A0001 Mock EXAM

P1 $A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}$ $\qquad \underset{2\times 2}{AB} = \underset{2\times 2}{BA} \Rightarrow B$ is 2×2.

$$B = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \Rightarrow \beta_{11} = \gamma \,,\ \beta_{12} = \delta$$

$$\beta_{21} = 0 \,,\ \beta_{22} = \gamma - \delta$$

P2 $A, B :\ A^T = A^{-1} \ ;\ B^T = B^{-1}$

$$(AB)^T = B^T A^T = B^{-1} A^{-1} = (AB)^{-1}$$

$$A^T A = A^{-1} A = I \ ;\ A A^T = A A^{-1} = I$$

$$A^T A^{-T} = A^{-1} A^{-T} = I \ ;\ \ldots$$

P3 (a) No, $\operatorname{rank}(A) = 2$.
Compute the row echelon form.

(b) Yes. For instance $x = (2, -\frac{1}{3}, 0)^T$
is the solution of
$Ax = b$.

P4 (a) Any LU is OK.

(b) $|[\underline{a}_1, \underline{a}_2, \underline{a}_3]| = \left| \det(\cdots) \right| = 6$

P5 $p(A) = -\lambda^3 + 3\lambda^2 + 2\lambda - 4 = 0$

$\lambda_1 = 1 \quad ; \quad \lambda_{2,3} = 1 \pm \sqrt{5}$

$(A - \lambda_1 I)x = 0 \quad \Rightarrow \quad v_1 = (-1, 0, 2)^T$

$(A - \lambda_2 I)x = 0 \quad \Rightarrow \quad v_2 = (2, \sqrt{5}, 1)^T$

$\cdots \quad \Rightarrow \quad v_3 = (2, -\sqrt{5}, 1)^T$

P6 $Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$A = Q \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} Q^T$

$\lim_{k \to \infty} A^k = Q \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} Q^T = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$