ELEC-E3510 Basics of IC Design

Lecture 2:

MOS transistor models

Lecture outline:

- threshold voltage of MOS transistor
- drain current equation
- parasitic capacitors and resistors
- short channel effects (extra material)
- weak inversion
- SPICE parameters
- small signal model

MOS transistor



Modes of operation

| Cut-off | $V_{GS} - V_T \leq 0$ | $I_{DS} = 0$ |
|----------------------------------|--|--|
| Linear region (Triode region) | $0 < V_{\rm DS} \leq (V_{\rm GS} - V_{\rm T})$ | $I_{D} = \frac{\mu_{0}C_{ox}W}{L} \left[(V_{GS} - V_{T}) - \frac{V_{DS}}{2} \right] V_{DS} (1 + \lambda V_{DS})$ $k = (K')\frac{W}{L} \cong (\mu_{0}C_{ox})\frac{W}{L} \text{ (amps/volt}^{2})$ |
| Saturation | $0 < (V_{GS} - V_T) \leq V_{DS}$ | $I_{D} = \frac{\mu_{0}C_{ox}W}{2L}(V_{GS} - V_{T})^{2}(1 + \lambda V_{DS})$ |
| Pinch-off | $V_{\rm DS,SAT} = V_{\rm GS} - V_{\rm T}$ | $I_{\rm DS,LIN} = I_{\rm DS,SAT}$ |



a) I_D as a function of V_{DS}

b) $\sqrt{I_D}$ as a function of V_{GS} (for V_{DS} = 5V)

NMOS Enhancement Transistor: W = $100\mu m$, L = $20\mu m$

MOS transistor modes of operation



MOS transistor modes of operation





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Threshold voltage



Bias independent part:

$$V_{TO} = \varphi_{GB} - 2\varphi_F - \frac{Q_{bO}}{C_{ox}} - \frac{Q_{ss}}{C_{ox}}$$



Threshold voltage

Source-bulk bias voltage dependence:

$$\begin{split} V_{T} &= V_{T0} + \gamma \Big(\sqrt{2 \left| \varphi_{F} \right| + V_{SB}} - \sqrt{2 \left| \varphi_{F} \right|} \Big) \\ V_{T0} &= V_{T} \left(V_{SB} = 0 \right) = V_{FB} + 2 \left| \varphi_{F} \right| + \sqrt{\frac{2q\epsilon_{si}N_{SUB}2 \left| \varphi_{F} \right|}{C_{OX}}} \end{split}$$



$$\gamma = \text{bulk threshold parameter (volts^{1/2})} = \frac{\sqrt{2\epsilon_{si}qN_{SUB}}}{C_{OX}}$$

$$\varphi_{F} = \text{strong inversion surface potential (volts)} = \frac{kT}{q} \ln\left(\frac{N_{SUB}}{n_{i}}\right)$$

$$V_{FB} = \text{flatband voltage (volts)} = \varphi_{GB} - \frac{Q_{SS}}{Q_{OX}}$$

$$\varphi_{GB} = \varphi_{F}(\text{substrate}) - \varphi_{F}(\text{gate})$$

$$\varphi_{F}(\text{substrate}) = \frac{kT}{q} \ln\left(\frac{n_{i}}{N_{SUB}}\right) [\text{NMOS with p - substrate}]$$

$$\varphi_{F}(\text{gate}) = \frac{kT}{q} \ln\left(\frac{N_{GATE}}{n_{i}}\right) [\text{NMOS with n^{+} polysilicon gate}]$$

$$Q_{SS} = \text{oxide charge} = N_{SS}q$$

Back bias effects on MOS transistor



• Increases threshold voltage

• Decreases drain current

Derivation of drain current equation

The charge per unit area in the channel:

$$Q_{1}(y) = C_{OX}[V_{GS} - V(y) - V_{T}]$$

Resistance of the channel per unit of length:

$$dR = \frac{dy}{\mu_n Q_1(y)W} \qquad R = \frac{\rho I}{A} \quad \text{with} \quad \rho = \frac{1}{\mu n q} \qquad A = Wz \qquad , \quad nq = \frac{Q_1(y)}{z}$$
 and

The voltage drop along the channel (y-direction):

$$dv(y) = I_D dR = \frac{I_D dy}{\mu_n Q_1(y)W} \Longrightarrow I_D dy = W\mu_n Q_1(y)dv(y)$$

Integrate this from source to drain, i.e. y = 0 to y = L

$$\int_{0}^{L} I_{D} dy = \int_{0}^{V_{DS}} W \mu_{n} Q_{1}(y) dv(y) = \int_{0}^{V_{DS}} W \mu_{n} C_{OX} [V_{GS} - v(y) - V_{T}] dv(y) \implies$$
$$\implies I_{D} = \frac{\mu_{n} C_{OX} W}{L} \int_{0}^{V_{DS}} (V_{GS} - V_{T}) v(y) - \frac{v(y)^{2}}{2}$$



$$I_{D} = \frac{\mu_{n}C_{OX}W}{2L} \Big[2(V_{GS} - V_{T})V_{DS} - V_{DS}^{2} \Big]$$

Pinch-off

In pinch-off inversion layer at drain end is lost due to higher V_{DS} over V_{GS} - V_T and drain current is saturated

$$I_{D} = \frac{\mu_{o}C_{ox}W}{L} \left[(V_{GS} - V_{T}) - \left(\frac{V_{DS}}{2}\right) \right] V_{DS}$$
$$0 < V_{DS} \le (V_{GS} - V_{T})$$



Saturation voltage

$$\rm V_{\rm DS,sat} = \rm V_{\rm GS} - \rm V_{\rm T}$$

Saturation current

$$I_{D} = \frac{\mu_{0}C_{ox}W}{2L}(V_{GS} - V_{T})^{2}(1 + \lambda V_{DS})$$
$$0 < (V_{GS} - V_{T}) \pounds V_{DS}$$



MOS Spice model (level 1)



Planar conductor resistance

 $=\frac{1}{3}$

W



Resistance of a conductor

$$R = \frac{\rho L}{A}(\Omega)$$
; $\rho = \frac{1}{q\mu_n n}$ resistivity

Insert area A = WT

 $R = \frac{\rho L}{WT} (\Omega)$ $R = \frac{L}{W} R_{\Box} (\Omega)$

where resistance per square is

 $R_{\Box} = \frac{\rho}{1}$

L/W determines the number of squares

Capacitance of pn-diodes



Capacitance of a reverse-biased diode (Abrupt Junction)

$$C_{j} = \frac{C_{j0}}{\sqrt{1 + \frac{V_{R}}{\Phi_{0}}}}$$
; V_{R} = reverse bias voltage

, where
$$C_{j0}$$
 is zero bias capacitance ($V_R = 0$)

$$C_{j0} = \sqrt{\frac{qK_s\epsilon_0}{2\Phi_0}\frac{N_DN_A}{N_A + N_D}} \qquad C_{j0} = \sqrt{\frac{qK_s\epsilon_0N_D}{2\Phi_0}} \quad \text{, if } N_A >> N_D$$

and Φ_0 is junction potential

$$\Phi_{o} = \frac{kT}{q} ln \left(\frac{N_{A}N_{D}}{n_{j}^{2}} \right)$$

A simplified model of a diode

Drain and source junction capacitances CDB, CSB

$$C_{BX} = C_{BXO}A_{BX}\left[1 - \left(\frac{V_{BX}}{PB}\right)\right]^{-MJ}, V_{BX} \leq (FC)(PB)$$

 $A_{BX} = junction areas$

$$C_{BXO} = C_{BX} \text{ (when } v_{BX} = 0) \cong \sqrt{\frac{(q\epsilon_{si}N_{SUB})}{PB}}$$

PB = bulk junction potential

FC = forward - bias nonideal junction - capacitance coefficient($\cong 0.5$)

MJ = bulk - junction grading coefficient (½ for step junctions and $\frac{1}{3}$ for graded junctions)

To ease numerical solution of the simulator

$$C_{BX} = \frac{C_{BXO}A_{BX}}{(1 - FC)^{1 + MJ}} \left[1 - (1 + MJ)FC + MJ\frac{V_{BX}}{PB} \right], \quad V_{BX} > (FC)(PB)$$
(2)



Drain and source junction capacitances CDB, CSB



Drain bottom = ABCD

$$C_{BX} = \frac{(CJ)(AX)}{\left[1 - \left(\frac{V_{BX}}{PB}\right)\right]^{MJ}} + \frac{(CJSW)(PX)}{\left[1 - \left(\frac{V_{BX}}{PB}\right)\right]^{MJSW}} \qquad C_{BX} = \frac{(CJ)(AX)}{(1 - FC)^{1+MJ}} \left[1 - (1 + MJ)FC + MJ\frac{V_{BX}}{PB}\right] + \frac{(CJSW)(PX)}{(1 - FC)^{1+MJSW}} \left[1 - (1 + JMSW)FC + \frac{V_{BX}}{PB}(MJSW) + \frac{V_{BX}}{(1 - FC)^{1+MJSW}}\right]$$

AX = area of the source (X = S) or drain (X = D)PX = perimeter of the source (X = S) or drain (X = D)CJSW = zero-bias, bulk-source/drain sidewall capacitance MJSW = bulk-source/drain sidewall grading coefficient

MOS transistor gate capacitance

Gate capacitance (linear region)

 $C_2 = W_{eff} (L - 2LD) C_{ox} = W_{eff} (L_{eff})C_{ox}$

Gate overlap capacitances

 $C_1 = C_3 \cong LDW_{eff}C_{ox} = W_{eff}CGXO$; $CGXO [F/m] = LD \cdot C_{OX}$





MOS transistor gate capacitance

Cut-off

 $C_{GB} = C_{2} + 2C_{5} = C_{ox} (W_{eff}L_{eff}) + 2CGBO(L_{eff})$ $C_{GS} = C_{1} \cong C_{ox} (LDW_{eff}) = CGSO(W_{eff})$ $C_{GD} = C_{3} \cong C_{ox} (LDW_{eff}) = CGDO(W_{eff})$

Saturation

$$C_{GB} = \frac{(C_2 + 2C_5)C_4}{C_2 + 2C_5 + C_4} \cong C_4 \cong 0$$

$$C_{GS} = C_1 + \frac{2}{3}C_2 = C_{ox}(LD + 0.67L_{eff})(W_{eff})$$

$$= CGSO(W_{eff}) + 0.67C_{ox}(W_{eff}L_{eff})$$

$$C_{GD} = C_3 \cong C_{ox}(LDW_{eff}) = CGDO(W_{eff})$$

Nonsaturated

$$C_{GB} = \frac{(C_2 + 2C_5)C_4}{C_2 + 2C_5 + C_4} \cong C_4 \cong 0$$

$$C_{GS} = C_1 + \frac{1}{2}C_2 = C_{ox}(LD + 0.5L_{eff})(W_{eff})$$

$$= (CGSO + 0.5C_{ox}L_{eff})W_{eff}$$

$$C_{GD} = C_3 + 0.5C_2 = C_{ox}(LD + 0.5L_{eff})(W_{eff})$$

$$= (CGDO + 0.5C_{ox}L_{eff})W_{eff}$$



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Charge conservation model and short channel effects

Charge conservation model

Charge conservation model (level 2):

 $dV_{c} = I_{d}dR = \frac{I_{d}dy}{W\mu_{n}Q_{n}'(y)}$ Channel change: $Q_{n}' = -C_{ox}'(V_{G} - V_{T})$ Threshold voltage: $V_{T} = f(V_{C})$ $V_{T} = V_{FB} + V_{C} + 2|\phi_{p}| + \frac{1}{C_{ox}'}\sqrt{2\varepsilon_{s}qN_{a}(2|\phi_{p}| + V_{C} - V_{B})}$

Channel change: $Q_n' = f(V_c)$ $Q_n' = -C_{OX}'(V_G - V_{FB} - 2|\Phi_p| - V_c)$ $+ \sqrt{2\epsilon_s q N_a(2|\Phi_p| + V_c - V_B)}$

Integrating:

$$dV_{c} = I_{d}dR = \frac{I_{d}dy}{W\mu_{n}Q_{n}'(y)}$$
$$I_{D} = \int_{0}^{L} dy = I_{D}L = -\mu W \int_{V_{s}}^{V_{D}}Q_{n}'(V_{c})dV_{c}$$

Drain current equation:

$$\begin{split} & \sum_{D} = \mu_{n} \frac{W}{L} C_{OX}' \left\{ C_{OX}' \left[V_{G} - V_{FB} - 2 \left| \varphi_{p} \right| \right] \\ & - \frac{1}{2} V_{D} - \frac{1}{2} V_{S} \right] (V_{D} - V_{S}) \\ & - \frac{2}{3} \sqrt{2\epsilon_{s} q N_{a}} \left[\left(2 \left| \varphi_{p} \right| + V_{D} - V_{B} \right)^{2} - \left(2 \left| \varphi_{p} \right| + V_{S} - V_{B} \right)^{2} \right] \right] \end{split}$$

Pinch-off:

$$Q_{n}'(L) = 0 = -C_{OX}'(V_{G} - V_{FB} - 2|\varphi_{p}| - V_{Dsat})$$

$$+ \sqrt{2\varepsilon_{s}qN_{a}(2|\varphi_{s}| + V_{Dsat} - V_{B})}$$

Saturation voltage:

$$V_{Dsat} = V_{G} - V_{FB} - 2 \left| \varphi_{p} \right|$$
$$- \frac{\varepsilon_{s} q N_{a}}{C_{OX}'^{2}} \left[\sqrt{1 + \frac{2 C_{OX}'^{2}}{\varepsilon_{s} q N} (V_{G} - V_{FB} - V_{B})} - 1 \right]$$

Channel length modulation



Charge carrier velocity (NMOS):

Charge carrier mobility (NMOS):





Electric field



 U_{CRIT} = critical field for mobility degradation and is the limit at which m_s starts decreasing.

 U_{TRA} = transverse field coefficient effecting mobility.

 U_{EXP} = critical field exponent for mobility degradation.



Linear Dependence on V_{GS} in saturation

Short channel effects



channel length (µm)

Cross section of a short channel transistor showing several depletion areas which affect each other.

Short channel effect on the threshold voltage V_T of an nMOS transistor with and without a DV_T implantation.

Short channel effects



Weak inversion behaviour

MOS transistor operates in the 'weak inversion' region when its gate-source voltage (V_{GS}) is just below its threshold voltage (V_T).



0.0

V_T 1.0

2.0

 $V_{GS}(V)$

If V_T is low (< 0,6V) then there is always a sub-threshold current when $V_{GS} = 0V$. There is a considerable stand-by current.

3.0



Spice transistor parameters

| Parameter name | Symbol | SPICE Name | Units | Default Value |
|-----------------------------|--------|------------|-------|---------------|
| Drawn Length | L | L | m | - |
| Effective width | W | W | m | - |
| Source Area | AREA | AS | m² | 0 |
| Drain Area | AREA | AD | m² | 0 |
| Source Perimeter | PERIM | PS | m | 0 |
| Drain Perimeter | PERIM | PD | m | 0 |
| Squares of Source Diffusion | | NRS | - | 1 |
| Squares of Drain Diffusion | | NRD | - | 1 |

Spice parameters for parasitics

| Parameter name | Symbol | SPICE Name | Units | Default Value |
|---------------------------------------|-------------------|------------|------------------|---------------|
| Source resistance | R _s | RS | Ω | 0 |
| Drain resistance | R _D | RD | Ω | 0 |
| Sheet resistance (Source/Drain) | R _□ | RSH | Ω/□ | 0 |
| Zero Bias Bulk Junction Cap | R _{JO} | CJ | F/m ² | 0 |
| Bulk Junction Grading Coeff. | m | MJ | - | 0.5 |
| Zero Bias Side Wall Junction Cap | C _{JSW0} | CJSW | F/m | 0 |
| Side Wall Grading Coeff. | m _{sw} | MJSW | - | 0.3 |
| Gate-Bulk Overlap Capacitance | C _{GBO} | CGBO | F/m | 0 |
| Gate-Source Overlap Capacitance | C _{GSO} | CGSO | F/m | 0 |
| Gate-Drain Overlap Capacitance | C _{GDO} | CGDO | F/m | 0 |
| Bulk Junction Leakage Current | ۱ _s | IS | А | 0 |
| Bulk Junction Leakage Current Density | J _S | JS | A/m ² | 1E-8 |
| Bulk Junction Potential | φ _o | PB | V | 0.8 |

Main MOS Spice parameters

| Parameter name | Symbol | SPICE Name | Units | Default Value |
|---|-------------------|------------|------------------|---------------|
| SPICE Model Index | | LEVEL | - | 1 |
| Zero-Bias Threshold Voltage | VTO | VTO | V | 0 |
| Process Transconductance | k' | KP | A/V ² | 2.E-5 |
| Body-Bias Parameter | g | GAMMA | V0.5 | 0 |
| Channel Modulation | I. | LAMBDA | 1/V | 0 |
| Oxide thickness | tox | ТОХ | m | 0 |
| Lateral Diffusion | xd | LD | m | 0 |
| Metallurgical Junction Depth | xj | XJ | m | 0 |
| Surface Inversion Potential | 2 fF | PHI | V | 0.6 |
| Substrate Doping | NA, ND | NSUB | cm ⁻³ | 0 |
| Surface State Density | Q _{ss/q} | NSS | cm ⁻³ | 0 |
| Fast Surface State Density | | NFS | cm ⁻³ | 0 |
| Total Channel Charge Coefficient | | NEFF | - | 1 |
| Type of Gate Material | | TPG | - | 1 |
| Surface Mobility | m0 | UO | cm²/Vs | 600 |
| Maximum Drift Velocity | umax | VMAX | m/s | 0 |
| Mobility Critical Field | xcrit | UCRIT | V/cm | 1.0E-4 |
| Critical Field Exponent in Mobility Degradation | | UEXP | - | 0 |
| Transverse Field Exponent (mobility) | | UTRA | - | 0 |

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Matching manual and Spice models



MOS transistor model characteristics



Variation of the drain current with model parameter VTO, for the LEVEL1 model.



Variation of the drain current with model parameter KP, for the LEVEL1 model.



Variation of the drain current with model parameter TOX, for the LEVEL1 model.



Variation of the drain current with model parameter LAMBDA, for the LEVEL1 model.

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MOS transistor characteristics



NMOS transfer characteristic of a typical wafer, W/L = 30/6, VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.



PMOS transfer characteristic of a typical wafer, W/L = 30/6, -VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.

12 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, W/L = 2/30, VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.



PMOS transfer characteristic of a typical wafer, W/L = 2/30, -VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.

1.2 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, W/L = 30/1.2, VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.



PMOS transfer characteristic of a typical wafer, W/L = 30/1.2, -VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.

1.2 um CMOS Process Parameters



NMOS transfer characteristic of a typical wafer, W/L = 30/1.2, VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model. PMOS transfer characteristic of a typical wafer, W/L = 30/1.2, -VGS = 1.5, 2, 3, 4, 5 V. o measured, solid line = MOS2 model, dashed line = AMS model.

Small-signal model

Small signal model



Drain $g_m = \frac{\partial i_D}{\partial v_{cs}}$ (at the quiescent point) Transconductance: Drain-source conductance: $g_{ds} = \frac{\partial i_D}{\partial V_{Ds}}$ (at the quiescent point) C_{gd} \mathbf{g}_{bd} $g_{mbs} = \frac{\partial i_{D}}{\partial v_{ps}}$ (at the quiescent point) $g_{\rm m}V_{\rm gs}$ **g**ds $g_{mbs}V_{bs}$ Bulk modulation: Gate \mathbf{g}_{bs} Bulk C_{gs} Drain and source diode conductances: rs C_{ab} $g_{bd} = \frac{\partial i_{BD}}{\partial V_{RD}}$ (at the quiescent point) ≈ 0 Source $g_{bs} = \frac{\partial i_{SB}}{\partial V_{SB}}$ (at the quiescent point) ≈ 0 41

Modes of operation

| // | | | |
|----|---------------|--|--|
| | Cut-off | $v_{gs} - V_T \le 0$ | $i_{D} = 0$ |
| | Linear region | $0 < v_{\rm DS} \leq (v_{\rm GS} - V_{\rm T})$ | $i_{D} = \frac{\mu_{0}C_{ox}W}{L} \left[(v_{GS} - V_{T}) - \frac{v_{DS}}{2} \right] v_{DS} (1 + \lambda v_{DS})$ $\beta = (K')\frac{W}{L} \cong (\mu_{0}C_{ox})\frac{W}{L} \text{ (amps/volt}^{2})$ |
| | Saturation | $0 < (v_{gs} - V_T) \le v_{ds}$ | $i_{D} = \frac{\mu_{0}C_{ox}W}{2L}(v_{GS} - V_{T})^{2}(1 + \lambda v_{DS})$ |
| | Pinch-off | $v_{DS}(sat.) = v_{GS} - V_{T}$ | |

Saturation region

$$i_{D} = \frac{\mu_{O}C_{OX}W}{2L} (v_{GS} - V_{T})^{2} (1 + \lambda v_{DS})$$
$$V_{T} = V_{TO} + \gamma \left(\sqrt{\left|-2\varphi_{F} + v_{SB}\right|} - \sqrt{\left|-2\varphi_{F}\right|}\right)$$

Drain-source conductance:

$$\begin{split} g_{ds} &= \frac{\partial i_{D}}{\partial v_{DS}} \\ g_{ds} &= g_{o} = \frac{I_{D} \lambda}{1 + \lambda V_{DS}} \approx I_{D} \lambda \qquad \qquad \lambda \propto \frac{1}{L} \end{split}$$

Transconductance:

 $g_{m} = \frac{\partial i_{D}}{\partial v_{GS}} = \frac{\mu_{O}C_{OX}W}{2L} 2(v_{GS} - V_{T})(1 + \lambda v_{DS})$ $g_{m} = \sqrt{(2K'W/L)|I_{D}|}(1 + \lambda V_{DS}) \cong \sqrt{(2K'W/L)|I_{D}|}$ $k' = \mu * C_{OX}$

Bulk modulation:

$$\mathbf{g}_{mbs} = \frac{\partial \mathbf{i}_{D}}{\partial \mathbf{V}_{SB}} = \left(\frac{\partial \mathbf{i}_{D}}{\partial \mathbf{V}_{T}}\right) \left(\frac{\partial \mathbf{V}_{T}}{\partial \mathbf{V}_{SB}}\right)$$

$$\frac{\partial i_{D}}{\partial V_{T}} = \left(\frac{-\partial i_{D}}{\partial V_{GS}}\right)$$

$$g_{mbs} = g_m \frac{\gamma}{2(2|\varphi_F| + V_{SB})^{1/2}} = \eta g_m$$

MOS transistor small signal model



 $g_m = \frac{\partial I_D}{\partial V_{GS}} = \sqrt{2\mu_0 C_{OX} I_D} \frac{W}{L}$; V_E is the normalised Early voltage relative to L $g_{DS} = \frac{\partial I_D}{\partial V_{DS}} = \frac{I_D}{L^* V_E} = I_D \lambda \qquad ; L \sim 3-10 \ \mu$ $C_{GS} = \frac{2}{3} WL * C_{OX}$; $C_{OX} = \frac{\varepsilon_{OX}}{t}$ $C_{GD} = W\Delta\Delta^* C_{OX}$ $C_{DB} = WC_{DB} / \sqrt{V_{DB} + 2\varphi_{p}}$ 0,2µ $C_{SB} = WC_{SB} / \sqrt{v_{SB} + 2\varphi_p}$

; Δ L = lateral diffusion 0,1-