# ELEC-E3510 Basics of IC Design 

Lecture 2:

MOS transistor models

## Lecture outline:

- threshold voltage of MOS transistor
- drain current equation
- parasitic capacitors and resistors
- short channel effects (extra material)
- weak inversion
- SPICE parameters
- small signal model


## MOS transistor




NMOS




## Modes of operation

| Cut-off | $V_{G S}-V_{T} \leq 0$ | $I_{D S}=0$ |
| :---: | :---: | :---: |
| Linear region <br> (Triode region) | $0<V_{D S} \leq\left(V_{G S}-V_{T}\right)$ | $I_{0}=\frac{\mu_{0} C_{0 x} W}{L}\left[\left(V_{G S}-V_{T}\right)-\frac{V_{D S}}{2}\right] V_{D S}\left(1+\lambda V_{D S}\right)$ |
| $k=\left(K^{\prime}\right) \frac{W}{L} \cong\left(\mu_{0} C_{0 \times}\right) \frac{W}{L}\left(a m p s / v o l t^{2}\right)$ |  |  |
| Saturation | $0<\left(V_{G S}-V_{T}\right) \leq V_{D S}$ | $I_{D}=\frac{\mu_{0} C_{o x} W}{2 L}\left(V_{G S}-V_{T}\right)^{2}\left(1+\lambda V_{D S}\right)$ |
| Pinch-off | $V_{D S, S A T}=V_{G S}-V_{T}$ | $I_{D S, L I N}=I_{D S, S A T}$ |


a) $I_{D}$ as a function of $V_{D S}$

b) $\sqrt{I_{D}}$ as a function of $\mathrm{V}_{\mathrm{GS}}$ (for $\mathrm{V}_{\mathrm{DS}}=5 \mathrm{~V}$ )

NMOS Enhancement Transistor: W = 100 $\mu \mathrm{m}, \mathrm{L}=20 \mu \mathrm{~m}$

## MOS transistor modes of operation



## MOS transistor modes of operation

Linear region
$V_{G S}>V_{T}>0$
$0<V_{D S}<V_{G S}-V_{T}$

$$
\begin{gathered}
\text { Pinch-off } \\
V_{G S}>V_{T}>0 \\
V_{D S}>V_{G S}-V_{T}
\end{gathered}
$$

Saturation
$V_{G S}>V_{T}>0$
$V_{D S}>V_{G S}-V_{T}$


B



## Threshold voltage

gate-bulk work function diff.
formation of inversion layer

$$
\begin{aligned}
V_{T} & =\left[\phi_{G B}\right]+\left[-2 \phi_{F}-\frac{Q_{b}^{\prime}}{C_{o x}}\right]+\left[\frac{-Q_{S S}}{C_{o x}}\right] \\
& =\phi_{G B}-2 \phi_{F}-\frac{Q_{b 0}}{C_{o x}}-\frac{Q_{\text {Ss }}}{C_{o x}}-\frac{Q_{b}-Q_{b 0}}{C_{o x}}
\end{aligned}
$$

Bias independent part:

$$
V_{T 0}=\phi_{G B}-2 \phi_{\mathrm{F}}-\frac{\mathrm{Q}_{\mathrm{b} 0}}{\mathrm{C}_{\mathrm{ox}}}-\frac{\mathrm{Q}_{\mathrm{ss}}}{\mathrm{C}_{\mathrm{ox}}}
$$



## Threshold voltage

Source-bulk bias voltage dependence:
$V_{T}=V_{T 0}+\nu\left(\sqrt{2\left|\phi_{F}\right|+V_{S B}}-\sqrt{2\left|\phi_{F}\right|}\right)$
$V_{T 0}=V_{T}\left(V_{S B}=0\right)=V_{F B}+2\left|\phi_{F}\right|+\sqrt{\frac{2 q \varepsilon_{s} N_{\text {SUB }} 2 \mid \phi_{F}}{C_{0 X}}}$

$\gamma=$ bulk threshold parameter $\left(\right.$ volts $\left.^{1 / 2}\right)=\frac{\sqrt{2 \varepsilon_{\text {si }} \mathrm{qN}_{\text {sUB }}}}{\mathrm{C}_{\mathrm{ox}}}$
$\phi_{F}=$ strong inversionsurface potential $($ volts $)=\frac{k T}{q} \ln \left(\frac{N_{\text {SUB }}}{n_{i}}\right)$
$V_{F B}=$ flatbandvoltage $($ volts $)=\phi_{G B}-\frac{\mathrm{Q}_{\mathrm{SS}}}{\mathrm{Q}_{\mathrm{OX}}}$
$\phi_{G B}=\phi_{F}($ substrate $)-\phi_{F}$ (gate)
$\phi_{F}($ substrate $)=\frac{k T}{q} \ln \left(\frac{n_{i}}{N_{\text {SUB }}}\right)$ [NMOS with p - substrate]
$\phi_{F}($ gate $)=\frac{k T}{q} \ln \left(\frac{N_{\text {GATE }}}{n_{i}}\right)$ [NMOS with $n^{+}$polysilicon gate $]$
$Q_{S S}=$ oxide charge $=N_{S S} q$

## Back bias effects on MOS transistor



- Increases threshold voltage

- Decreases drain current


## Derivation of drain current equation

The charge per unit area in the channel:

$$
Q_{1}(y)=C_{o x}\left[V_{G S}-V(y)-V_{T}\right]
$$

Resistance of the channel per unit of length:

$$
d R=\frac{d y}{\mu_{n} Q_{1}(y) W} \quad R=\frac{\rho 1}{A} \quad \begin{aligned}
& \text { with } \\
& \text { and }
\end{aligned} \quad \rho=\frac{1}{\mu n q} \quad A=W z \quad, n q=\frac{Q_{1}(y)}{z}
$$

The voltage drop along the channel ( y -direction):

$$
d v(y)=I_{0} d R=\frac{I_{0} d y}{\mu_{n} Q_{1}(y) W} \Rightarrow I_{0} d y=W \mu_{n} Q_{1}(y) d v(y)
$$



Integrate this from source to drain, i.e. $y=0$ to $y=L$

$$
\begin{aligned}
& \int_{0}^{L} I_{D} d y=\int_{0}^{V_{D S}} W \mu_{n} Q_{1}(y) d v(y)=\int_{0}^{V_{D S}} W \mu_{n} C_{o x}\left[V_{G S}-v(y)-V_{T}\right] d v(y) \Rightarrow I_{D}=\frac{\mu_{n} C_{O X} W}{2 L}\left[2\left(V_{G S}-V_{T}\right) V_{D S}-V_{D S}^{2}\right] \\
& \Rightarrow I_{D}=\left.\frac{\mu_{n} C_{0 x} W}{L}\right|_{0} ^{V_{D S}}\left(V_{G S}-V_{T}\right) v(y)-\frac{v(y)^{2}}{2}
\end{aligned}
$$

## Pinch-off

In pinch-off inversion layer at drain end is lost due to higher $\mathrm{V}_{\mathrm{DS}}$ over $\mathrm{V}_{G S}-\mathrm{V}_{\mathrm{T}}$ and drain current is saturated

$$
\begin{gathered}
I_{D}=\frac{\mu_{0} C_{0 x} W}{L}\left[\left(V_{G S}-V_{T}\right)-\left(\frac{V_{D S}}{2}\right)\right] V_{D S} \\
0<V_{D S} \leq\left(V_{G S}-V_{T}\right)
\end{gathered}
$$



Saturation voltage

$$
V_{D S, \text { sat }}=V_{G S}-V_{T}
$$

Saturation current

$$
\begin{gathered}
\mathrm{I}_{\mathrm{D}}=\frac{\mu_{0} \mathrm{C}_{\mathrm{ox}} \mathrm{~W}}{2 \mathrm{~L}}\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{T}\right)^{2}\left(1+\lambda \mathrm{V}_{\mathrm{DS}}\right) \\
0<\left(\mathrm{V}_{\mathrm{GS}} \quad \mathrm{~V}_{\mathrm{T}}\right) \quad \mathrm{V}_{\mathrm{DS}}
\end{gathered}
$$



## MOS Spice model (level 1)



## Planar conductor resistance



Current flow in conductive bar

6 squares in series:


3 squares in parallel:


L

Resistance of a conductor

$$
R=\frac{\rho L}{A}(\Omega) \quad ; \rho=\frac{1}{q \mu_{n} n} \text { resistivity }
$$

Insert area $\mathrm{A}=\mathrm{WT}$

$$
\begin{aligned}
& R=\frac{\rho L}{W T}(\Omega) \\
& R=\frac{L}{W} R_{\square}(\Omega)
\end{aligned}
$$

where resistance per square is

$$
R_{\square}=\frac{\rho}{t}
$$

L/W determines the number of squares

## Capacitance of pn-diodes

Planar diode:


Capacitance of a reverse-biased diode (Abrupt Junction)

$$
C_{j}=\frac{C_{j 0}}{\sqrt{1+\frac{V_{R}}{\Phi_{0}}}} ; V_{R}=\text { reverse bias voltage }
$$

, where $\mathrm{C}_{\mathrm{j} 0}$ is zero bias capacitance $\left(\mathrm{V}_{\mathrm{R}}=0\right)$

$$
C_{j 0}=\sqrt{\frac{q K_{s} \varepsilon_{0}}{2 \Phi_{0}} \frac{N_{D} N_{A}}{N_{A}+N_{D}}} \quad C_{j 0}=\sqrt{\frac{q K_{s} \varepsilon_{0} N_{D}}{2 \Phi_{0}}} \text {, if } N_{A} \gg N_{D}
$$

and $\Phi_{0}$ is junction potential

$$
\Phi_{0}=\frac{k T}{q} \ln \left(\frac{\mathrm{~N}_{\mathrm{A}} \mathrm{~N}_{\mathrm{D}}}{\mathrm{n}_{\mathrm{j}}^{2}}\right)
$$

## Drain and source junction capacitances CDB, CSB

$$
\begin{equation*}
C_{B X}=C_{B X 0} A_{B X}\left[1-\left(\frac{v_{B X}}{P B}\right)\right]^{-M J}, \quad v_{B X} \leq(F C)(P B) \tag{1}
\end{equation*}
$$

$A_{B X}=$ junction areas
$C_{B X 0}=C_{B X}\left(\right.$ when $\left.v_{B X}=0\right) \cong \sqrt{\frac{\left(q \varepsilon_{\text {si }} \mathrm{N}_{\text {SUB }}\right)}{P B}}$
$\mathrm{PB}=$ bulk junction potential
FC = forward - bias nonideal junction - capacitance coefficient $(\cong 0.5)$
$\mathrm{MJ}=$ bulk - junction grading coefficient ( $1 / 2$ for step junctions
and $\frac{1}{3}$ for graded junctions)


To ease numerical solution of the simulator
$C_{B X}=\frac{C_{B X 0} A_{B X}}{(1-F C)^{1+M J}}\left[1-(1+M J) F C+M J \frac{V_{B X}}{P B}\right], \quad V_{B X}>(F C)(P B)$

## Drain and source junction capacitances CDB, CSB



Drain bottom $=A B C D$

$$
\begin{aligned}
C_{B X}=\frac{(C J)(A X)}{\left[1-\left(\frac{V_{B X}}{P B}\right)\right]^{M J}}+\frac{(C J S W)(P X)}{\left[1-\left(\frac{V_{B X}}{P B}\right)\right]^{M S S W}} & C_{B X}
\end{aligned}=\frac{(C J)(A X)}{(1-F C)^{1+M J}}\left[1-(1+M J) F C+M J \frac{V_{B X}}{P B}\right] \quad \begin{aligned}
& +\frac{(C J S W)(P X)}{(1-F C)^{1+M S W}}\left[1-(1+J M S W) F C+\frac{V_{B X}}{P B}(M J S W)\right]
\end{aligned}
$$

$A X=$ area of the source $(X=S)$ or drain ( $X=D$ ) $P X=$ perimeter of the source $(X=S)$ or drain ( $X=D$ ) CJSW = zero-bias, bulk-source/drain sidewall capacitance MJSW = bulk-source/drain sidewall grading coefficient

$$
v_{B X} \leq(F C)(P B)
$$

$$
\mathrm{v}_{\mathrm{BX}} \geq(\mathrm{FC})(\mathrm{PB})
$$

## MOS transistor gate capacitance

Gate capacitance (linear region)

$$
C_{2}=W_{\text {eff }}(L-2 L D) C_{o x}=W_{\text {eff }}\left(L_{\text {eff }}\right) C_{o x}
$$

Gate overlap capacitances

$$
\mathrm{C}_{1}=\mathrm{C}_{3} \cong \mathrm{LDW}_{\text {eff }} \mathrm{C}_{\mathrm{ox}}=\mathrm{W}_{\text {eff }} \mathrm{CGXO} ; \mathrm{CGXO}[\mathrm{~F} / \mathrm{m}]=\mathrm{LD} \cdot \mathrm{C}_{\mathrm{ox}}
$$



## MOS transistor gate capacitance

## Cut-off

$$
\begin{aligned}
& C_{G B}=C_{2}+2 C_{5}=C_{\text {ox }}\left(W_{\text {eff }} L_{\text {eff }}\right)+2 \operatorname{CGBO}\left(L_{\text {eff }}\right) \\
& C_{G S}=C_{1} \cong C_{\text {ox }}\left(L D W_{\text {eff }}\right)=\operatorname{CGSO}\left(W_{\text {eff }}\right) \\
& C_{G D}=C_{3} \cong C_{\text {ox }}\left(L D W_{\text {eff }}\right)=\operatorname{CGDO}\left(W_{\text {eff }}\right)
\end{aligned}
$$

## Saturation

$$
\begin{aligned}
C_{G B} & =\frac{\left(C_{2}+2 C_{5}\right) C_{4}}{C_{2}+2 C_{5}+C_{4}} \cong C_{4} \cong 0 \\
C_{G S} & =C_{1}+\frac{2}{3} C_{2}=C_{\text {ox }}\left(L D+0,67 L_{\text {eff }}\right)\left(W_{\text {eff }}\right) \\
& =C G S O\left(W_{\text {eff }}\right)+0,67 C_{\text {ox }}\left(W_{\text {eff }} L_{\text {eff }}\right) \\
C_{G D} & =C_{3} \cong C_{o x}\left(L D W_{\text {eff }}\right)=\operatorname{CGDO}\left(W_{\text {eff }}\right)
\end{aligned}
$$

## Nonsaturated

$$
\begin{aligned}
& \mathrm{C}_{6 \mathrm{~B}}=\frac{\left(\mathrm{C}_{2}+2 \mathrm{C}_{5}\right) \mathrm{C}_{4}}{\mathrm{C}_{2}+2 \mathrm{C}_{5}+\mathrm{C}_{4}} \cong \mathrm{C}_{4} \cong 0 \\
& C_{G S}=C_{1}+\frac{1}{2} C_{2}=C_{o x}\left(L D+0,5 L_{\text {eff }}\right)\left(W_{\text {eff }}\right) \\
& =\left(\mathrm{CGSO}+0,5 \mathrm{C}_{\mathrm{ox}} \mathrm{~L}_{\text {eff }}\right) \mathrm{W}_{\text {eff }} \\
& C_{60}=C_{3}+0,5 C_{2}=C_{o x}\left(L D+0,5 L_{\text {eff }}\right)\left(W_{\text {eff }}\right) \\
& =\left(C G D O+0,5 C_{\text {ox }} L_{\text {eff }}\right) W_{\text {eff }}
\end{aligned}
$$



## Charge conservation model and short channel effects

## Charge conservation model

Charge conservation model (level 2):

$$
d V_{c}=I_{d} d R=\frac{I_{d} d y}{W \mu_{n} Q_{n}^{\prime}(y)}
$$

Channel change: $\quad Q_{n}{ }^{\prime}=-C_{o x}{ }^{\prime}\left(V_{G}-V_{T}\right)$
Threshold voltage: $V_{T}=f\left(V_{c}\right)$

$$
V_{T}=V_{F B}+V_{C}+2\left|\phi_{\mathrm{p}}\right|+\frac{1}{C_{0 X}{ }^{\prime}} \sqrt{2 \varepsilon_{\mathrm{s}} \mathrm{q} \mathrm{~N}_{\mathrm{a}}\left(2\left|\phi_{\mathrm{p}}\right|+\mathrm{V}_{\mathrm{C}}-\mathrm{V}_{\mathrm{B}}\right)}
$$

Channel change: $\quad Q_{n}{ }^{\prime}=f\left(v_{c}\right)$

$$
\begin{aligned}
\mathrm{Q}_{n}^{\prime} & =-C_{0 X}{ }^{\prime}\left(V_{G}-V_{F B}-2\left|\phi_{p}\right|-V_{C}\right) \\
& +\sqrt{2 \varepsilon_{\mathrm{s}} 9 N_{\mathrm{a}}\left(2\left|\phi_{\mathrm{p}}\right|+\mathrm{V}_{\mathrm{C}}-\mathrm{V}_{B}\right)}
\end{aligned}
$$

Integrating:

$$
\begin{aligned}
& d V_{C}=I_{d} d R=\frac{I_{d} d y}{W \mu_{n} Q_{n}^{\prime}(y)} \\
& I_{D}=\int_{0}^{L} d y=I_{D} L=-\mu W \int_{V_{S}}^{V_{0}} Q_{n}^{\prime}\left(V_{c}\right) d V_{c}
\end{aligned}
$$

Drain current equation:

$$
\begin{aligned}
\mathrm{I}_{\mathrm{D}}= & \mu_{\mathrm{n}} \frac{\mathrm{~W}}{\mathrm{~L}} \mathrm{C}_{\mathrm{OX}} \cdot\left\{\mathrm { C } _ { \mathrm { ox } } ^ { \prime } \cdot \left[\mathrm{~V}_{G}-\mathrm{V}_{\mathrm{FB}}-2\left|\phi_{\mathrm{p}}\right|\right.\right. \\
& \left.-\frac{1}{2} \mathrm{~V}_{\mathrm{D}}-\frac{1}{2} \mathrm{~V}_{\mathrm{S}}\right]\left(\mathrm{V}_{\mathrm{D}}-\mathrm{V}_{\mathrm{S}}\right) \\
& \left.-\frac{2}{3} \sqrt{2 \varepsilon_{\mathrm{s}} \mathrm{GN}}\left[\left(2\left|\phi_{\mathrm{p}}\right|+\mathrm{V}_{\mathrm{D}}-\mathrm{V}_{B}\right)^{\frac{2}{3}}-\left(2\left|\phi_{\mathrm{p}}\right|+\mathrm{V}_{\mathrm{S}}-\mathrm{V}_{B}\right)^{\frac{2}{3}}\right]\right\}
\end{aligned}
$$

Pinch-off:

$$
\begin{aligned}
\mathrm{Q}_{n}^{\prime}(L)= & =-C_{o x}\left(V_{\mathrm{G}}-V_{\text {FB }}-2\left|\phi_{\mathrm{p}}\right|-V_{\text {osat }}\right) \\
& +\sqrt{2 \varepsilon_{s} g N_{\mathrm{a}}\left(2\left|\phi_{s}\right|+V_{\text {Dsat }}-V_{B}\right)}
\end{aligned}
$$

Saturation voltage:

$$
\begin{aligned}
V_{\text {Dsat }} & =V_{G}-V_{F B}-2\left|\phi_{p}\right| \\
& -\frac{\varepsilon_{s} q N_{a}}{C_{O X}{ }^{\prime 2}}\left[\sqrt{1+\frac{2 C_{0 X}{ }^{\prime 2}}{\varepsilon_{s} q N}\left(V_{G}-V_{F B}-V_{B}\right)}-1\right]
\end{aligned}
$$

## Channel length modulation

Basic model

$$
L_{\text {mod }}=L_{\text {eff }}\left(1-\lambda v_{\text {DS }}\right) \quad L_{\text {eff }}=L-2(L D)
$$

More accurate charge conservation model

$\lambda=\frac{1}{L_{\text {eff }} v_{D S}}\left[\frac{2 \varepsilon_{\text {si }}}{\mathrm{qN}_{\text {SUB }}}\right]^{\frac{1}{2}}\left\{\frac{\mathrm{v}_{\text {DS }}-\mathrm{v}_{\mathrm{DS}}(\mathrm{sat} .)}{4}+\left[1+\left(\frac{\mathrm{v}_{\mathrm{DS}}-\mathrm{v}_{\mathrm{DS}}(\mathrm{sat} .)}{4}\right)^{2}\right]^{\frac{1}{2}}\right\}$
$\mathrm{v}_{\mathrm{DS}}(\mathrm{sat})=.\frac{\mathrm{v}_{G S}-\mathrm{V}_{\mathrm{BIN}}}{\theta}+\frac{1}{2}\left(\frac{\gamma \mathrm{~s}}{\theta}\right)^{2}\left\{1-\left[1+\left(\frac{2 \theta}{\gamma s}\right)^{2}\left(\frac{\mathrm{v}_{G S}-\mathrm{V}_{\mathrm{BIN}}}{\theta}+2\left|\phi_{\mathrm{F}}\right|+\mathrm{v}_{\mathrm{SB}}\right)\right]^{\frac{1}{2}}\right\}$


## Charge carrier velocity saturation

Charge carrier velocity (NMOS):


Charge carrier mobility (NMOS):


## Charge carrier velocity saturation

Charge carrier velocity (NMOS):


Charge carrier mobility (NMOS):


## Charge carrier velocity saturation

I: No velocity saturation $|\bar{E}|<E_{\text {crit }}$
$\mu_{\mathrm{s}}=\mu_{0}$
conditions

1) $V_{D S}$ small
2) strong inversion $V_{G S}>V_{T}$
3) $|\bar{E}|<E_{\text {crit }}$

II: With velocity saturation $|\bar{E}|>E_{\text {crit }}$

$$
\begin{aligned}
& \mu_{s}=\mu_{0}\left[\frac{\left(U_{C R T}\right) E_{S i}}{C_{o x}\left[V_{G S}-V_{T}-\left(U_{T R A}\right) V_{D S}\right]}\right]^{U_{E X P}} \text { when } \\
& \frac{\left(U_{C R T T}\right) E_{S i}}{C_{\text {ox }}}<V_{G S}-V_{T}-\left(U_{T R A}\right) V_{D S}
\end{aligned}
$$



The parameters are defined as
$U_{\text {CRIT }}=$ critical field for mobility degradation and is the limit at which $m_{s}$ starts decreasing.
$U_{\text {TRA }}=$ transverse field coefficient effecting mobility.
$U_{\text {EXP }}=$ critical field exponent for mobility degradation.

## Charge carrier velocity saturation



## Short channel effects



Cross section of a short
channel transistor showing several depletion areas
which affect each other.


Short channel effect on the threshold voltage $\mathrm{V}_{T}$ of an nMOS transistor with and without a $\mathrm{DV}_{\mathrm{T}}$ implantation.

## Short channel effects

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{S}}=\sqrt{\frac{2 \varepsilon_{\mathrm{si}}}{\mathrm{qN} \mathrm{~S}_{\text {SUB }}}} \sqrt{2\left|\phi_{\mathrm{F}}\right|+\mathrm{v}_{\mathrm{SB}}} \\
& \mathrm{~W}_{\mathrm{D}}=\sqrt{\frac{2 \varepsilon_{\mathrm{Si}}}{\mathrm{qN}}} \sqrt{2\left|\phi_{\mathrm{F}}\right|+\mathrm{V}_{\mathrm{SB}}+\mathrm{v}_{\mathrm{DS}}}
\end{aligned}
$$


$i_{D}=\frac{\mu_{S} C_{\text {ox }} W}{L_{\text {mod }}}\left\{\left[\mathrm{v}_{\text {GS }}-\mathrm{V}_{\text {BIN }}-\frac{\theta \mathrm{v}_{\text {DS }}}{2}\right] \mathrm{v}_{\text {DS }}-\frac{2}{3} \gamma S\left[\left(2\left|\phi_{F}\right|+\mathrm{v}_{\mathrm{DS}}+\mathrm{v}_{\text {SB }}\right)^{1,5}-\left(2\left|\phi_{F}\right|+\mathrm{v}_{\mathrm{BS}}\right)^{1,5}\right]\right\}$
$V_{\text {BIN }}=V_{F B}+2\left|\phi_{F}\right|+\frac{\pi \varepsilon_{S i}}{4 C_{\text {ox }} W}\left(2\left|\phi_{F}\right|+V_{S B}\right)$
$\theta=1+\frac{\pi \varepsilon_{\mathrm{si}}}{4 \mathrm{C}_{\mathrm{ox}} W}$
$\gamma_{\mathrm{s}}=\gamma\left(1-\alpha_{\mathrm{S}}-\alpha_{\mathrm{D}}\right)$
$\alpha_{s}=\frac{X J}{2 L}\left[\sqrt{1+\left(\frac{2 W_{S}}{X J}\right)}-1\right]$
$\alpha_{D}=\frac{X J}{2 L}\left[\sqrt{1+\left(\frac{2 W_{D}}{X J}\right)}-1\right]$


## Weak inversion behaviour

MOS transistor operates in the 'weak inversion' region when its gate-source voltage $\left(\mathrm{V}_{\mathrm{GS}}\right)$ is just below its threshold voltage $\left(\mathrm{V}_{\mathrm{T}}\right)$.

$$
\mathrm{I}_{\text {Duub }}=\frac{\mathrm{W}}{\mathrm{~L}} \cdot \mathrm{Cl}_{\mathrm{D}} \mathrm{e}^{\frac{\mathrm{V}_{G B}}{\mathrm{mU}}}
$$

$$
\begin{aligned}
C & =e^{\frac{-V_{S B}}{U_{T}}}-e^{\frac{-V_{0 B}}{U_{T}}}(\text { Cis constanthere }) \\
U_{T} & =\frac{\mathrm{kT}}{\mathrm{q}} \approx 25 \mathrm{mV} \text { atroom temperature } \\
\mathrm{I}_{\text {oso }} & =\text { characteristic current at } \mathrm{V}_{G B}=0 \mathrm{~V} \\
\mathrm{~m} & =\text { slope } \approx 1,5
\end{aligned}
$$

If $\mathrm{V}_{T}$ is low $(<0,6 \mathrm{~V})$ then there is always
a sub-threshold current when $\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}$. There is a considerable stand-by
 current.

## Matching of weak and strong inversion models



## Spice transistor parameters

| Parameter name | Symbol | SPICE Name | Units | Default Value |
| :--- | :---: | :---: | :---: | :---: |
| Drawn Length | L | L | m | - |
| Effective width | W | W | m | - |
| Source Area | AREA | AS | $\mathrm{m}^{2}$ | 0 |
| Drain Area | AREA | AD | $\mathrm{m}^{2}$ | 0 |
| Source Perimeter | PERIM | PS | m | 0 |
| Drain Perimeter | PERIM | PD | m | 0 |
| Squares of Source Diffusion |  | NRS | - | 1 |
| Squares of Drain Diffusion |  | NRD | - | 1 |

## Spice parameters for parasitics

| Parameter name | Symbol | SPICE Name | Units | Default Value |
| :--- | :---: | :---: | :---: | :---: |
| Source resistance | $\mathrm{R}_{\mathrm{S}}$ | RS | $\Omega$ | 0 |
| Drain resistance | $\mathrm{R}_{\mathrm{D}}$ | RD | $\Omega$ | 0 |
| Sheet resistance (Source/Drain) | $\mathrm{R}_{\mathrm{a}}$ | RSH | $\Omega / \square$ | 0 |
| Zero Bias Bulk Junction Cap | $\mathrm{R}_{\mathrm{Jo}}$ | CJ | $\mathrm{F} / \mathrm{m}^{2}$ | 0 |
| Bulk Junction Grading Coeff. | m | MJ | - | 0.5 |
| Zero Bias Side Wall Junction Cap | $\mathrm{C}_{\text {JSwo }}$ | CJSW | $\mathrm{F} / \mathrm{m}$ | 0 |
| Side Wall Grading Coeff. | $\mathrm{m}_{\text {Sw }}$ | MJSW | - | 0.3 |
| Gate-Bulk Overlap Capacitance | $\mathrm{C}_{\text {GBO }}$ | CGBO | $\mathrm{F} / \mathrm{m}$ | 0 |
| Gate-Source Overlap Capacitance | $\mathrm{C}_{\text {GSO }}$ | CGSO | $\mathrm{F} / \mathrm{m}$ | 0 |
| Gate-Drain Overlap Capacitance | $\mathrm{C}_{\text {GDO }}$ | CGDO | $\mathrm{F} / \mathrm{m}$ | 0 |
| Bulk Junction Leakage Current | $\mathrm{I}_{\mathrm{S}}$ | IS | A | 0 |
| Bulk Junction Leakage Current Density | $\mathrm{J}_{\mathrm{S}}$ | JS | $\mathrm{A} / \mathrm{m}^{2}$ | $1 \mathrm{E}-8$ |
| Bulk Junction Potential | $\phi_{0}$ | PB | V | 0.8 |

## Main MOS Spice parameters

| Parameter name | Symbol | SPICE Name | Units | Default Value |
| :---: | :---: | :---: | :---: | :---: |
| SPICE Model Index |  | LEVEL | - | 1 |
| Zero-Bias Threshold Voltage | VTO | VTO | V | 0 |
| Process Transconductance | $k^{\prime}$ | KP | A/V ${ }^{2}$ | 2.E-5 |
| Body-Bias Parameter | g | GAMMA | V0.5 | 0 |
| Channel Modulation | 1 | LAMBDA | 1/V | 0 |
| Oxide thickness | tox | TOX | m | 0 |
| Lateral Diffusion | xd | LD | m | 0 |
| Metallurgical Junction Depth | xj | XJ | m | 0 |
| Surface Inversion Potential | 2\|fF| | PHI | V | 0.6 |
| Substrate Doping | NA, ND | NSUB | $\mathrm{cm}^{-3}$ | 0 |
| Surface State Density | $\mathrm{O}_{5 s / q}$ | NSS | $\mathrm{cm}^{-3}$ | 0 |
| Fast Surface State Density |  | NFS | $\mathrm{cm}^{-3}$ | 0 |
| Total Channel Charge Coefficient |  | NEFF | - | 1 |
| Type of Gate Material |  | TPG | - | 1 |
| Surface Mobility | mo | U0 | $\mathrm{cm}^{2} / \mathrm{Vs}$ | 600 |
| Maximum Drift Velocity | umax | VMAX | $\mathrm{m} / \mathrm{s}$ | 0 |
| Mobility Critical Field | xcrit | UCRIT | $\mathrm{V} / \mathrm{cm}$ | 1.0E-4 |
| Critical Field Exponent in Mobility Degradation |  | UEXP | - | 0 |
| Transverse Field Exponent (mobility) |  | UTRA | - | 0 |

## Matching manual and Spice models



## MOS transistor model characteristics



Variation of the drain current with model parameter VTO, for the LEVEL1 model.



Variation of the drain current with model parameter TOX, for the LEVEL1 model.


## MOS transistor characteristics



NMOS transfer characteristic of a typical wafer, W/L = 30/6, VGS = 1.5, 2, 3, 4, 5 V . o measured, solid line $=$ MOS2 model, dashed line $=$ AMS model.


PMOS transfer characteristic of a typical wafer, W/L = 30/6, -VGS = 1.5, 2, 3, 4, 5 V . o measured, solid line $=$ MOS2 model, dashed line $=$ AMS model.

## 12 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, $W / L=2 / 30, V G S=1.5,2,3,4,5 \mathrm{~V}$. o measured, solid line $=$ MOS2 model, dashed line $=$ AMS model.


PMOS transfer characteristic of a typical wafer, $W / L=2 / 30,-V G S=1.5,2,3,4,5 \mathrm{~V}$. o measured, solid line $=$ MOS2 model, dashed line $=$ AMS model.

## 1.2 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, $W / L=30 / 1.2, ~ V G S=1.5,2,3,4,5 \mathrm{~V}$. o measured, solid line = MOS2 model, dashed line
= AMS model.


PMOS transfer characteristic of a typical wafer, $W / L=30 / 1.2,-V G S=1.5,2,3,4,5 \mathrm{~V} .0$ measured, solid line = MOS2 model, dashed line = AMS model.

## 1.2 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, $W / L=30 / 1.2, V G S=1.5,2,3,4,5 \mathrm{~V}$. o measured, solid line = MOS2 model, dashed line = AMS model.


PMOS transfer characteristic of a typical wafer, $W / L=30 / 1.2,-V G S=1.5,2,3,4,5 \mathrm{~V} .0$ measured, solid line $=$ MOS2 model, dashed line = AMS model.

## Small-signal model

## Small signal model

Small-signal model is a linearised model at the operating point
Transconductance: $\quad \mathrm{g}_{\mathrm{m}}=\frac{\partial \mathrm{i}_{\mathrm{D}}}{\partial \mathrm{v}_{G S}}$ (at the quiescent point)

Drain-source conductance: $g_{d s}=\frac{\partial i_{D}}{\partial \mathrm{v}_{\mathrm{DS}}}$ (at the quiescent point)

Bulk modulation:

$$
\mathrm{g}_{\mathrm{mbs}}=\frac{\partial \mathrm{i}_{\mathrm{D}}}{\partial \mathrm{v}_{\mathrm{BS}}} \text { (at the quiescent point) }
$$

Drain and source diode conductances:
$\mathrm{g}_{\mathrm{bd}}=\frac{\partial \mathrm{i}_{\mathrm{BD}}}{\partial \mathrm{v}_{\mathrm{BD}}}$ (at the quiescent point) $\approx 0$

$\mathrm{g}_{\mathrm{bs}}=\frac{\partial \mathrm{i}_{\mathrm{SB}}}{\partial \mathrm{v}_{\mathrm{SB}}}$ (at the quiescent point) $\approx 0$

## Modes of operation

| Cut-off | $V_{G S}-V_{T} \leq 0$ | $i_{D}=0$ |
| :---: | :---: | :---: |
| Linear region | $0<V_{D S} \leq\left(V_{G S}-V_{T}\right)$ | $i_{D}=\frac{\mu_{0} c_{0 X} W}{L}\left[\left(v_{G S}-V_{T}\right)-\frac{v_{0 S}}{2}\right] v_{D S}\left(1+\lambda v_{D S}\right)$ <br> $\beta=\left(K^{\prime}\right) \frac{W}{L} \cong\left(\mu_{0} C_{o x}\right) \frac{W}{L}\left(a m p s / v o l t^{2}\right)$ |
| Saturation | $0<\left(V_{G S}-V_{T}\right) \leq V_{D S}$ | $i_{D}=\frac{\mu_{0} C_{0 \times} W}{2 L}\left(v_{G S}-V_{T}\right)^{2}\left(1+\lambda v_{D S}\right)$ |
| Pinch-off | $V_{D S}($ sat. $)=V_{G S}-V_{T}$ |  |

## Saturation region

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{D}}=\frac{\mu_{O} C_{O X} W}{2 \mathrm{~L}}\left(\mathrm{~V}_{G S}-\mathrm{V}_{T}\right)^{2}\left(1+\lambda \mathrm{V}_{\mathrm{DS}}\right) \\
& \mathrm{V}_{T}=\mathrm{V}_{T O}+\gamma\left(\sqrt{\left|-2 \phi_{F}+\mathrm{V}_{S B}\right|}-\sqrt{\left|-2 \phi_{F}\right|}\right)
\end{aligned}
$$

Transconductance:
$g_{m}=\frac{\partial i_{D}}{\partial v_{G S}}=\frac{\mu_{0} C_{O X} W}{2 L} 2\left(v_{G S}-V_{T}\right)\left(1+\lambda \mathrm{v}_{\mathrm{DS}}\right)$
$g_{m}=\sqrt{\left(2 K^{\prime} W / L\right)| |_{D} \mid}\left(1+\lambda V_{D S}\right) \cong \sqrt{\left(2 K^{\prime} W / L\right)| |_{D} \mid}$
$k^{\prime}=\mu * C_{o x}$

## Drain-source conductance:

$$
\begin{aligned}
& g_{d s}=\frac{\partial i_{D}}{\partial v_{D S}} \\
& g_{d s}=g_{o}=\frac{I_{D} \lambda}{1+\lambda V_{D S}} \approx I_{D} \lambda \quad \lambda \propto \frac{1}{L}
\end{aligned}
$$

Bulk modulation:

$$
\begin{aligned}
& g_{m b s}=\frac{\partial i_{D}}{\partial v_{S B}}=\left(\frac{\partial i_{D}}{\partial V_{T}}\right)\left(\frac{\partial V_{T}}{\partial v_{S B}}\right) \quad \frac{\partial i_{D}}{\partial V_{T}}=\left(\frac{-\partial i_{D}}{\partial v_{G S}}\right) \\
& g_{m b s}=g_{m} \frac{Y}{2\left(2\left|\phi_{F}\right|+V_{S B}\right)^{1 / 2}}=\eta g_{m}
\end{aligned}
$$

## MOS transistor small signal model



