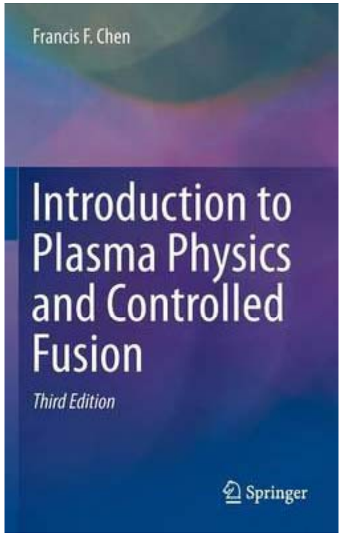
The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Outline

① What is $\pi\lambda\acute{\alpha}\sigma\mu\alpha$? The (A) world of plasmas.

② Overview: what makes a plasma a plasma?

③ How to tame plasmas for fusion: motion in magnetic fields



Introduction to Plasma Physics and Controlled Fusion

by Francis F. Chen

Springer (3rd edition, 2016)

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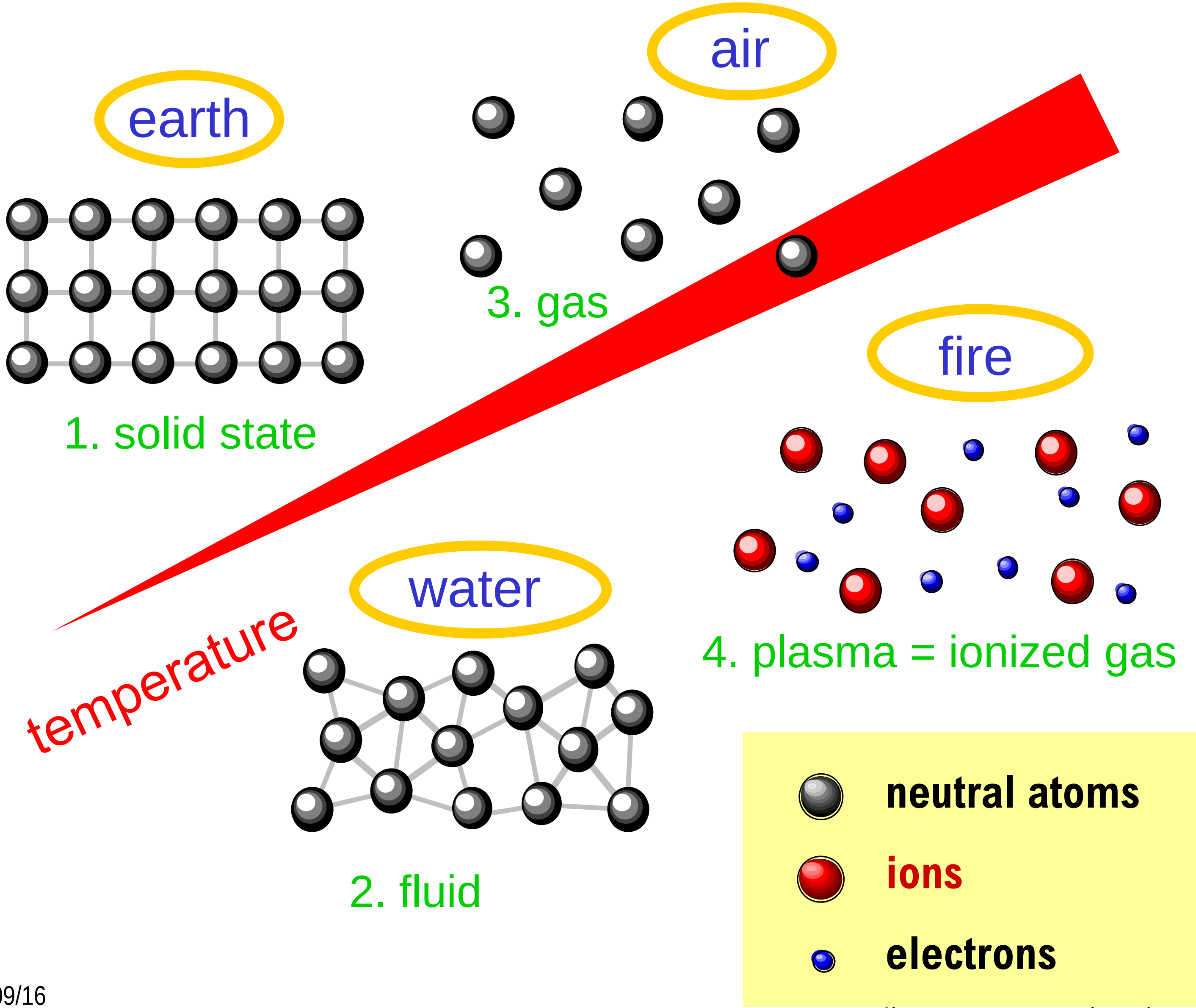
A world of plasmas

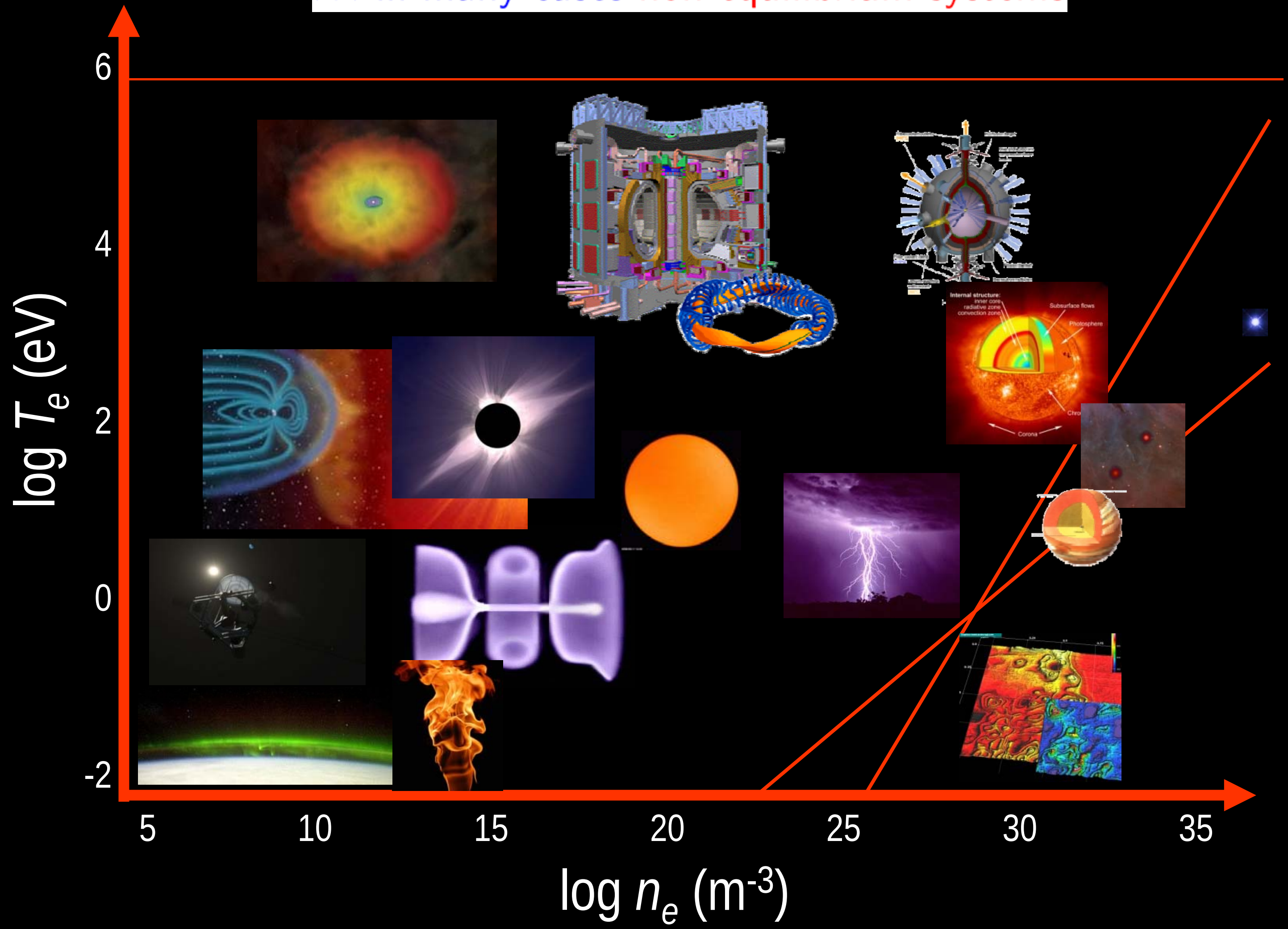
Empedokles
494-432 B.C

World consists of four elements



The world of plasmas

... in many cases non-equilibrium systems



$1\mathrm{eV} = e/k_B\,\mathrm{K} \approx 11.600\,\mathrm{K}$

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Outline

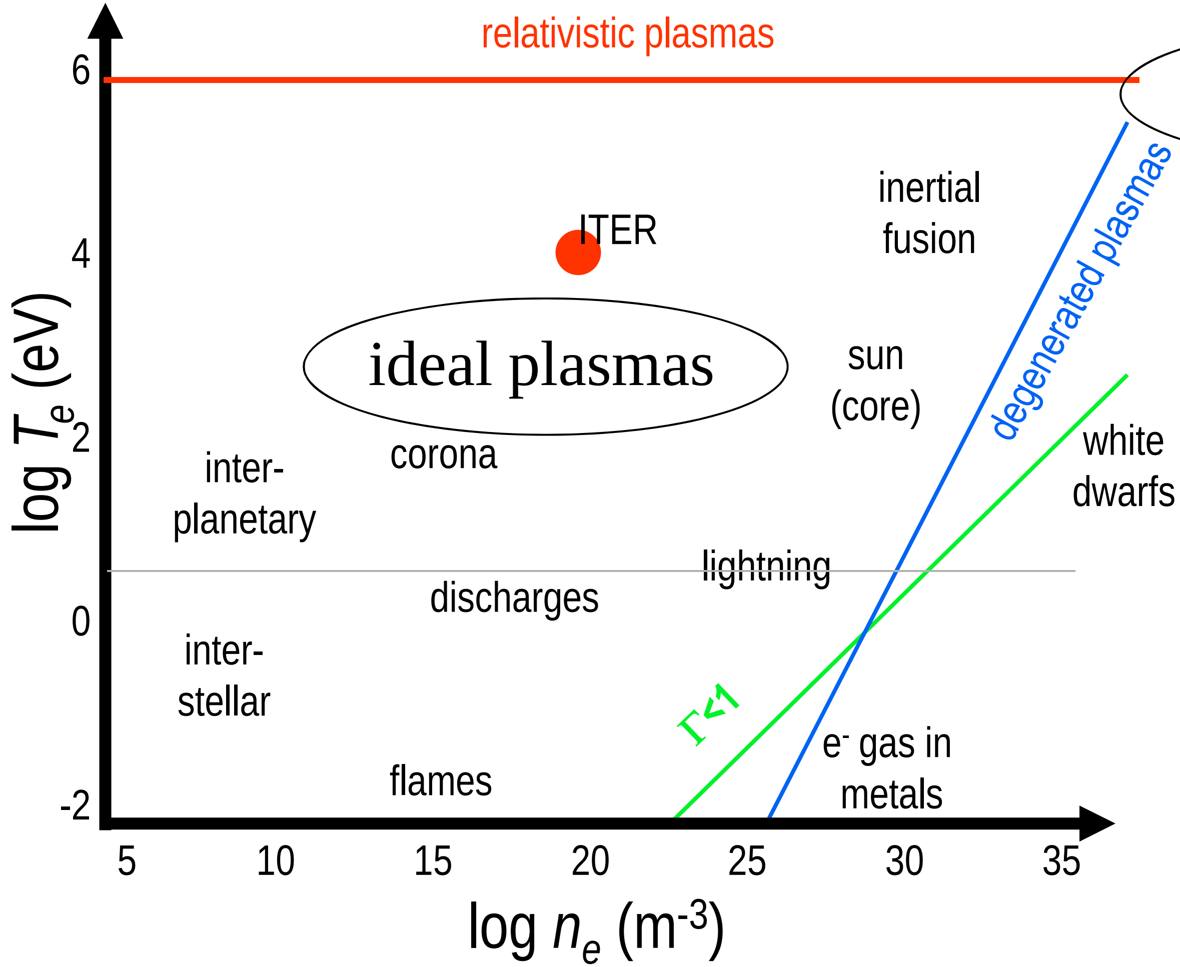
① What is $\pi\lambda\acute{\alpha}\sigma\mu\alpha$? The (A) world of plasmas.

② Overview: what makes a plasma a plasma?

- ① limits: relativity, quantum effects, ionization …
- ② time scales - plasma frequency
- ③ lengths - Debye shielding
- ④ Coulomb collisions and transport quantities

③ How to tame plasmas for fusion: motion in magnetic fields

Plasma map



ideal plasmas:

$k_B T << e^-$ mass at rest

$3/2 k_B T = m_{0e} c^2$

$k_B T >>$ Fermi energy

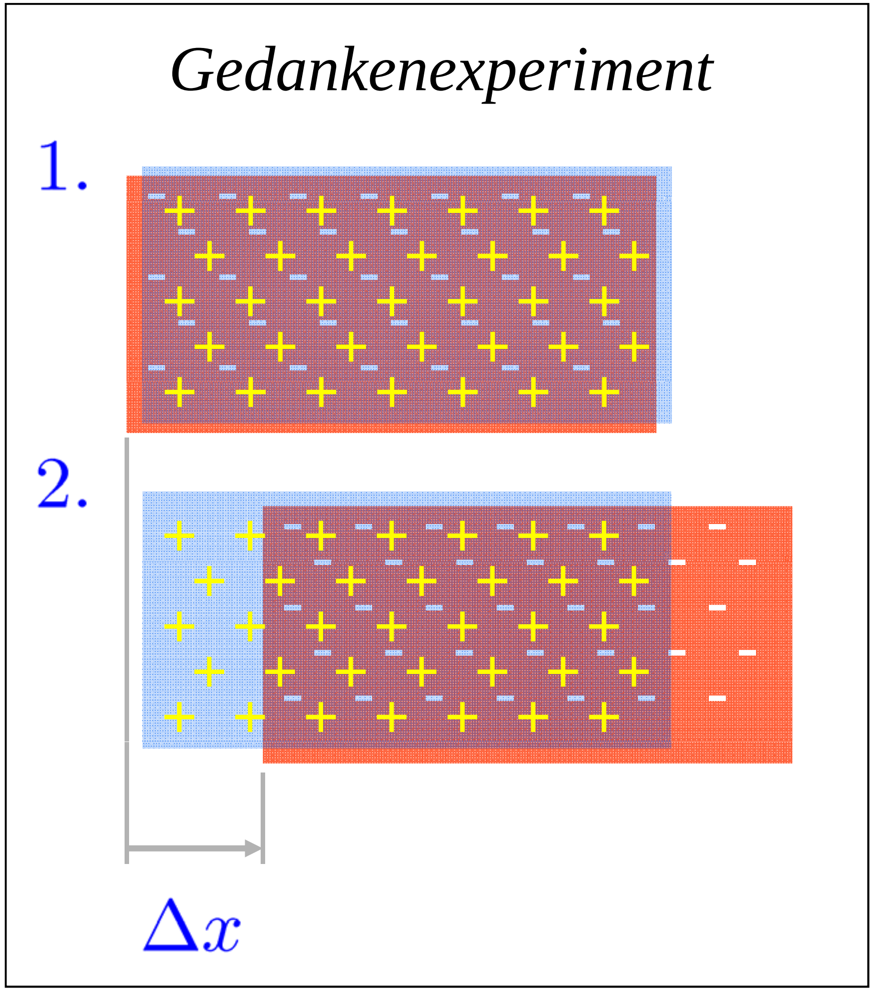
$$\frac{\hbar^2 (3\pi^2 n)^{2/3}}{2 m_e} = 3/2 k_B T$$

$k_B T >>$ electrost. potential

$$\frac{e^2}{4\pi\varepsilon_0} n^{1/3} = k_B T$$

... tought in this course!

Plasma frequency



displacement of charges

$\Rightarrow$ electric field

$\Rightarrow$ repelling force

$\Rightarrow$ oscillation

$$\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{\mathrm{e}\,\mathrm{n_e}\,\Delta\mathrm{x}}{\varepsilon_0}}$$

figures:

$\Delta x = 1\,\mathrm{mm}$

$n_e = 1 \times 10^{20}\,\mathrm{m}^{-3}$

$\Rightarrow E \approx 1.8 \times 10^{9}\,\mathrm{Vm}^{-1}$

`quasi-neutrality`

Plasma frequency

ions at rest, small perturbation:

+ equation of motion

$$m_e \frac{\partial v_1}{\partial t} = -\mathrm{e}\mathrm{E}_1$$

+ continuity equation

$$\frac{\partial n_1}{\partial t} + n_e \frac{\partial v_1}{\partial x} = 0$$

+ Poisson's equation

$$\varepsilon_0 \frac{\partial E_1}{\partial x} = -\mathrm{e}\,\mathrm{n}_1$$

+ harmonic Ansatz (ω, k):

$$-i\, m_e\, \omega\, v_1 = -i\, \frac{n_e \mathrm{e}^2}{\varepsilon_0 \omega}\, v_1$$

- v_1 (T) cancels out, n_e not!
- $\partial\omega/\partial k = 0$: Langmuir oscillation

$$\omega_p = \left(\frac{n_e \mathrm{e}^2}{\varepsilon_0 m_e}\right)^{1/2}$$

$$\boxed{f_p(Hz) \approx 9\sqrt{n(\mathrm{m}^{-3})}}$$

figures:

$$n_e = 1 \times 10^{20}\,\mathrm{m}^{-3}$$

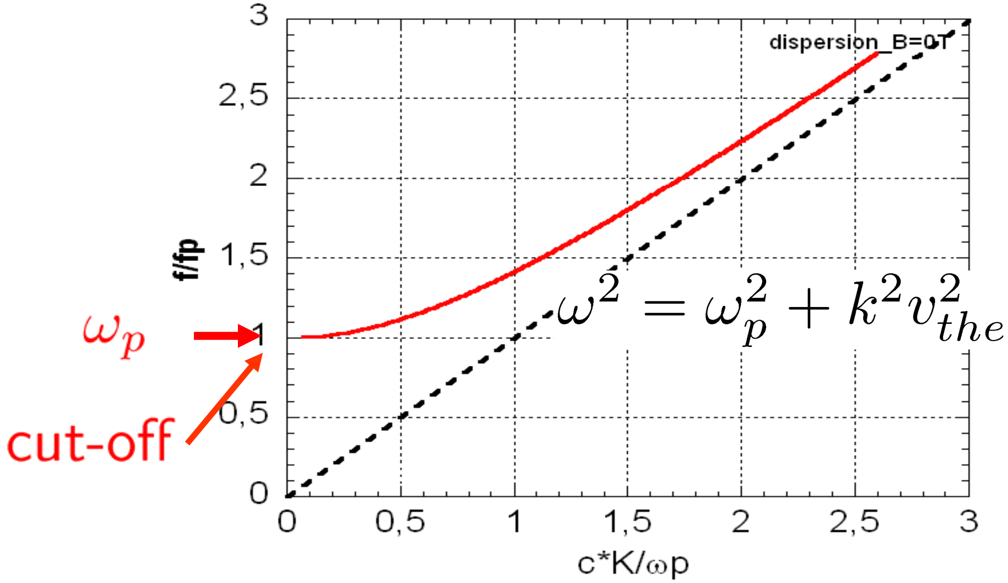
$$\Rightarrow\ f_p \approx 90\,\mathrm{GHz}$$

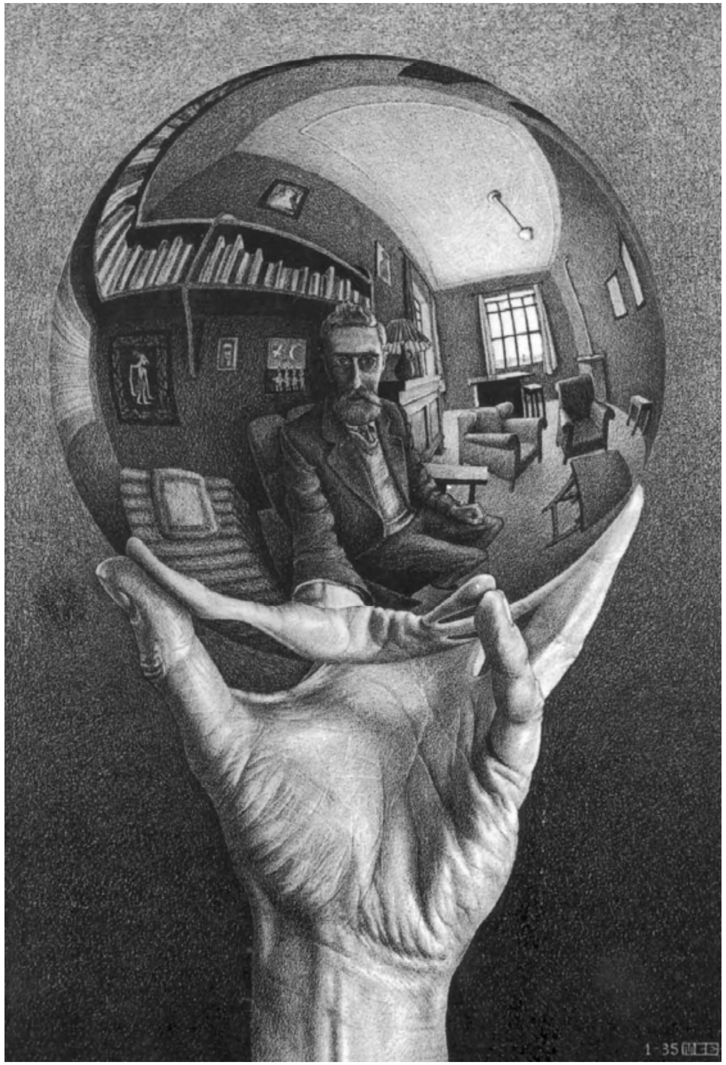
Plasma frequency: Apps.

wave propagation and reflection

dispersion $\omega = \omega(k)$

include thermal motion of electrons





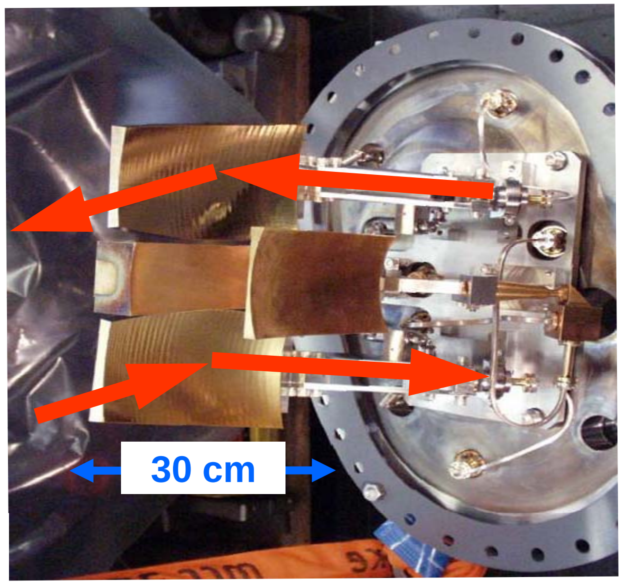
in metals, e.g. $n_e^{Ag} \approx 6 \times 10^{28}\,\mathrm{m}^{-3}$

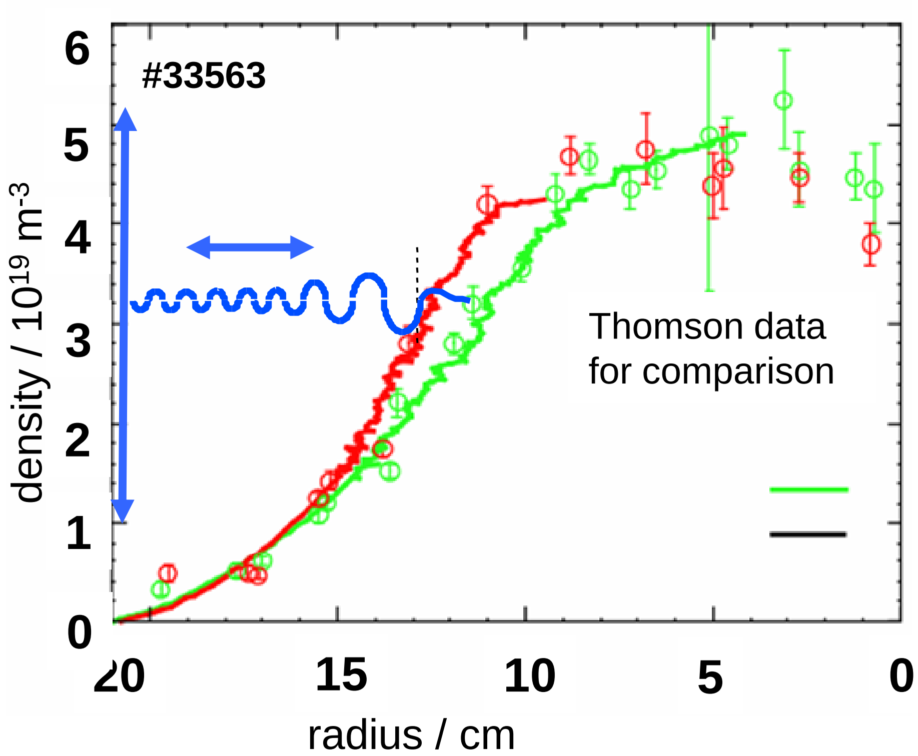
$\lambda_{prop} < 135\,\mathrm{nm}$, i.e. reflection above

Plasma frequency: Apps.

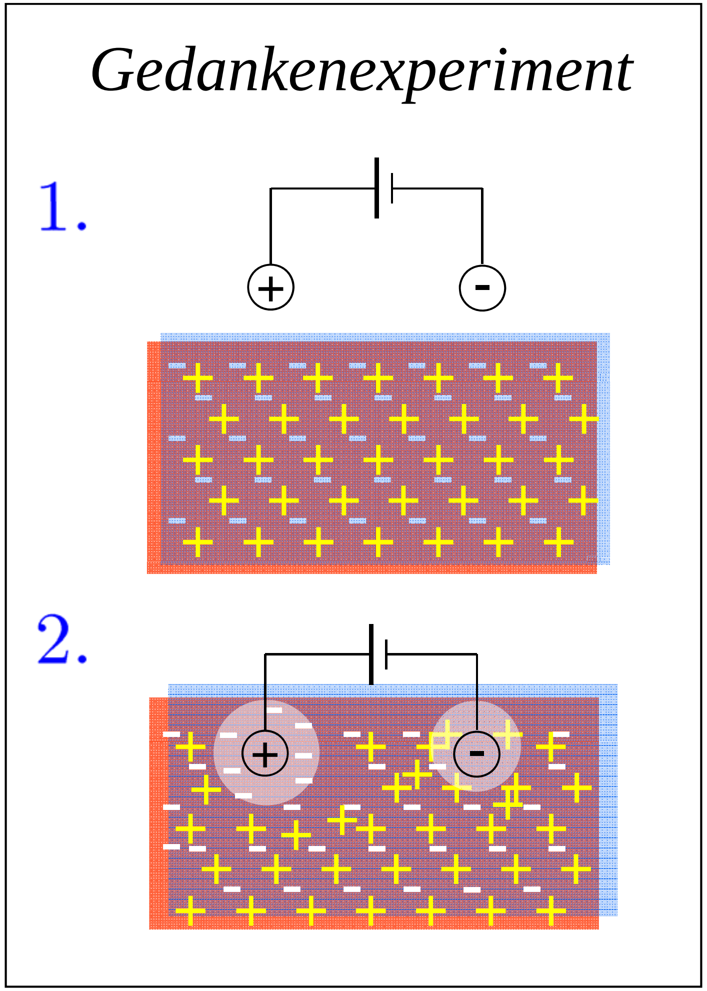
Reflectometry:

Density profiles: radar-like (time-of-flight) detection of cut-off layer

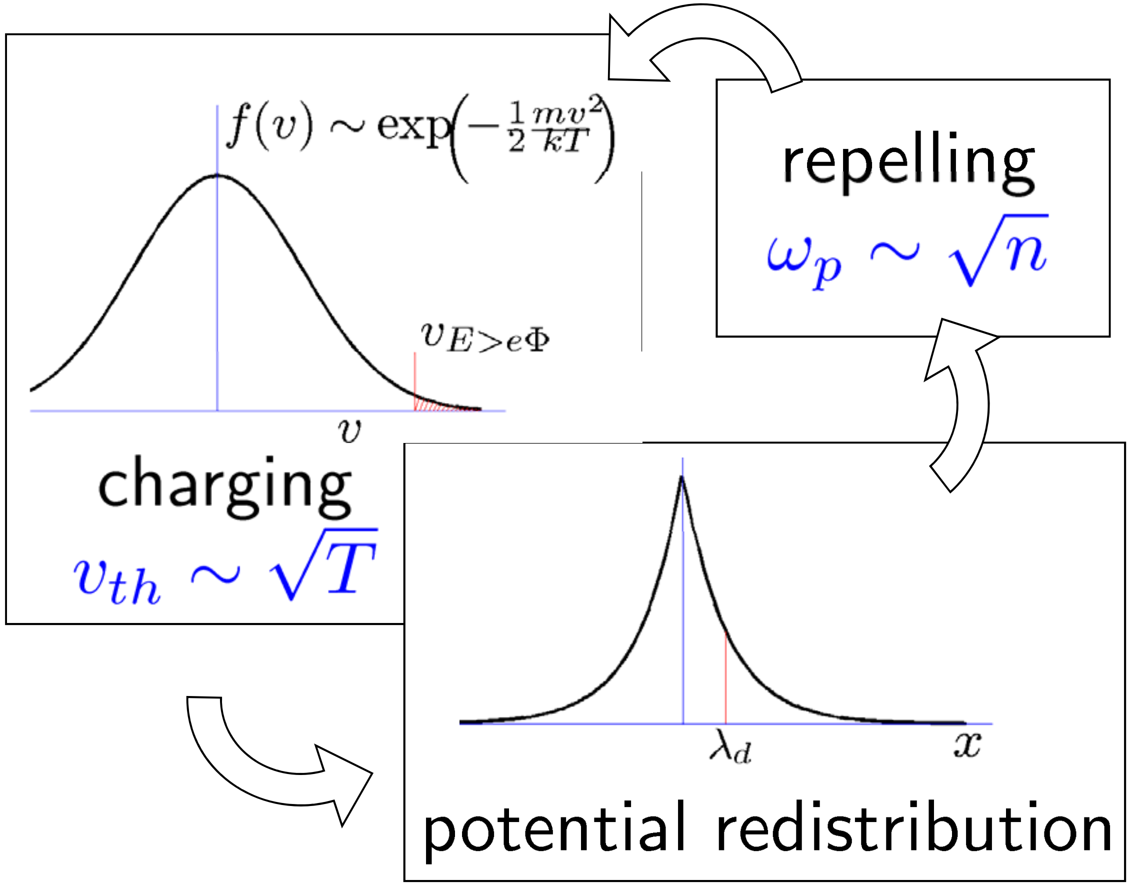




Debye shielding



perturbation in a plasma



$\Rightarrow$ shielding

Debye length

shielding:

heavy ions ($M/m_e = \infty$):

$+$ Maxwellian:

$f(v) \sim \exp\left(-\frac{1/2mv^2+q\Phi}{k_B T_e}\right)$

$\longrightarrow$ Boltzmann distribution

$n_e = n_\infty \exp\left(\frac{q\Phi}{k_B T_e}\right)$

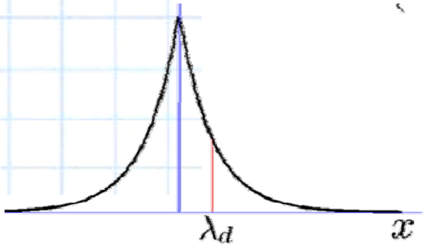
in Poisson's equation

$\varepsilon_0 \frac{d^2\Phi}{dx^2} = -\mathrm{e}(\mathrm{n_i} - \mathrm{n_e})$

gives Debye-Hückel potential

$\Phi = \Phi_0 \exp\left(-\frac{|x|}{\lambda_d}\right)$

- sheath develops



$$\lambda_d = \left(\frac{\varepsilon_0 k_B T_e}{n\,\mathrm{e}^2}\right)^{1/2}$$

$$\boxed{\lambda_d(m) \approx 7430 \sqrt{\frac{T\,(eV)}{n\,(\mathrm{m}^{-3})}}^{(1)}}$$

figures:

$$n_e = 1 \times 10^{20}\,\mathrm{m}^{-3}$$
$$T_e = 10\,\mathrm{keV}$$
$$\Rightarrow \lambda_d \approx 75\,\mu\mathrm{m}$$

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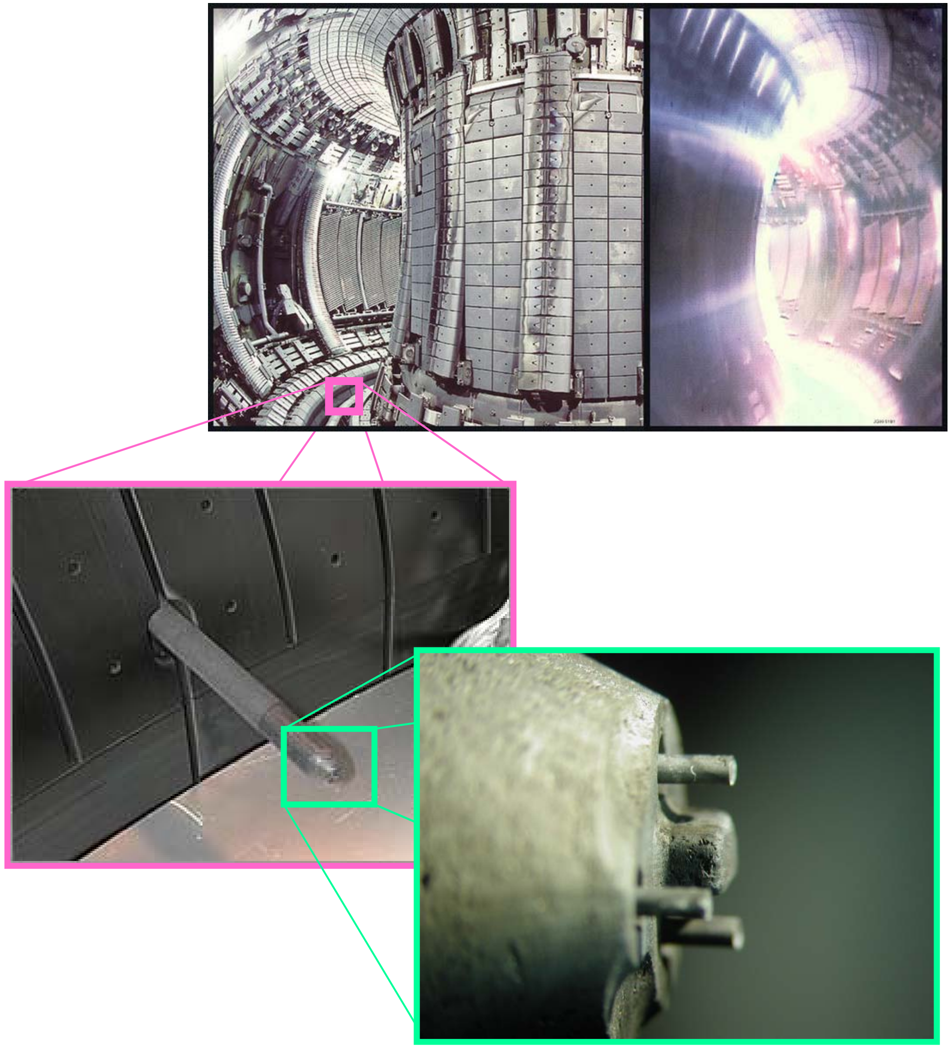
[1] $1\mathrm{eV} = \mathrm{e}/\mathrm{k_B}\,\mathrm{K} \approx 11.000\,\mathrm{K}$
$\mathrm{e} \cdot \mathrm{T(eV)} = (\mathrm{k_B T})\,(\mathrm{J})$

Debye length: Apps.



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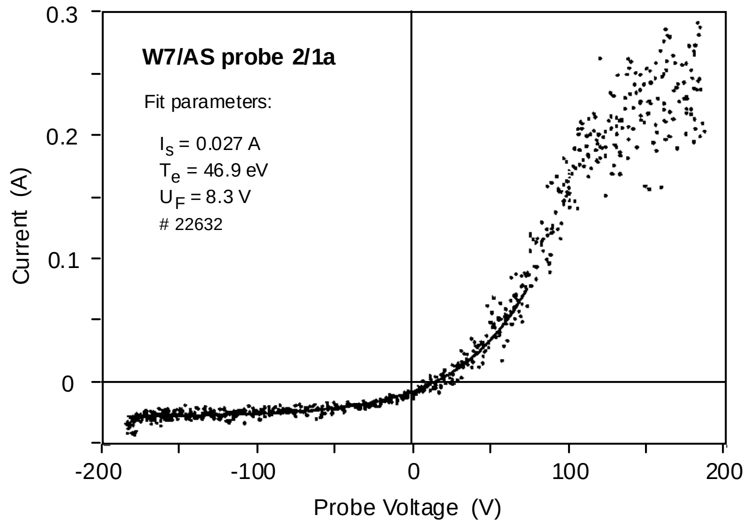


Langmuir probe

current-voltage characteristics:

$\longrightarrow$ electron density n_e

$\longrightarrow$ electron temperature T_e

in 'cold' plasmas

Outlook: turbulence in magnetized plasmas

Probe arrays

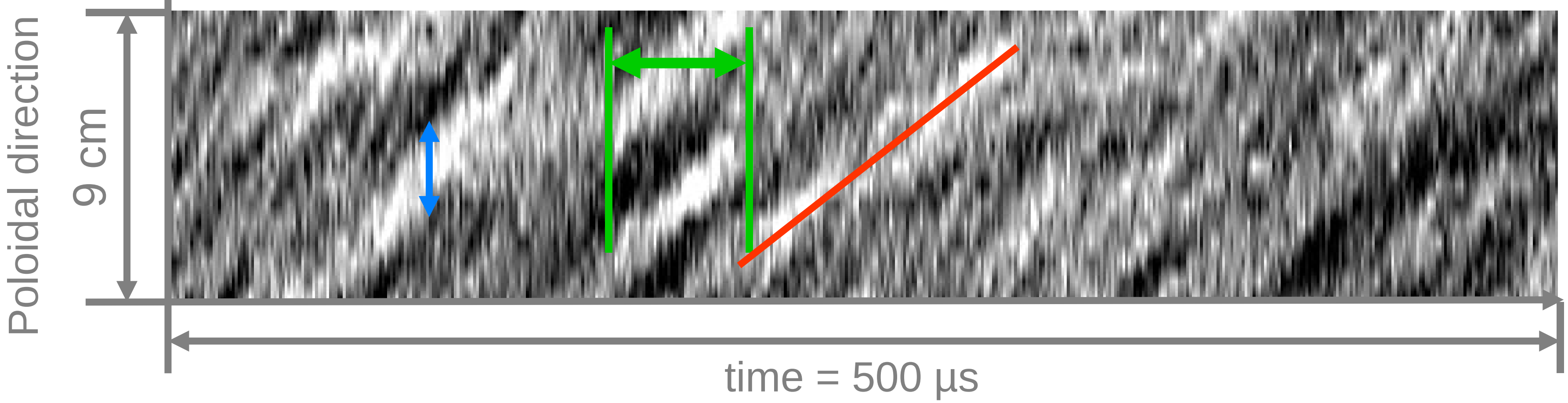
Turbulence affects transport, stability, …
the efficiency of a power plant

Turbulence is topical, basic science

Turbulence affected by
- external magnetic fieldsVerscherung
- internal elektric fields
- plasma boundaries

Example: propagation of turbulent structures

lifetime	about 10 µs
poloidal correlation length	1–5 cm
poloidale velocity	*O*(100)–*O*(1000) m/s



Plasma in a nutshell

Plasmas are quasi-neutral many-particle systems.

Collective effects of charged particles are conveyed by electromagnetic fields.

Criteria:

1. $\lambda_d \ll L, n/\nabla n, T/\nabla T, \ldots$
2. $N_d >>> 1$ $(N_d = 4\pi/3n\lambda_d^3)$ $(\Gamma_c = kT/E_{pot} > 1)$
3. $\omega_{Plasma}\,\tau_{\text{collisions}} > 1$

Collisions & elementary processes

3. $\omega_{Plasma}\,\tau_{\text{collisions}} > 1$

ionization 3-body recombination	$e^- + A^{+Z}$	$\rightarrow$ $\leftarrow$	$e^- + A^{+Z+1} + e^-$
collisional excitation coll. de-excitation	$e^- + A^{+Z}$	$\rightarrow$ $\leftarrow$	$e^- + (A^{Z+})^*$
photo ionization radiative recombination	$h\nu + A^{+Z}$	$\rightarrow$ $\leftarrow$	$(A^{+Z+1})^* + e^-$
absorption spontaneous emission	$h\nu + A^{+Z}$	$\rightarrow$ $\leftarrow$	$(A^{Z+})^*$
absorption Bremsstrahlung	$h\nu + e^- + A^{+Z}$	$\rightarrow$ $\leftarrow$	$A^{Z+} + (e^-)^*$
Coulomb collisions	$e^- + A^{+Z}$	$\rightleftarrows$	$(A^{Z+})' + (e^-)'$
fusion e.g.	$D + T$	$\rightleftarrows$	$n + \alpha + 17.6\,\mathrm{MeV}$

Collisions in hot fusion plasmas

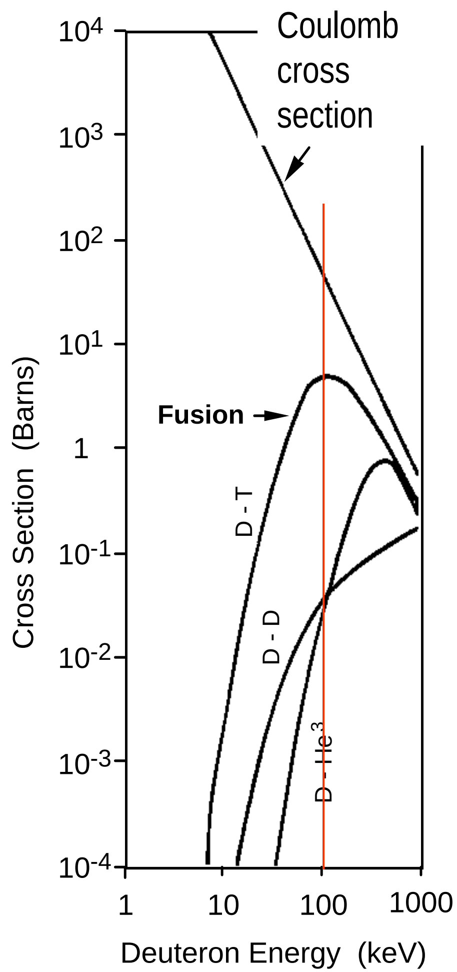
Coulomb collisions	$e^- + A^{+Z}$	$\rightleftarrows$	$(A^{Z+})' + (e^-)'$
fusion e.g.	$D + T$	$\rightleftarrows$	$n + \alpha + 17.6\,\mathrm{MeV}$

Effect of collisions? Reaction rate.

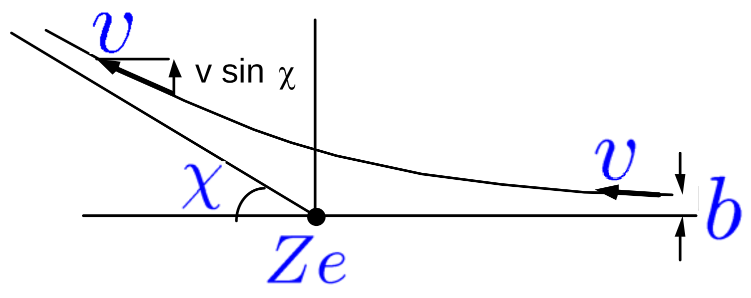
single collision - cross section: $\sigma\ (\mathrm{m})^2$

ensemble - rate coefficient : $\langle \sigma\, v \rangle\ (\mathrm{m}^3\mathrm{s}^{-1})$

$$\boxed{R = \underbrace{\underbrace{\langle \sigma v \rangle}_{\text{rate coefficient}} \times n}_{\text{reaction rate (reactions per second)}}}$$



Coulomb collisions



cross section

$$\sigma = \pi b^2$$

(the area the colliding particle sees)

Coulomb cross section (pedestrian's approach)

$$E_{pot} = E_{kin}$$

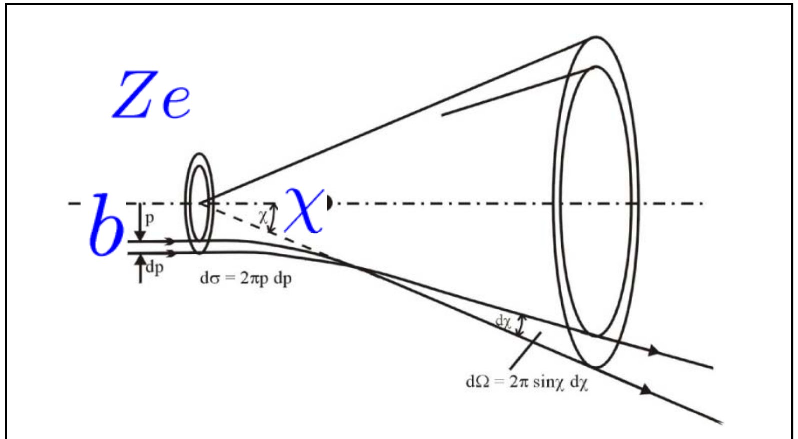
$$\frac{1}{4\pi\varepsilon_0}\,\frac{1}{b} = \frac{1}{2}\,mv^2 = \frac{3}{2}k_B T$$

$$\sigma_{CB} = \frac{1}{36\pi\varepsilon_0^2}\,\frac{e^4 Z^2}{k_B^2}\,\frac{1}{T^2}$$

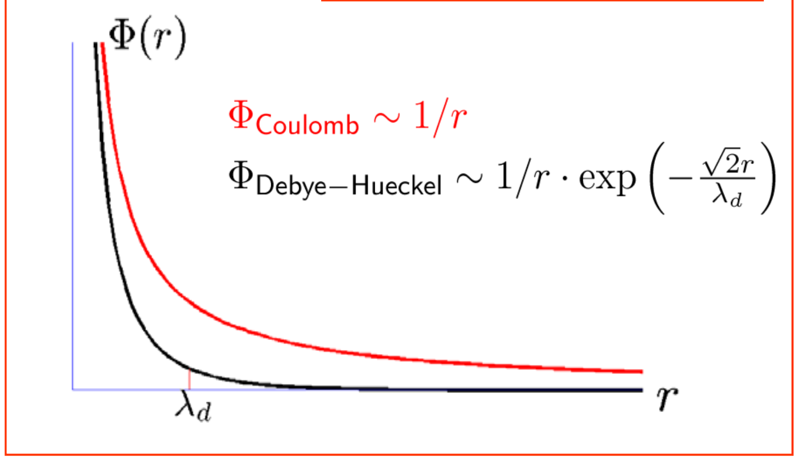
$$\langle\sigma_{CB}\, v\rangle \sim T^{-3/2}$$

...the hotter, the less collisions!

Coulomb collisions (contd.)



$$\Lambda = b_{max}/b_{min}$$



Coulomb cross section
(scattering theory)

$+$ integrate over all b (resp. χ)

but Coulomb: $1/r$ singularity

$+$ plasma: shielding $\rightarrow b_{max} \approx \lambda_d$
thermal motion $\rightarrow b_{min} \approx \lambda_{\text{deBrog.}}$

$$\boxed{\langle \sigma_{CB} \, v \rangle \sim T^{-3/2} \times \ln(\Lambda)}$$

$\ln(\Lambda) \approx 15 \ldots 20$ for fusion plasmas

small angle collisions are most effective
($\times$ 100 for fusion plasmas)!

Coulomb collisions (contnd.)

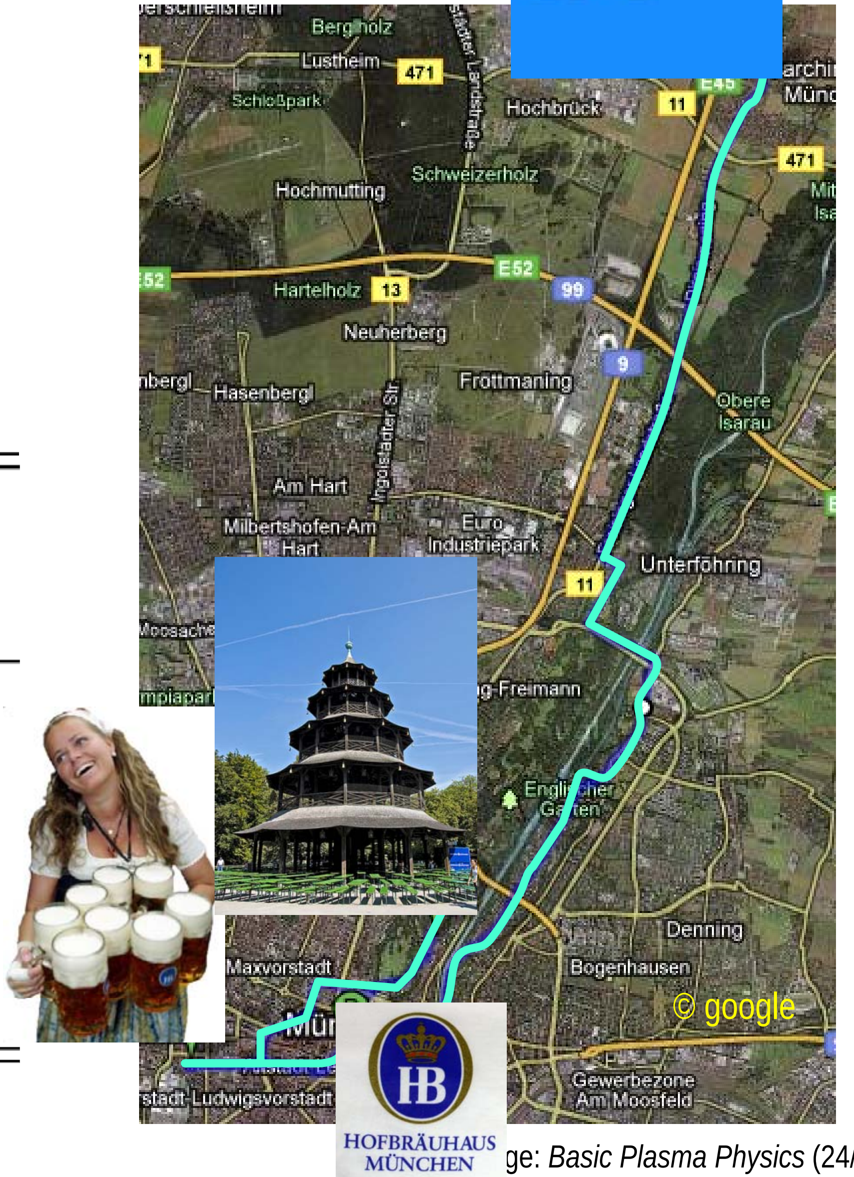
mean free path $\lambda = \frac{1}{n\sigma}$

collision time $\tau = \frac{\lambda}{v_{th}}$

$$\tau_{ee} \approx \tau_{ei} \approx \sqrt{\frac{m_e}{m_i}}\tau_{ii} \approx \frac{m_e}{m_t}\tau_{ie}$$

$T = 10\,\mathrm{keV},\ n = 1 \times 10^{20}\,\mathrm{m}^{-3}$

collision	τ	λ_{CB}
$e^- - T^+$	0.2 ms	9 km
$e^- - e^-$	0.3 ms	12 km
$D^+ - D^+$	18 ms	18 km
$T^+ - T^+$	23 ms	22 km



Coulomb collisions (contd.)

resistivity

$$E = \eta j$$

$$\eta \approx \frac{m_e}{ne^2} \frac{1}{\tau_{ei}}$$

$$\boxed{\eta(\Omega\mathrm{m}) \approx 5.2 \times 10^{-5} Z \ln\Lambda \; (T(\mathrm{eV}))^{-3/2}}$$

figures:

$$\eta(T = 1.3\mathrm{keV}) \approx 18\,\mathrm{n\Omega m} \approx \eta_{\mathrm{Copper}}$$

$$\eta(T = 10\mathrm{keV}) \approx 0.8\,\mathrm{n\Omega m} \approx 1/20 \times \eta_{\mathrm{Copper}}$$

hot plasmas have a very low ohmic resistance!

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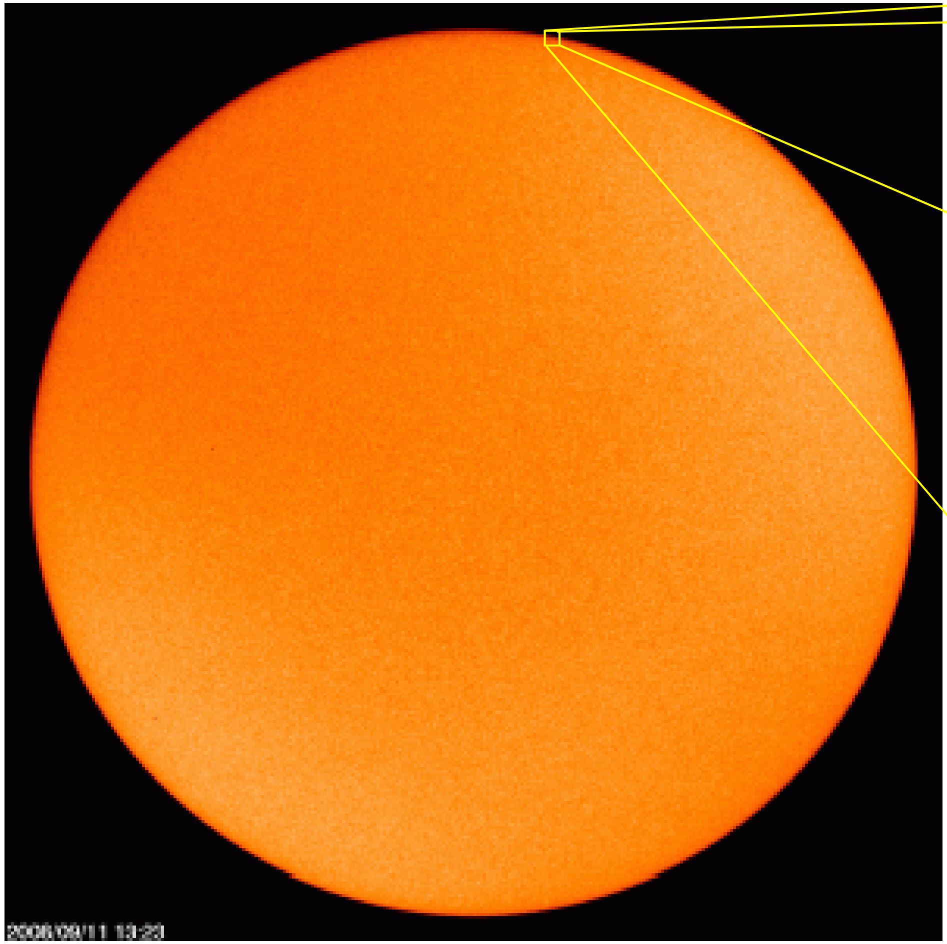
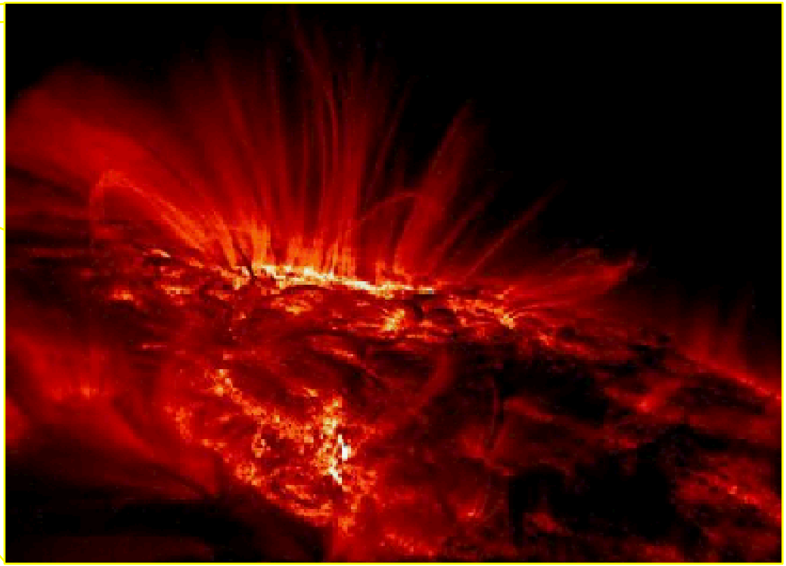
gravitational confinement

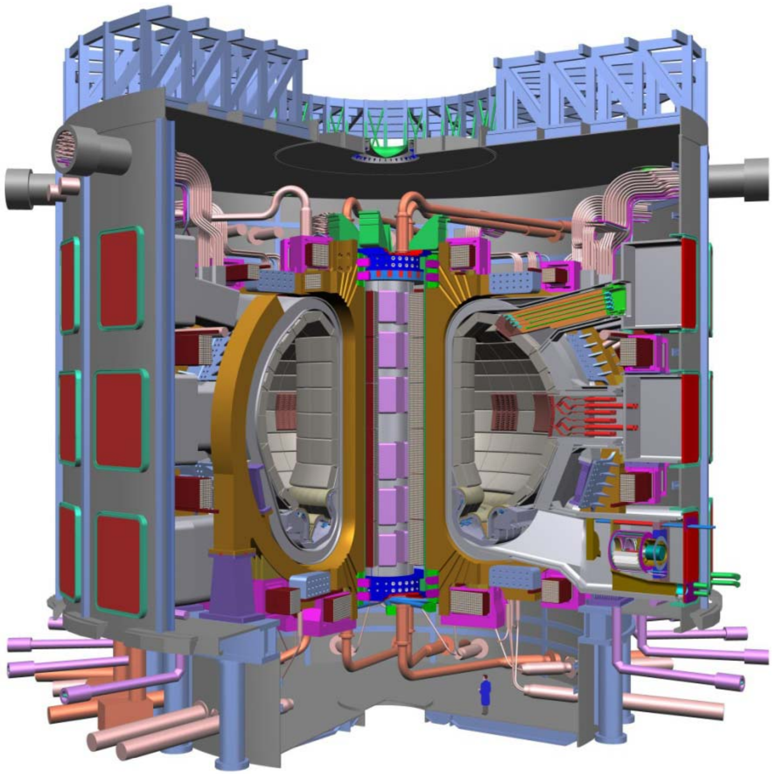




& electromagnetic forces

$$\vec{F} = m\,\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

How to get to a managable size?



force plasma particles on toroidal orbits

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Motion in electro-magnetic fields

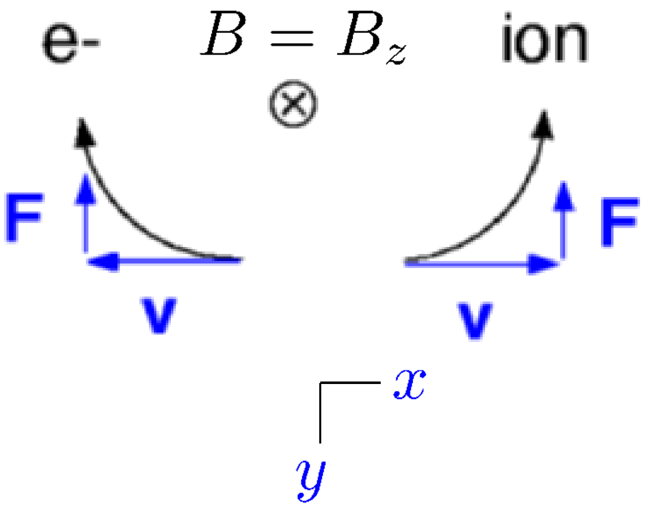
ideal plasma ($\underline{N_d >>> 1}$) vs. single particle motion?

$$\lambda_{CB} >>> L??$$

E = 0, B = const

- Lorentz force:

$$\vec{F} = m\,\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$



$$m\dot{v}_x = qB_z v_y$$

$$m\dot{v}_y = -qB_z v_x$$

$$m\dot{v}_z = 0$$

$$\ddot{v}_{x,\,y} = -\left(\frac{qB}{m}\right)^2 v_{x,\,y}$$

$$v_z = \text{const}$$

- cyclotron frequency

$$\omega_c = \frac{qB}{m}$$

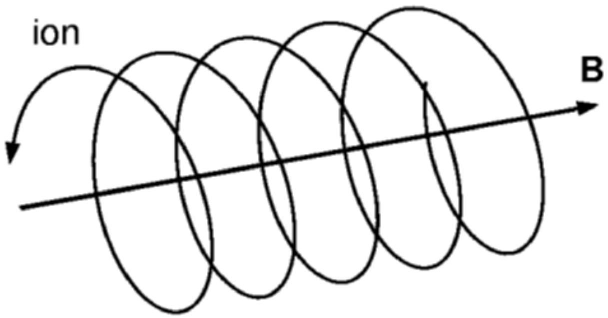
- Larmor radius

$$r_L = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{qB}$$

E = 0, B = const

- Lorentz force:

$$\vec{F} = m\,\frac{d\vec{v}}{dt} = q\vec{v}\times\vec{B}$$



helical trajectory along B

figures:

- cyclotron frequency for $B = 2.5\,\mathrm{t}$

$f_{ce} = \omega_{ce}/(2\pi) \approx 70\,\mathrm{GHz}$ $\longrightarrow$ heating

$f_{cD} = \omega_{cD}/(2\pi) \approx 20\,\mathrm{MHz}$

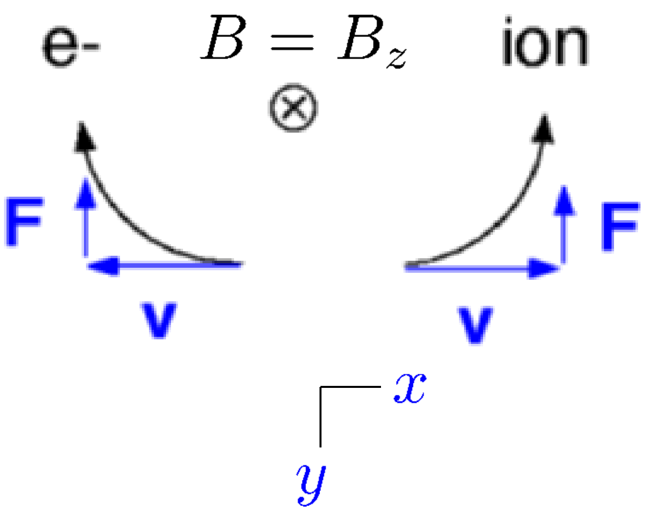
- Larmor radius for $T = 10\,\mathrm{keV}$

$$\left.\begin{array}{l} r_{Le} \approx 95\,\mu\mathrm{m} \\ r_{LD} \approx 6\,\mathrm{mm} \end{array}\right\} \ll L \ll \lambda_{CB}$$

E = 0, B = const

- Lorentz force:

$$\vec{F} = m\,\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$



Invariants:

- kinetic energy:

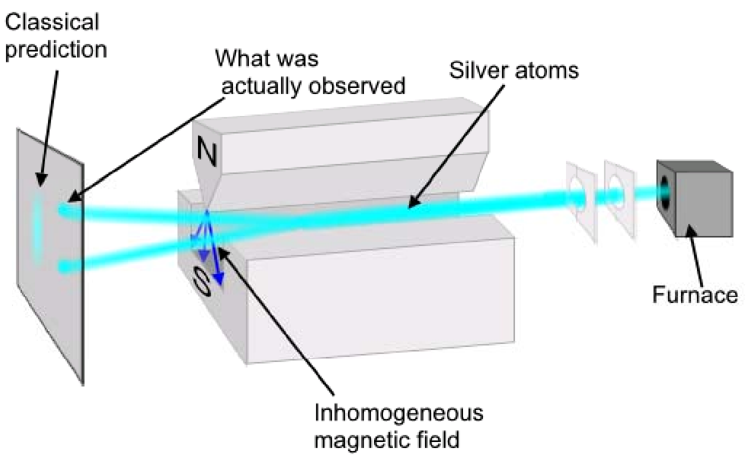
$$\frac{dW}{dt} = \frac{1}{2}m\,\frac{d}{dt}\vec{v}^{\,2} = m\,\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$= q\,\vec{v}\cdot\left(\vec{v}\times\vec{B}\right)$$

$$\frac{dW}{dt} = 0$$

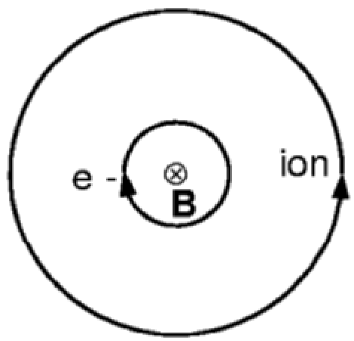
- W is a constant of motion ($\vec{B} =$ const)
- Lorentz forces guide, but do not work!
- in other words: $\vec{F}_L$ pins particles to $\vec{B}$
- inhomogeneous fields?

Stern-Gerlach experiment



$$\vec{F} = -\mu \nabla_{\parallel} B$$

gyro motion



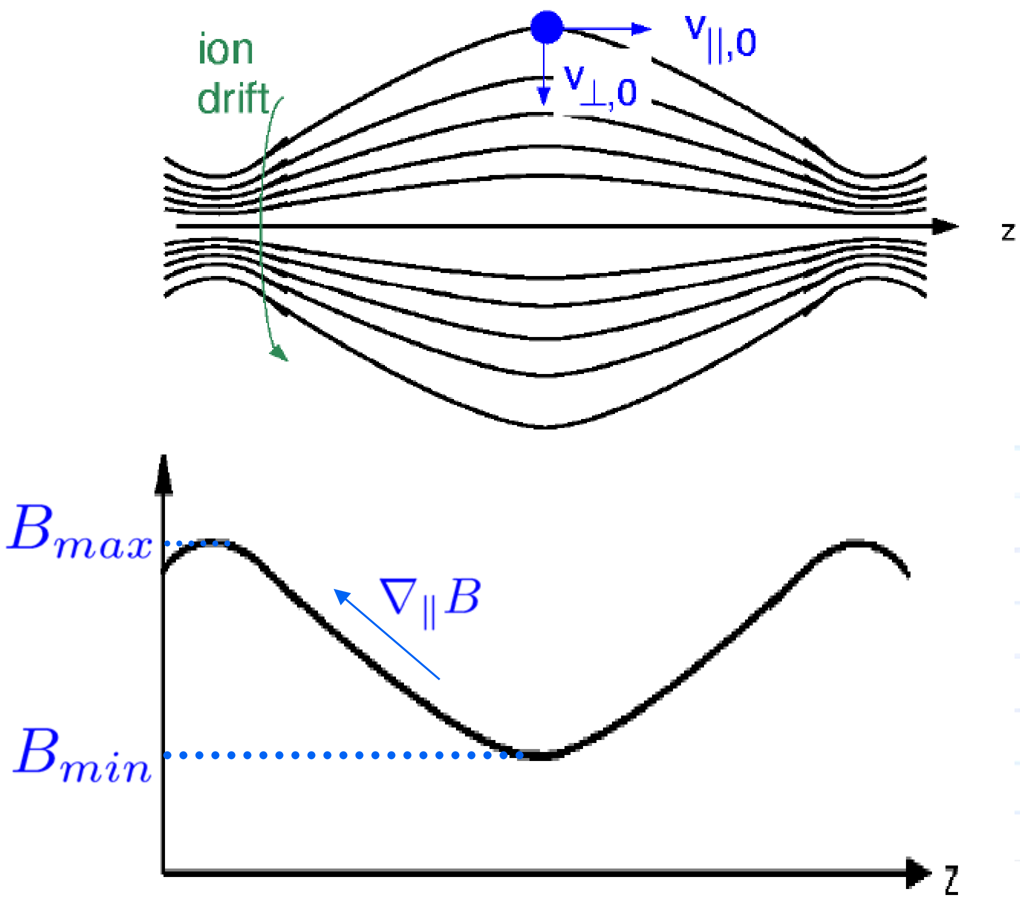
magnetic moment μ

$$\mu = IA$$

$$= \frac{e\omega_c}{2\pi} \pi r_L^2 = \frac{1}{2} \frac{m v_\perp^2}{B}$$

$E = 0$, $B \neq$ const – App.

Magnetic mirror



What's going on with μ?

$$m\dot{v}_{\|} = -\mu \partial_z B$$

$$m v_{\|} \dot{v}_{\|} = \frac{d}{dt}\left(\frac{1}{2} m v_{\|}^2\right) = -\mu \partial_z B \frac{ds}{dt}$$

+ energy conservation $\frac{d}{dt}\left(\frac{1}{2} m v_{\|}^2 + \mu B\right) = 0$

$$-\mu \frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

$$d\mu/dt = 0$$

μ is an *adiabatic invariant*

i.e. invariant for 'slow' $(1/\omega_c)$ proc.

Adiabatic invariants

periodic motion, slow changes

$$\oint p\,dq = \text{const}$$

p, q:

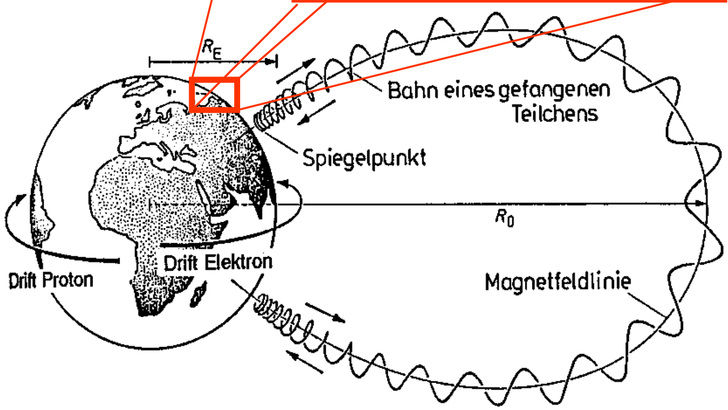
generalized momentum, coordinate

$\Rightarrow$ access to complex mechanics

applications in plasma theory:

- 1^{st} inv. μ: gyro-center motion
- 2^{nd} inv. J: bounce motion $\longrightarrow$
- 3^{rd} inv. ψ: precession

charged particles from **solar wind** are reflected in regions of stronger field strength and produce the polar light when exciting the atoms in the atmosphere



Adiabatic invariants

periodic motion, slow changes

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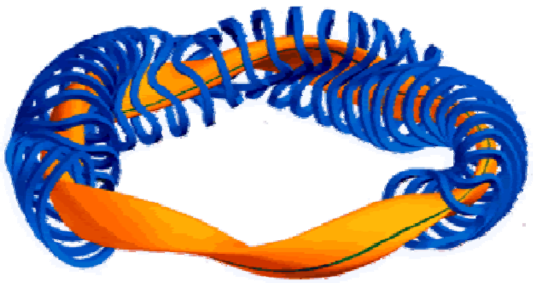
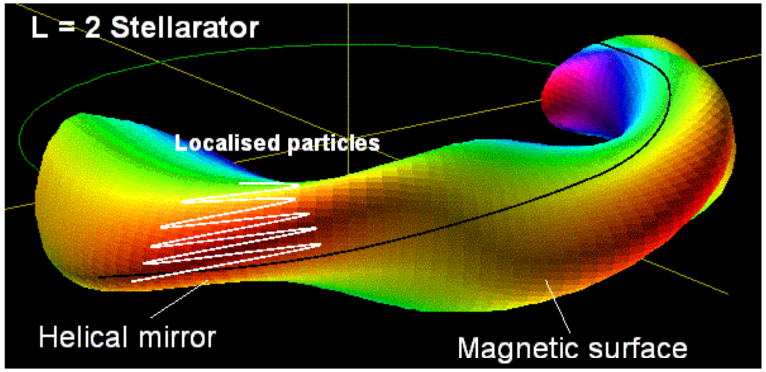
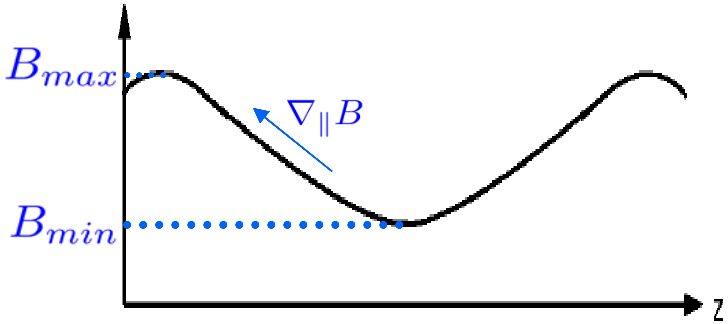
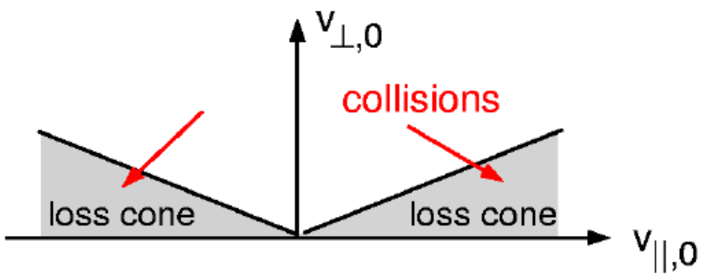




$E = 0$, $B \neq$ const – App.

Magnetic mirror





phase space

Confinement in magnetic mirrors?

$\mu = \text{const} : \frac{1}{2} m v_{\perp\, min}^2 m / B_{min} = \frac{1}{2} m v_{\perp\, r}^2 m / B_{refl.}$

$W = \text{const} : v_{\perp\, r}^2 = v_{\perp\, min}^2 + v_{\|\, min}^2$

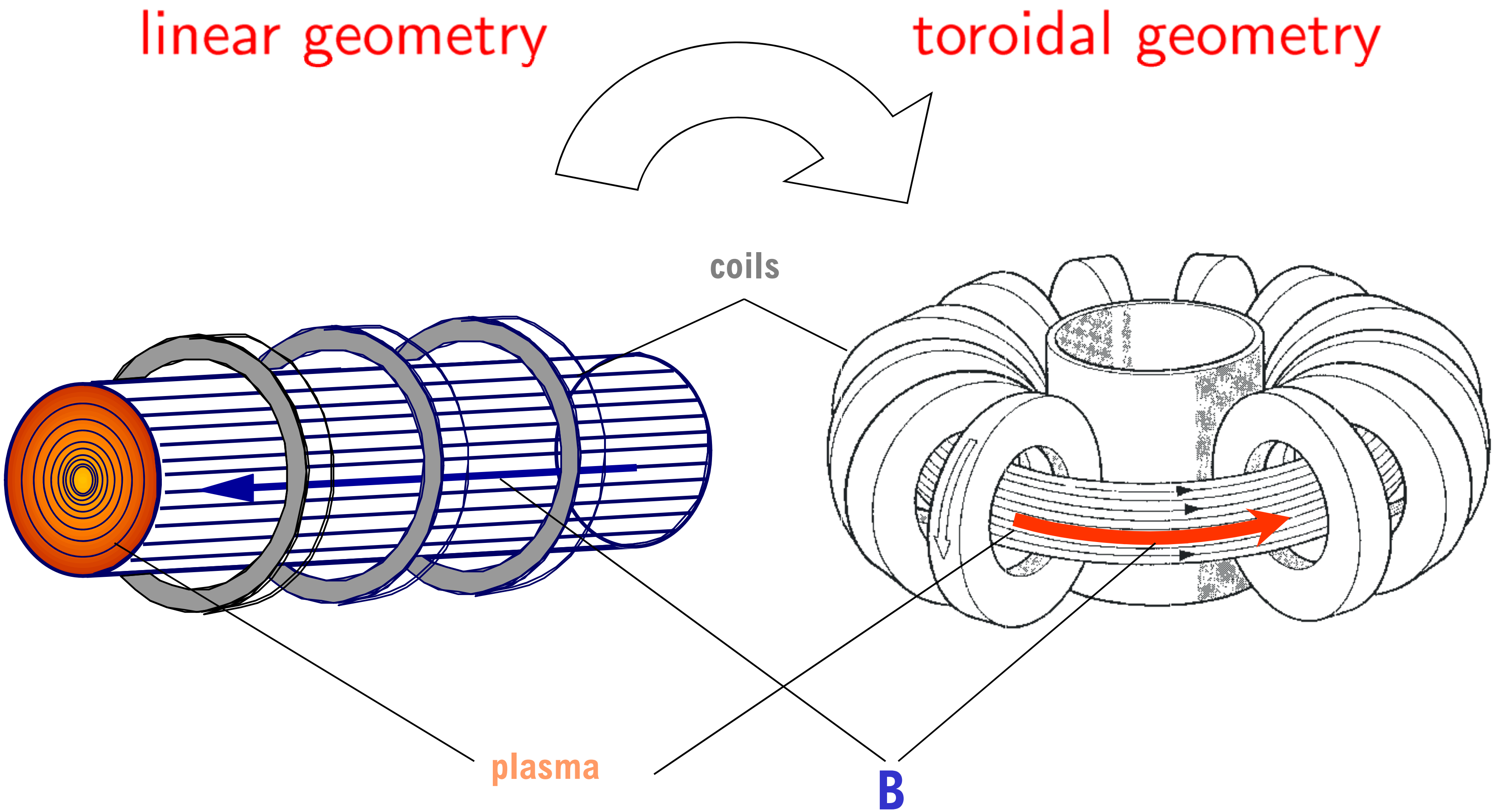
pitch angle: $\sin(\theta) = \frac{v_\perp}{v}$

$\sin^2(\theta_{critical}) = B_{min} / B_{max}$

$\Rightarrow$ plasma lost at high $v_{\|} / v_{\perp}$

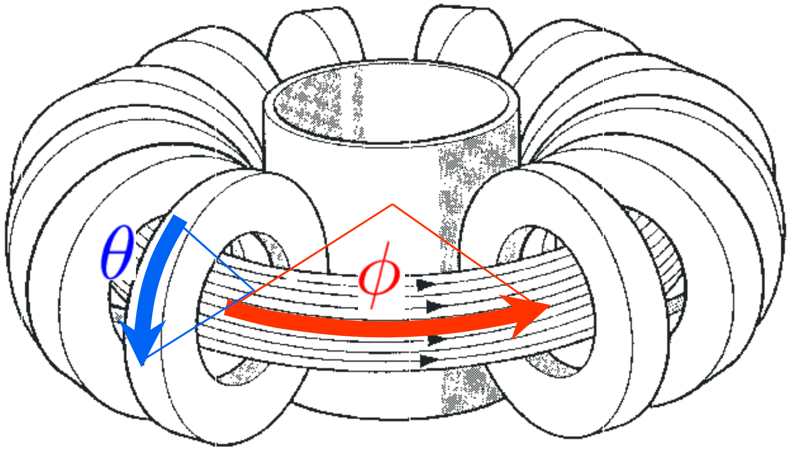
No - how to avoid end losses?

overcoming losses at the ends



overcoming losses at the ends

torus and toroidal geometry



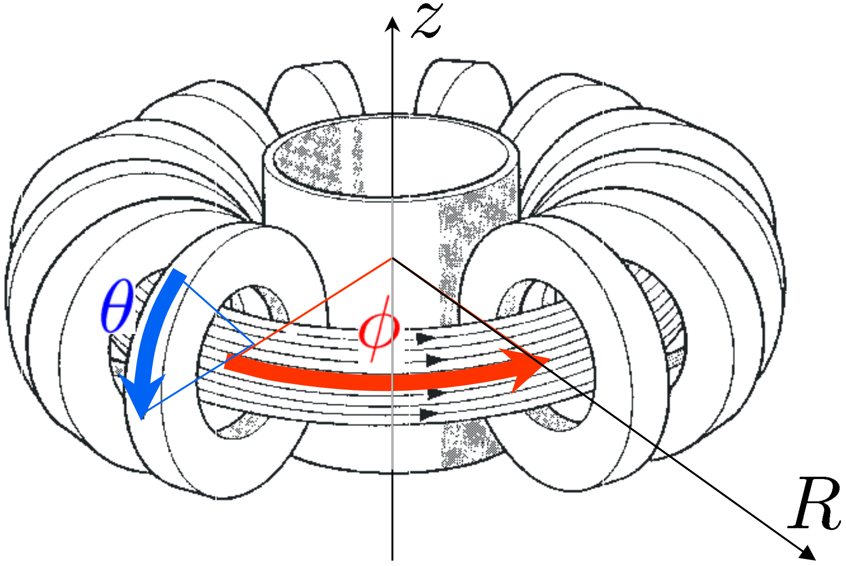
toroidal angle ϕ

poloidal angle θ

poloidal cross section ('cuts the pole')

toroidal cross section

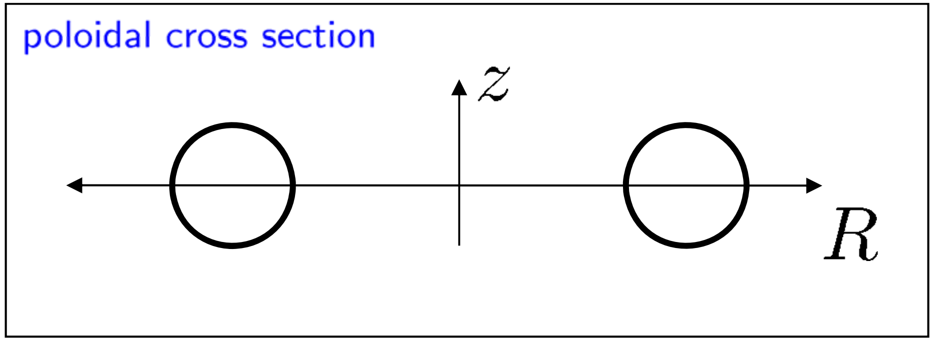
field variation in toroids

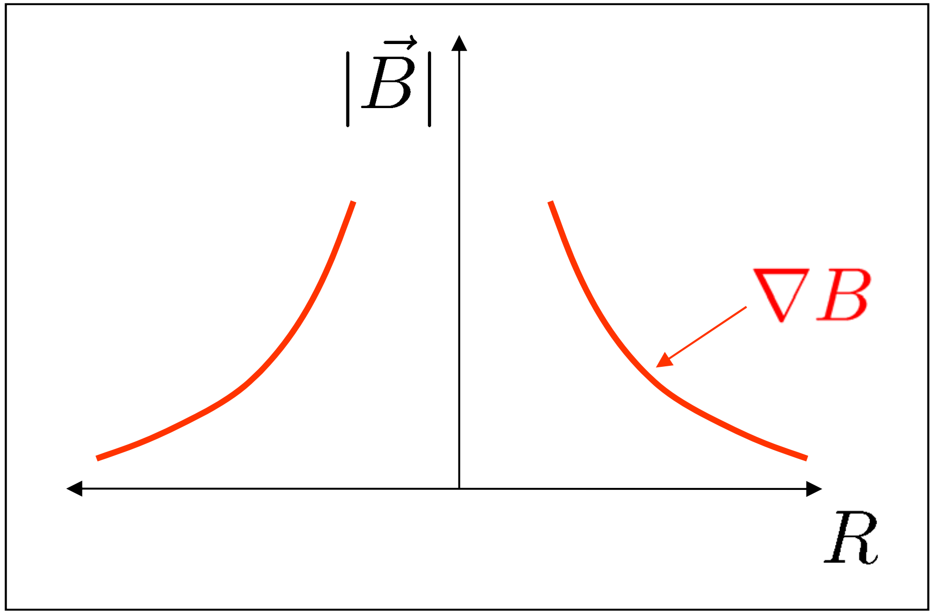


toroidal angle ϕ

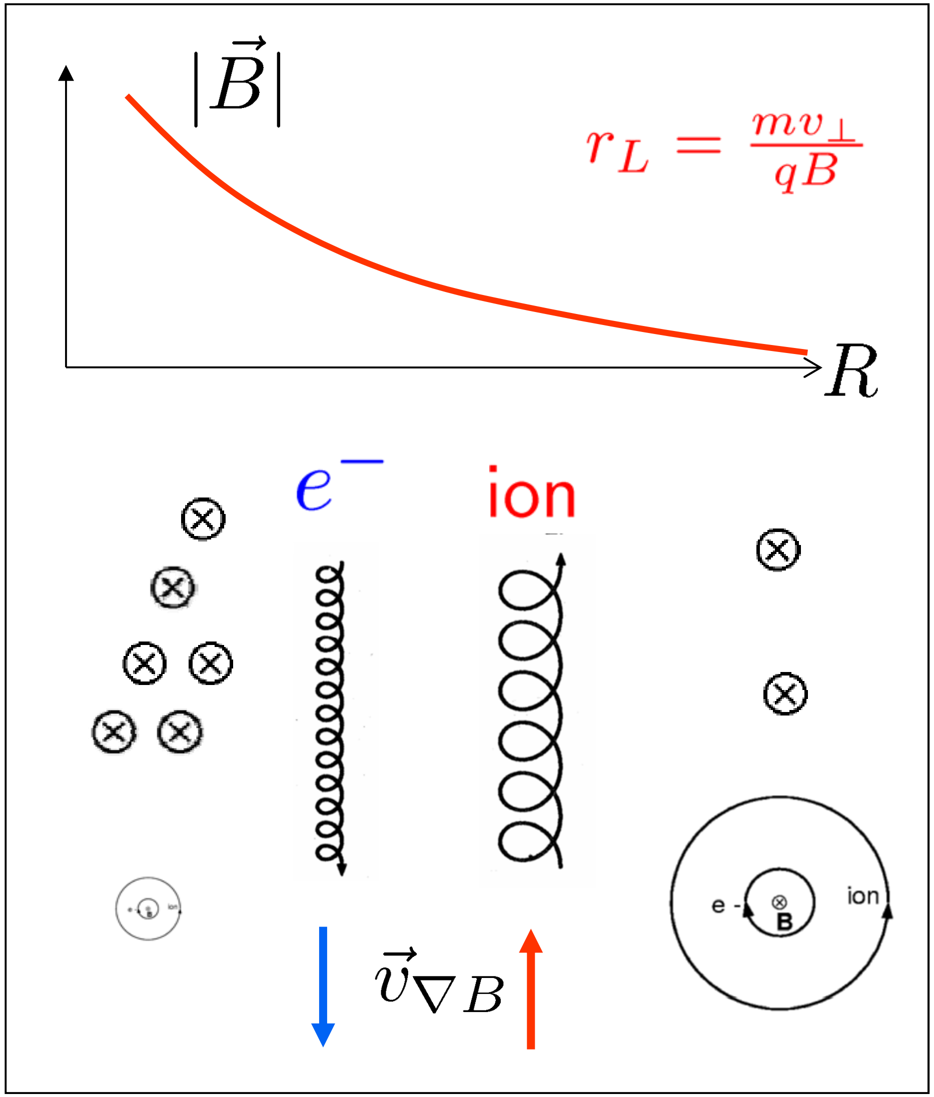
poloidal angle θ

radius R





∇B drift



force $\rightarrow$ guiding centre drift

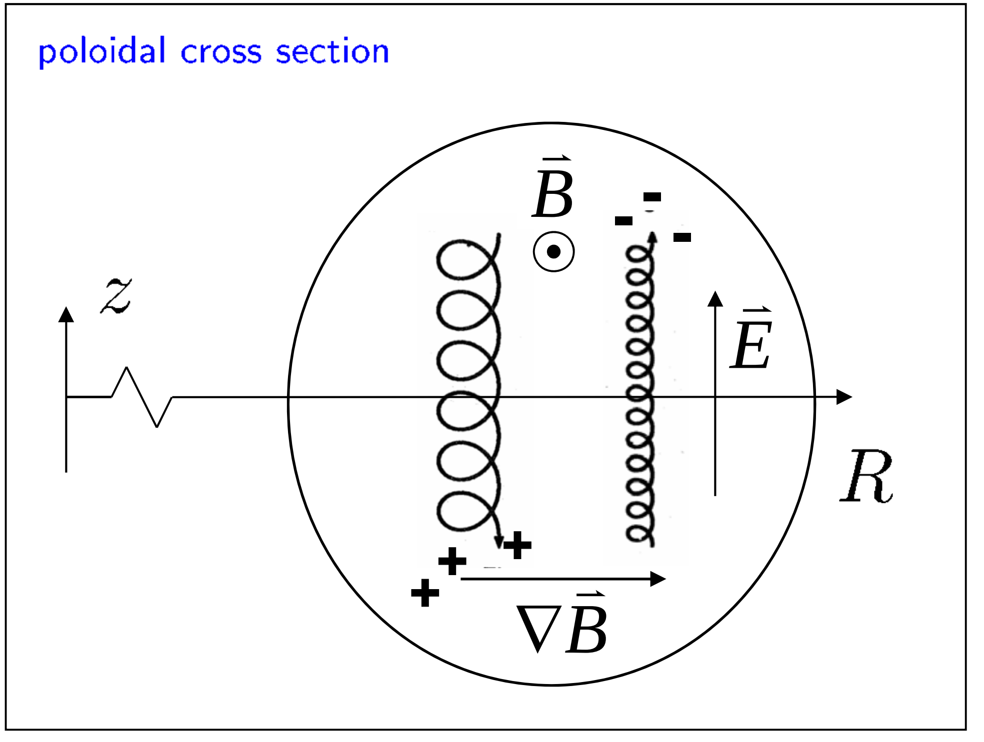
$$m\dot{\vec{v}} = \vec{F} + q\vec{v} \times \vec{B}$$

$$\boxed{\vec{v}_D = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}}$$

here: $\vec{F} = -\mu\nabla B$

$$\vec{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L \frac{\nabla B \times \vec{B}}{B^2}$$

∇B drift in toroids

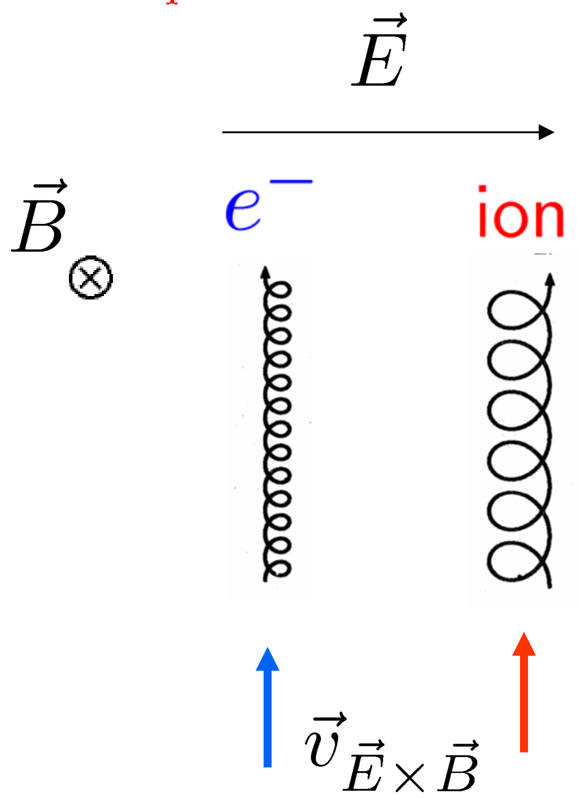


∇B-drift polarizes the plasma toroid.

$$\Rightarrow \vec{E} = E\,\vec{e}_z$$

$\vec{E}x\vec{B}$ drift

$$r_L = \frac{mv_\perp}{qB}$$



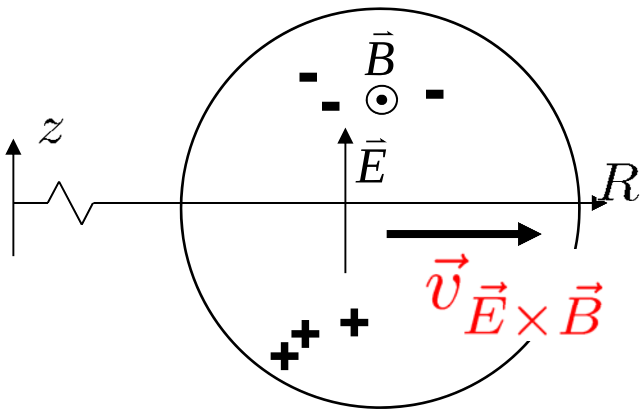
$$\vec{v}_D = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}$$

force depends on q

$$\vec{F} = q\vec{E}$$

$$\boxed{\vec{v}_{\vec{E}\times\vec{B}} = \frac{\vec{E}\times\vec{B}}{B^2}}$$

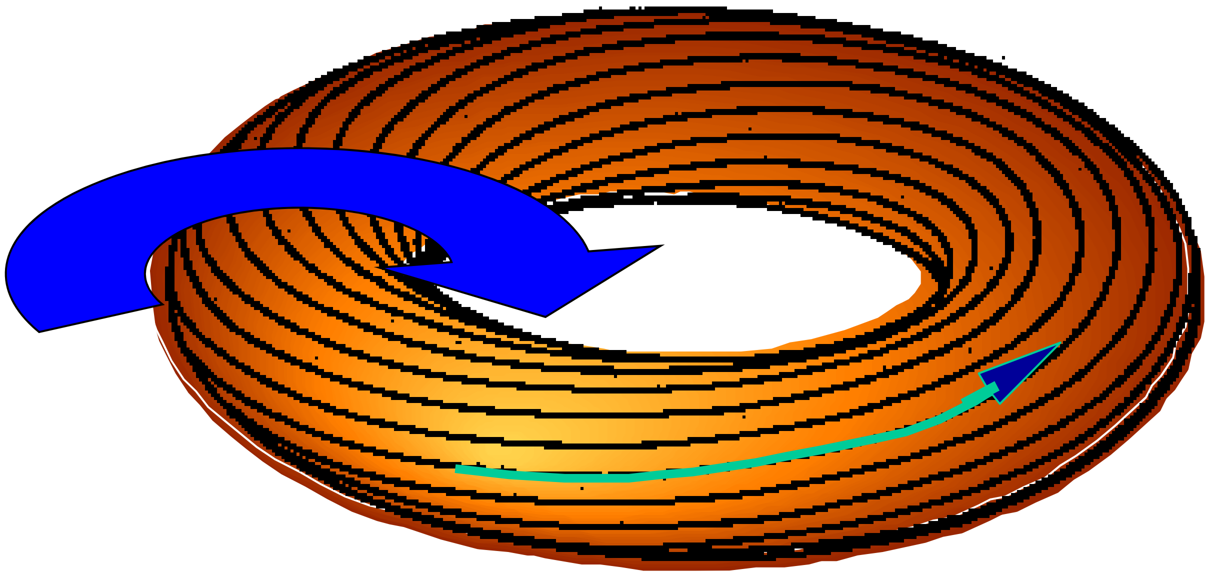
$\Rightarrow$ radial $\vec{E}\times\vec{B}$-drift in toroids



Compensating drifts in toroids: rotational transform

compensation for drifts: twist of $\vec{B}$

$$\vec{B} = \vec{B}_\phi + \vec{B}_\theta$$



$\vec{B}_\theta$ toroidal current $\longrightarrow$ tokamak

external coils $\longrightarrow$ stellarator

Compensating drifts in toroids: rotational transform

Summary (1/2)

A plasma is ...

- made of charged particles
- quasi-neutral many particles system
- roughly characterized by
  - plasma frequency $\omega_p \sim \sqrt{n}$
  - Debye length $\lambda_d \sim \sqrt{T/n}$

Summary (2/2)

A fusion plasma ...

- must be hot ($T_{DT} \approx 15\,\mathrm{keV}$)
- undergoes very rarely collisions
- can be confined by magnetic fields

Single-particle motion in magnetic fields:

- gyro-motion and (adiabatic) invariants
- drifts
- motion in toroidal fields