ELEC-E3510 Basics of IC Design

Lecture 6:

Noise and PSRR

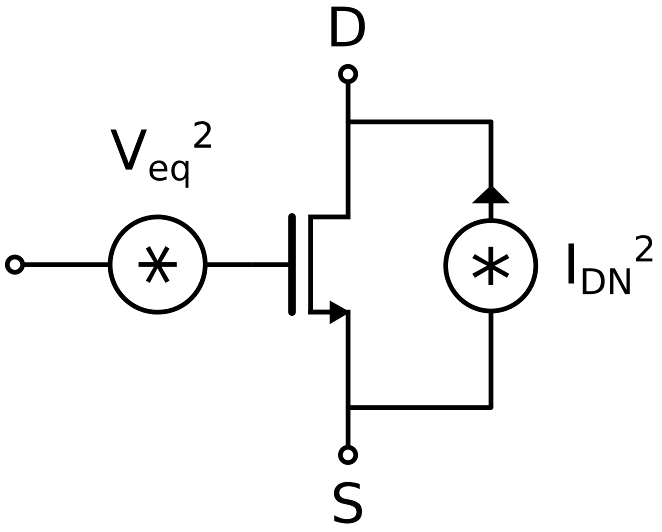
MOS transistor noise model

- Resistivity of conductive channel causes thermal noise

$$\frac{\partial I_{DN}^2}{\partial f} = \frac{8}{3} kTg_m$$

- Charge traps in oxide-silicon surface are slow
  → 1/f-noise, i.e. flicker noise

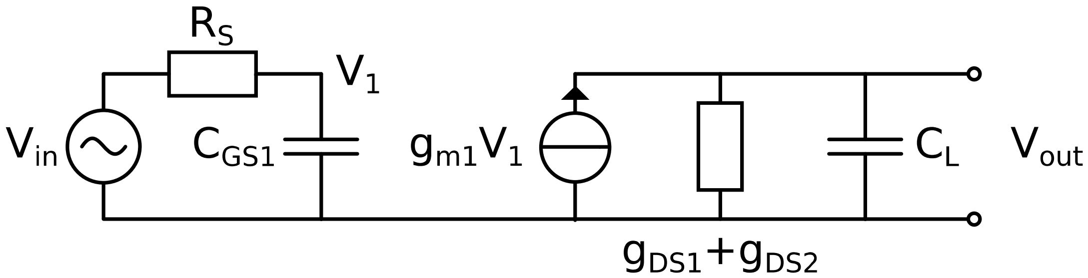
$$\frac{\partial V_{GN}^2}{\partial f} = \frac{B}{fW \cdot L}$$

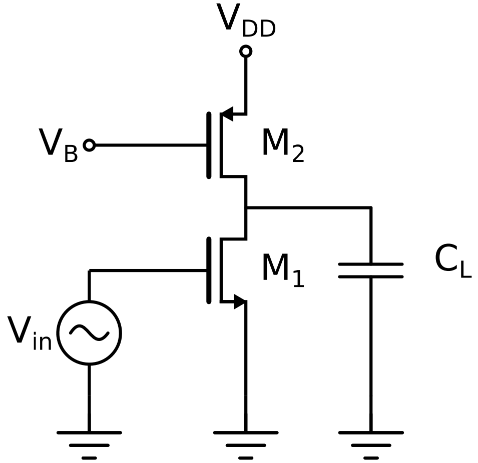


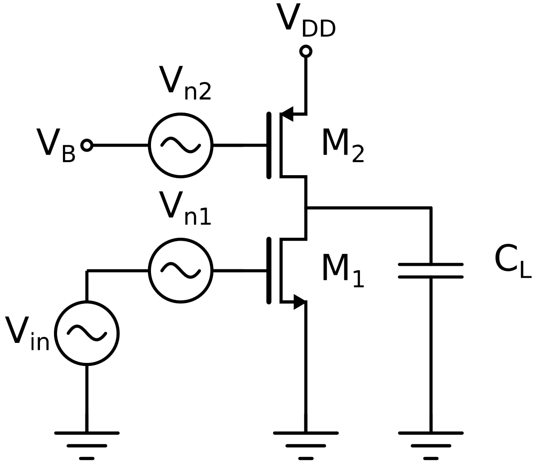
Inverter noise analysis

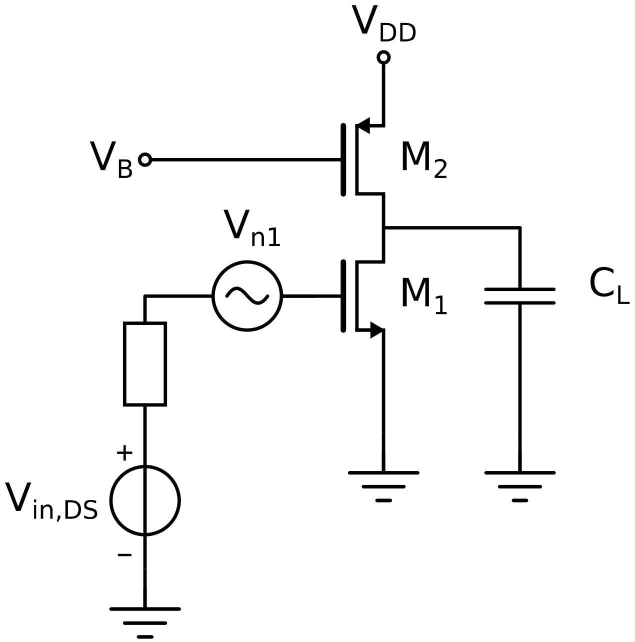
$$A(s) = \frac{-g_{m1}}{(sC_{GS1}R_S + 1)(sC_L + g_{DS1} + g_{DS2})}$$

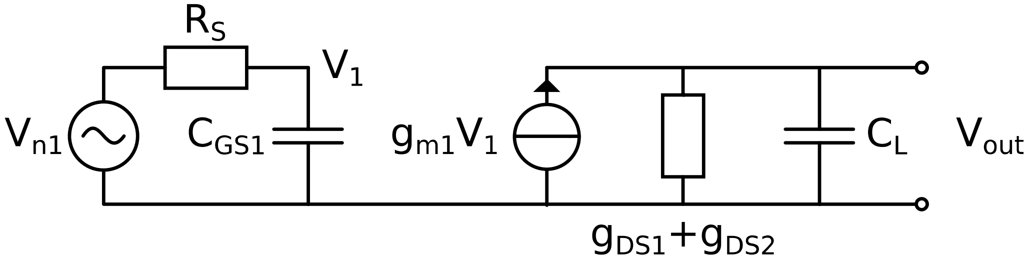
- Contribution of each noise source to the output noise is analysed separately by calculating the corresponding noise transfer functions
- Total output noise is calculated by summing quadratically the noise coming from each noise source





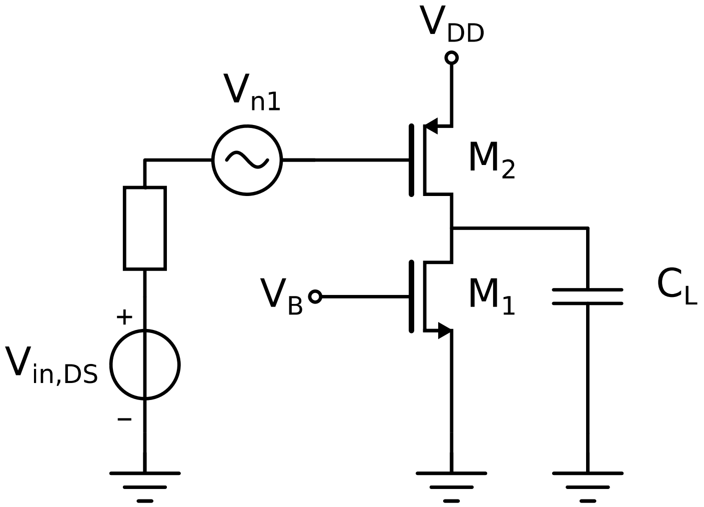


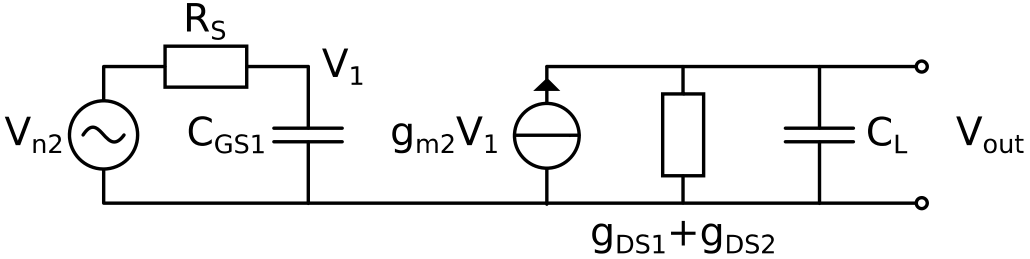




$$A_{n1} = \frac{v_{on1}}{v_{n1}} = \frac{-g_{m1}}{(sC_{GS1}R_S + 1)(sC_L + g_{DS1} + g_{DS2})}$$

$$v_{on1}^{\;2} = |A_{n1}|^2 \cdot v_{n1}^{\;2}$$





$$A_{n1} = \frac{v_{on2}}{v_{n2}} = \frac{-g_{m2}}{(sC_{GS2}R_B + 1)(sC_L + g_{DS1} + g_{DS2})}$$

$$v_{on2}^{\;2} = |A_{n2}|^2 \cdot v_{n2}^{\;2}$$

Output noise voltage:

$$v_{on}^2 = v_{on1}^2 + v_{on2}^2 = |A_{n1}|^2 v_{n1}^2 + |A_{n2}|^2 v_{n2}^2$$

Equivalent input noise voltage:

$$v_{on}^2 = |A|^2 \cdot v_{in,ek}^2$$

$$\Rightarrow v_{in,ek}^2 = \frac{v_{on}^2}{|A|^2} = \frac{|A_{n1}|^2}{|A|} v_{n1}^2 + \frac{|A_{n2}|^2}{|A|^2} v_{n2}^2$$

Assume $\frac{g_{DS1} + g_{DS2}}{C_L} << \frac{1}{R_S C_{GS1}}$ or $\frac{1}{R_B C_{GS2}}$ (single pole approximation)

$$A_{n1} \approx \frac{-g_{m1}}{(sC_L + g_{DS1} + g_{DS2})}$$

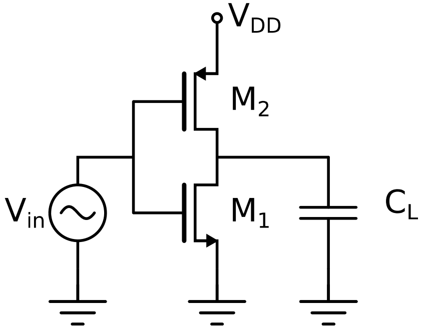
$$A_{n2} \approx \frac{-g_{m2}}{(sC_L + g_{DS1} + g_{DS2})}$$

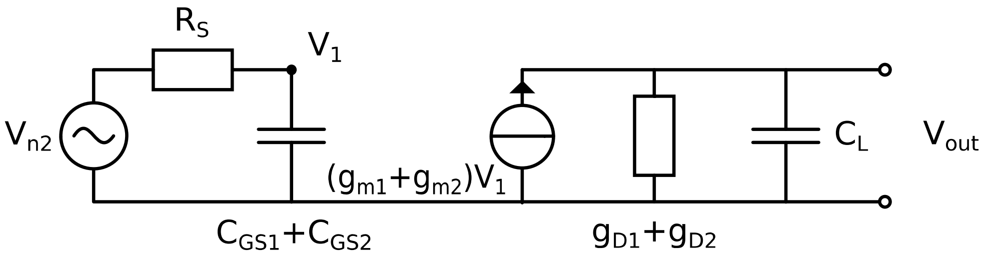
$$A \approx \frac{-g_{m1}}{(sC_L + g_{DS1} + g_{DS2})}$$

$$\boxed{\Rightarrow \quad v_{in,ek}^2 = v_{n1}^2 + \left(\frac{g_{m2}}{g_{m1}}\right)^2 v_{n2}^2}$$

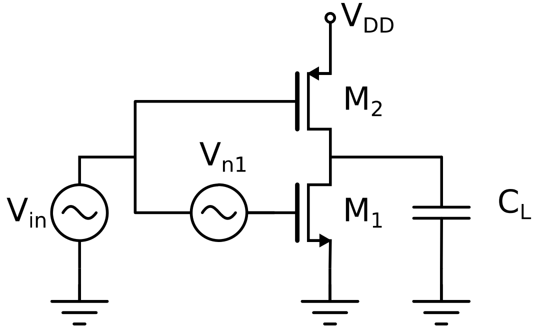
Note: This is also valid for diode load!

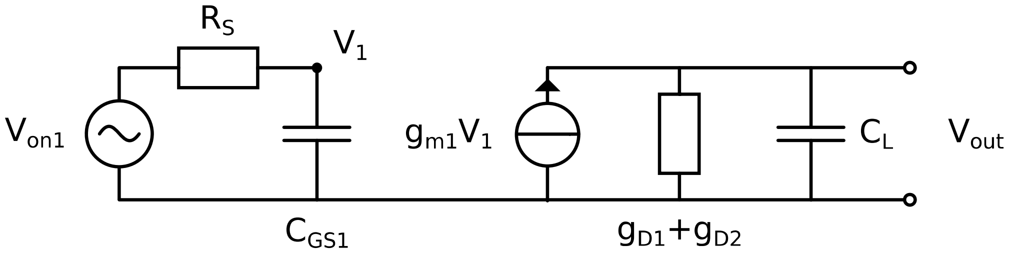
Push-pull inverter



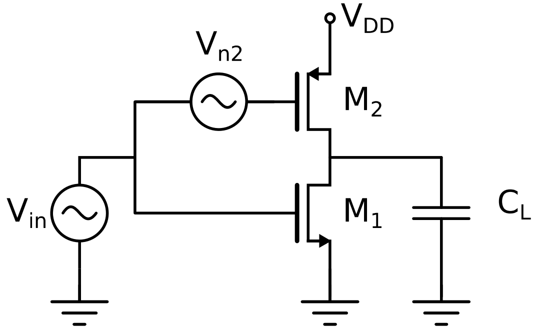


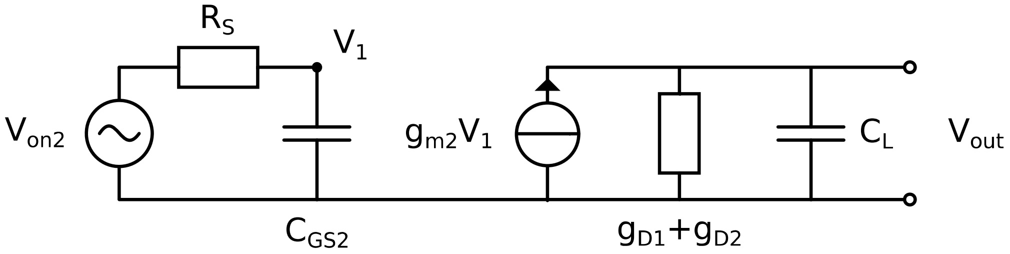
$$A(s)=\frac{-(g_{m1}+g_{m2})}{(s(C_{GS1}+C_{GS2})R_s+1)(sC_L+g_{D1}+g_{D2})}$$





$$A(s)=\frac{-g_{m1}}{(sC_{GS1}R_s+1)(sC_L+g_{D1}+g_{D2})}$$





$$A(s)=\frac{-g_{m2}}{(sC_{GS2}R_s+1)(sC_L+g_{D1}+g_{D2})}$$

Output noise voltage:

$$v_{on}^2 = v_{on1}^2 + v_{on2}^2 = |A_{n1}|^2 v_{n1}^2 + |A_{n2}|^2 v_{n2}^2$$

Equivalent input noise voltage:

$$v_{on}^2 = |A|^2 \cdot v_{in,ek}^2$$

$$\Rightarrow v_{in,ek}^2 = \frac{v_{on}^2}{|A|^2} = \frac{|A_{n1}|^2}{|A|} v_{n1}^2 + \frac{|A_{n2}|^2}{|A|^2} v_{n2}^2$$

Assume single pole approximation

$$\frac{g_{DS1} + g_{DS2}}{C_L} << \frac{1}{R_S C_{GS1}}$$

$$A(s) = -\frac{g_{m1} + g_{m2}}{sC_L + g_{D1} + g_{D2}}$$

$$A_{n1} = -\frac{g_{m1}}{sC_L + g_{D1} + g_{D2}}$$

$$A_{n2} = -\frac{g_{m2}}{sC_L + g_{D1} + g_{D2}}$$

Equivalent input noise voltage:

$$v_{in,ek}^2 = \frac{g_{m1}^2 v_{n1}^2 + g_{m2}^2 v_{n2}^2}{(g_{m1} + g_{m2})^2}$$

Assume:

$$g_{m1} = g_{m2} = g_m$$

$$v_{n1}^2 = v_{n2}^2$$

$$v_{in,ek}^2 = \frac{2|g_m|^2 v_n^2}{(2g_m)^2} = \frac{1}{2} v_n^2$$

Noise of differential input stage

Assume single pole approximation with the dominant pole

$$p_1 = \frac{g_{DS2} + g_{DS4}}{C_L}$$

and the transfer function of

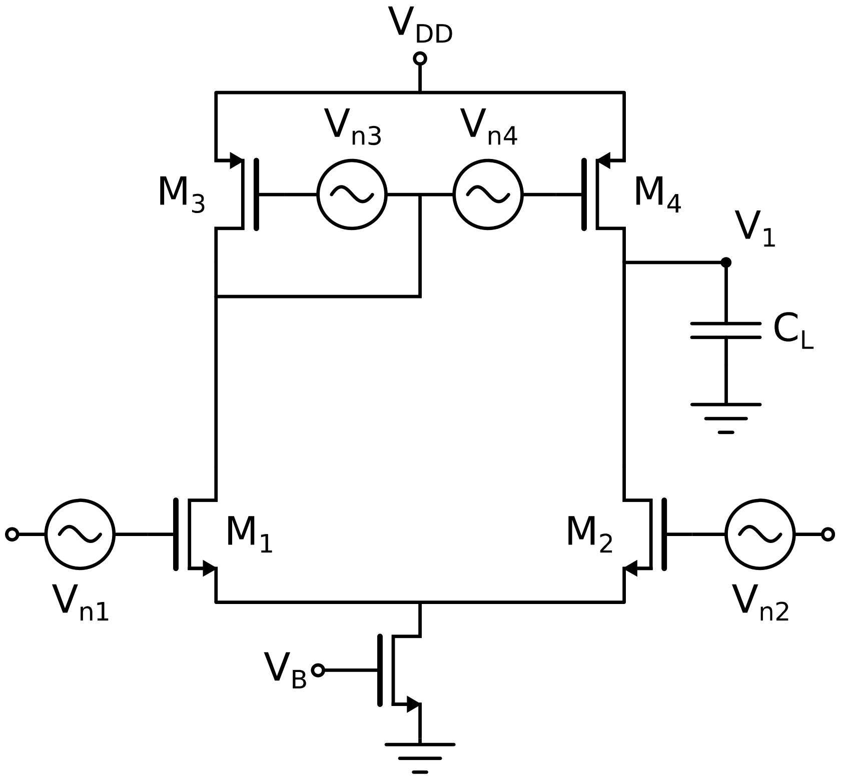
$$A(s) = \frac{V_1}{V_{in}} = -\frac{g_{m1}}{sC_L + g_{DS2} + g_{DS4}}$$

Noise transfer functions from sources V_{n1} and V_{n2}

$$A_{n1} = -A_{n2} = -\frac{g_{m1}}{sC_L + g_{DS2} + g_{DS4}}$$

and from sources V_{n3} and V_{n4}

$$A_{n3} = -A_{n4} = -\frac{g_{m3}}{sC_L + g_{DS2} + g_{DS4}}$$



Total output noise is

$$V_{on}^{2} = \left|A_{n1}\right|^{2}\left(V_{n1}^{2} + V_{n2}^{2}\right) + \left|A_{n3}\right|^{2}\left(V_{n3}^{2} + V_{n4}^{2}\right)$$

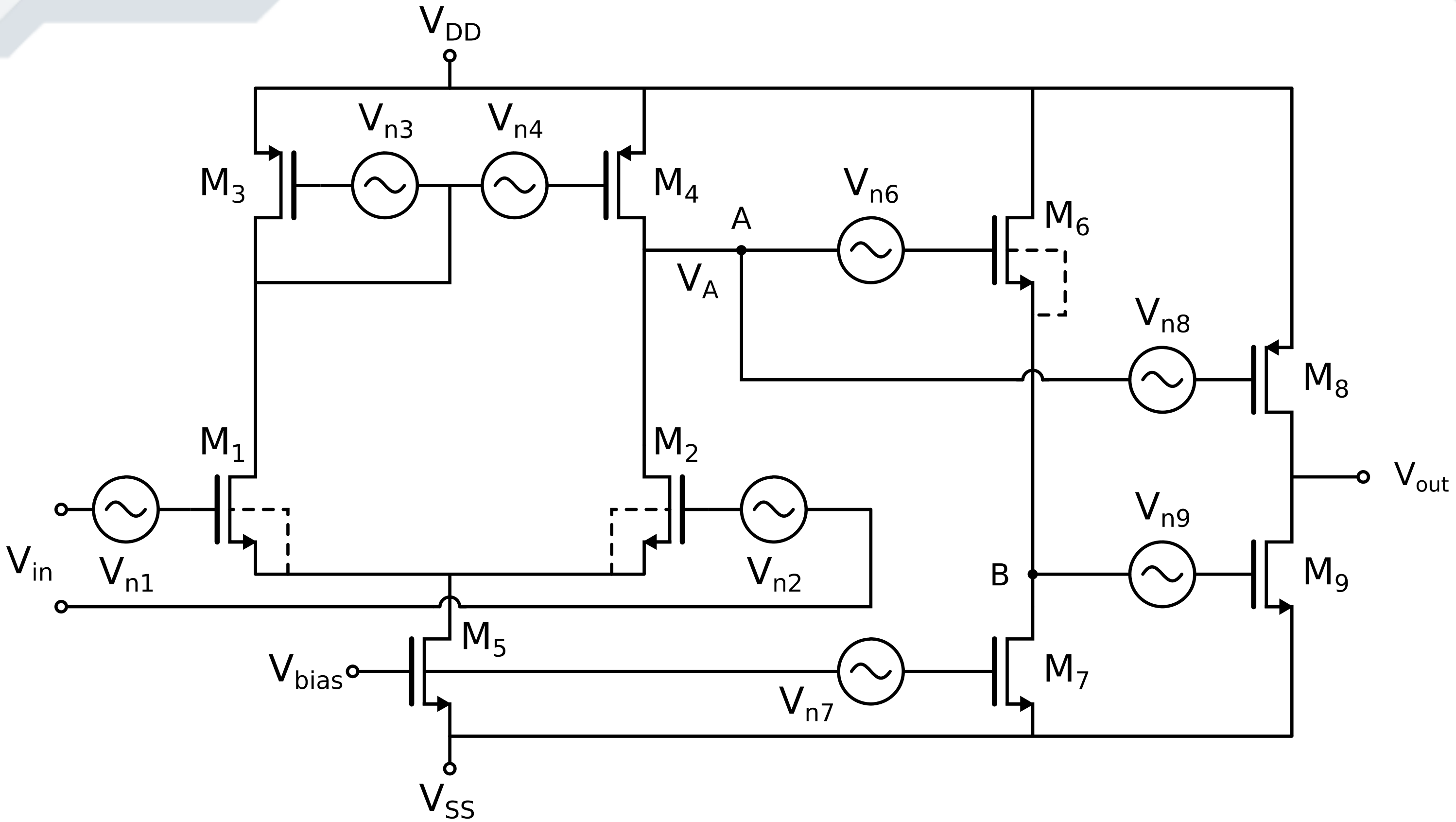
Equivalent noise source at the input is

$$V_{in,ek}^{2} = \frac{\left|V_{on}\right|^{2}}{\left|A\right|^{2}} = \frac{\left|A_{n1}\right|^{2}}{\left|A\right|^{2}}\left(V_{n1}^{2} + V_{n2}^{2}\right) + \frac{\left|A_{n3}\right|^{2}}{\left|A\right|^{2}}\left(V_{n3}^{2} + V_{n4}^{2}\right)$$

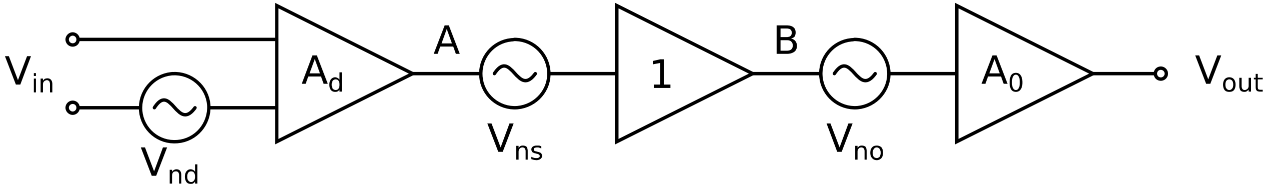
Insert the noise and signal transfer functions

$$\boxed{V_{in,ek}^{2} = V_{n1}^{2} + V_{n2}^{2} + \left(\frac{g_{m3}}{g_{m1}}\right)^{2}\left(V_{n3}^{2} + V_{n4}^{2}\right)}$$

Noise of two stage operational amplifier



Noise reduced to the inputs of different stages



Output noise of the amplifier (assume for source follower A = 1):

$$v_{on}^2 = |A_d A_0|^2 v_{nd}^2 + |A_0|^2 \left(v_{no}^2 + v_{ns}^2\right)$$

Equivalent input noise source:

$$v_{in,ek}^2 = \frac{v_{on}^2}{|A_d A_0|^2} = v_{nd}^2 + \frac{|A_0|^2}{|A_d A_0|^2}\left(v_{no}^2 + v_{ns}^2\right)$$

$$= v_{nd}^2 + \frac{1}{|A_d|^2}\left(v_{no}^2 + v_{ns}^2\right) \approx v_{nd}^2 \left(\approx v_{n1}^2 + v_{n2}^2\right)$$

Noise of input stage:

$$v_{nd}^2 = v_{n1}^2 + v_{n2}^2 + \left(\frac{g_{m3}}{g_{m1}}\right)^2 \left(v_{n3}^2 + v_{n4}^2\right)$$

Noise of source follower:

$$v_{ns}^2 = v_{n6}^2 + \left(\frac{g_{m7}}{g_{m6}}\right)^2 v_{n7}^2$$

Noise of outout stage:

$$v_{no}^2 = \frac{g_{m8}^2 v_{n8}^2 + g_{m9}^2 v_{n9}^2}{\left(g_{m8} + g_{m9}\right)^2}$$

Noise analysis of OTA

Amplification in the input stage

$$A_1 = \frac{g_{m1}}{g_{m3}}$$

A_1 is small → noise of output stage also has to be taken into account!

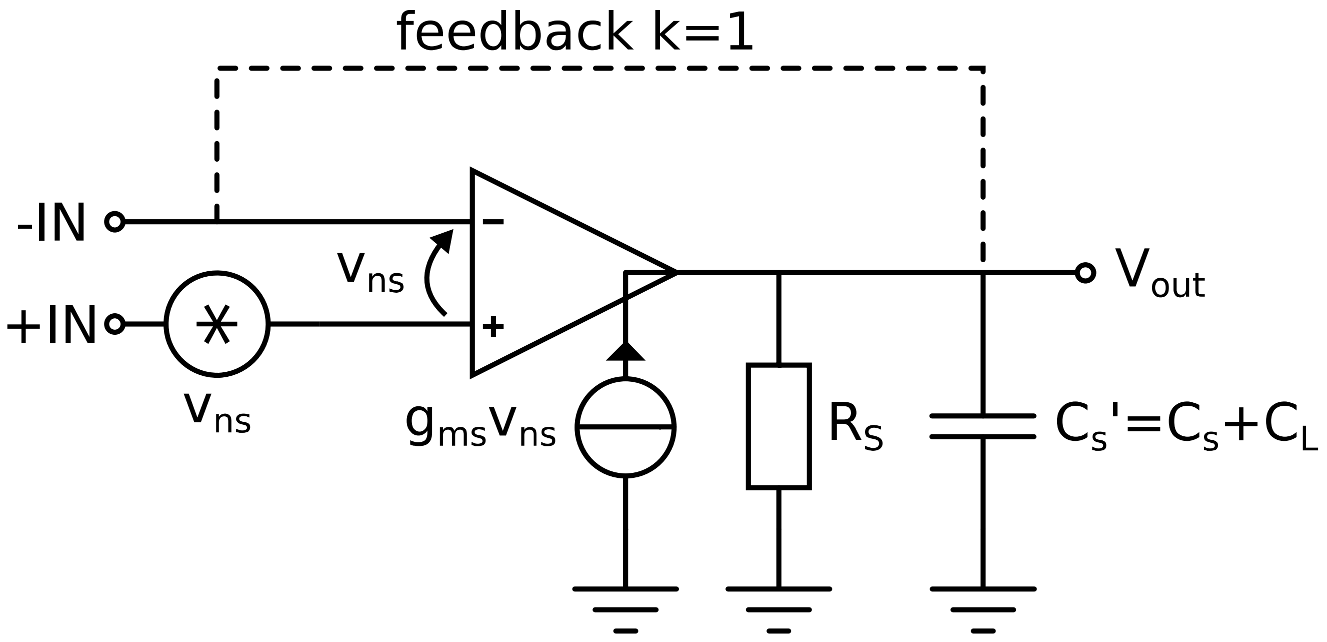
Equivalent input noise source:

$$V_{in,eq}^2 = v_{n1}^2 + v_{n2}^2 + \left(\frac{g_{m3}}{g_{m1}}\right)^2 \left(v_{n3}^2 + v_{n4}^2\right) + \left(\frac{g_{m5}}{g_{m1}}\right)^2 \left(v_{n5}^2 + v_{n6}^2\right) + \left(\frac{g_{m7}}{g_{m1}}\right)^2 \left(v_{n7}^2 + v_{n8}^2\right)$$

$$= v_{n1}^2 + v_{n2}^2 + \left(\frac{g_{m3}}{g_{m1}}\right)^2 \left(v_{n3}^2 + v_{n4}^2\right) + B^{-2} \left(\frac{g_{m3}}{g_{m1}}\right)^2 \left(v_{n5}^2 + v_{n6}^2 + v_{n7}^2 + v_{n8}^2\right)$$

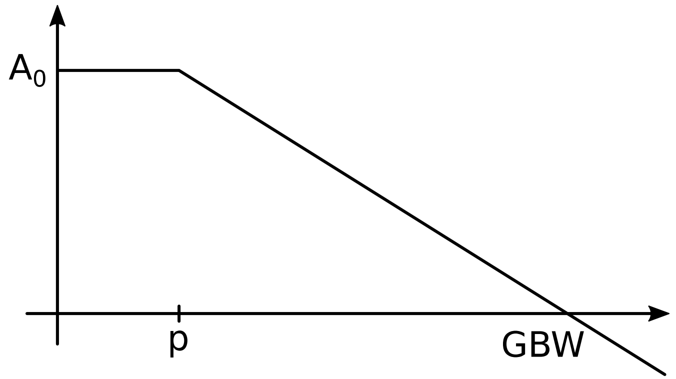
Assume $\begin{cases} B_1 = B_2 = B \\ B_3 = 1 \end{cases}$

Noise bandwidth of single stage amplifiers



Assume A(s) is the amplification of amplifier and k is the feedback factor

$$A(s)v_{ns} - kA(s)\cdot V_{out} = V_{out}$$
$$\Rightarrow V_{out} + kA(s)\cdot V_{out} = A(s)v_{ns}$$
$$\Rightarrow \frac{V_{out}}{v_{ns}} = \frac{A(s)}{1+kA(s)}$$



Assume single stage approximation

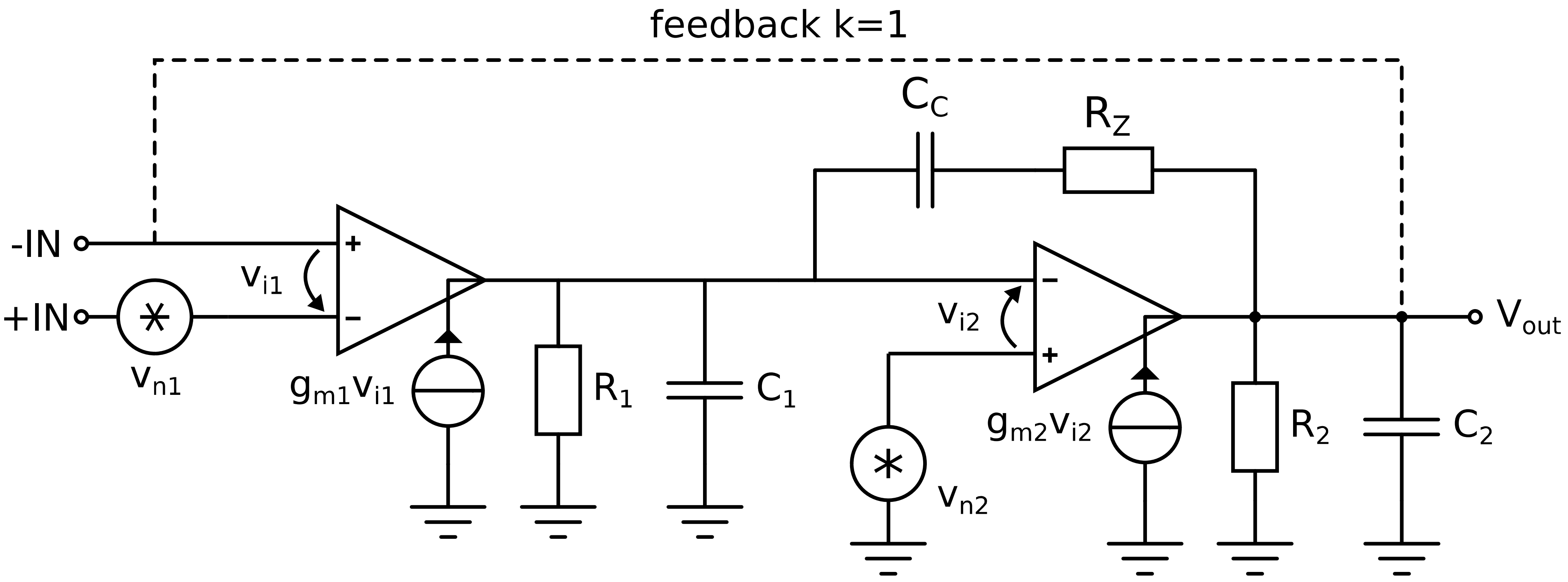
$$A(s) = \frac{g_{ms}R_S}{1+sR_SC_S} \qquad ;A_0 = g_{ms}R_S, p = R_SC_S, GBW = \frac{g_{ms}}{C_S}$$

and assume k = 1

$$\Rightarrow \frac{V_{out}}{v_{ns}} = \frac{g_{ms}R_S}{1+g_{ms}R_S+sR_SC_S} \quad ;A_0 = g_{ms}R_S >> 1$$
$$\approx \frac{g_{ms}R_S}{g_{ms}R_S+sR_SC_S} = \frac{1}{1+s\frac{C_S}{g_{ms}}} = \frac{1}{1+\frac{s}{GBW}}$$

$$\boxed{\frac{V_{out}}{v_{ns}} = \frac{1}{1+\frac{s}{GBW}}}$$

Noise bandwidth in 2-stage amplifiers



Open loop amplification:

$$A(s)=\frac{A_0\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

For feedback amplifier

A) Noise of input stage

$$\frac{V_{on1}}{V_{n1}}=\frac{A(s)}{1+k(A(s))}$$

$$=\frac{A_0\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)+A_0\left(1-\frac{s}{z}\right)}$$

$$\approx\frac{\left(1-sC_C\left(\frac{1}{g_{m2}}-R_z\right)\right)}{\underbrace{\left(1-\frac{sC_C}{g_{m1}}\right)}_{1-\frac{s}{GBW}}\left(1-\underbrace{\frac{sC_L}{g_{m2}}}_{p_2}\right)\left(1-\underbrace{sC_1R_z}_{p_3}\right)}$$

$$\approx\frac{\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{GBW}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}\qquad ;\ p_3=-z$$

B) Noise of output stage

$$\frac{V_{on2}}{V_{n2}}=\frac{A_2(s)}{1+k(A(s))}\qquad ;\ A_z(s)=\frac{A_2\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

$$=\frac{A_2\left(1+\frac{s}{p_1}\right)\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)+A_0\left(1-\frac{s}{z}\right)}$$

assume $p_3=z$

$$\Rightarrow\frac{V_{on2}}{V_{n2}}=\frac{A_2\left(1+\frac{s}{p_1}\right)\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1A_0}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)A_0}$$

$$=\frac{\left(1+\frac{s}{p_1}\right)}{\left(1-\frac{s}{GBW}\right)\left(1+\frac{s}{p_2}\right)A_1}=\frac{\left(1-s\frac{A_1}{GBW}\right)}{\left(1-\frac{s}{GBW}\right)\left(1-\frac{s}{p_2}\right)A_1}$$

$$=\frac{(1+sC_CR_1)}{\left(1-\frac{s}{GBW}\right)\left(1-\frac{sC_L}{g_{m2}}\right)A_1}$$

Denominator:

$$\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)+A_0\left(1-\frac{s}{z}\right)$$

Assume $-z = p_3$

$$\left[\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)+A_0\right]\left[\left(1+\frac{s}{p_3}\right)\right]$$

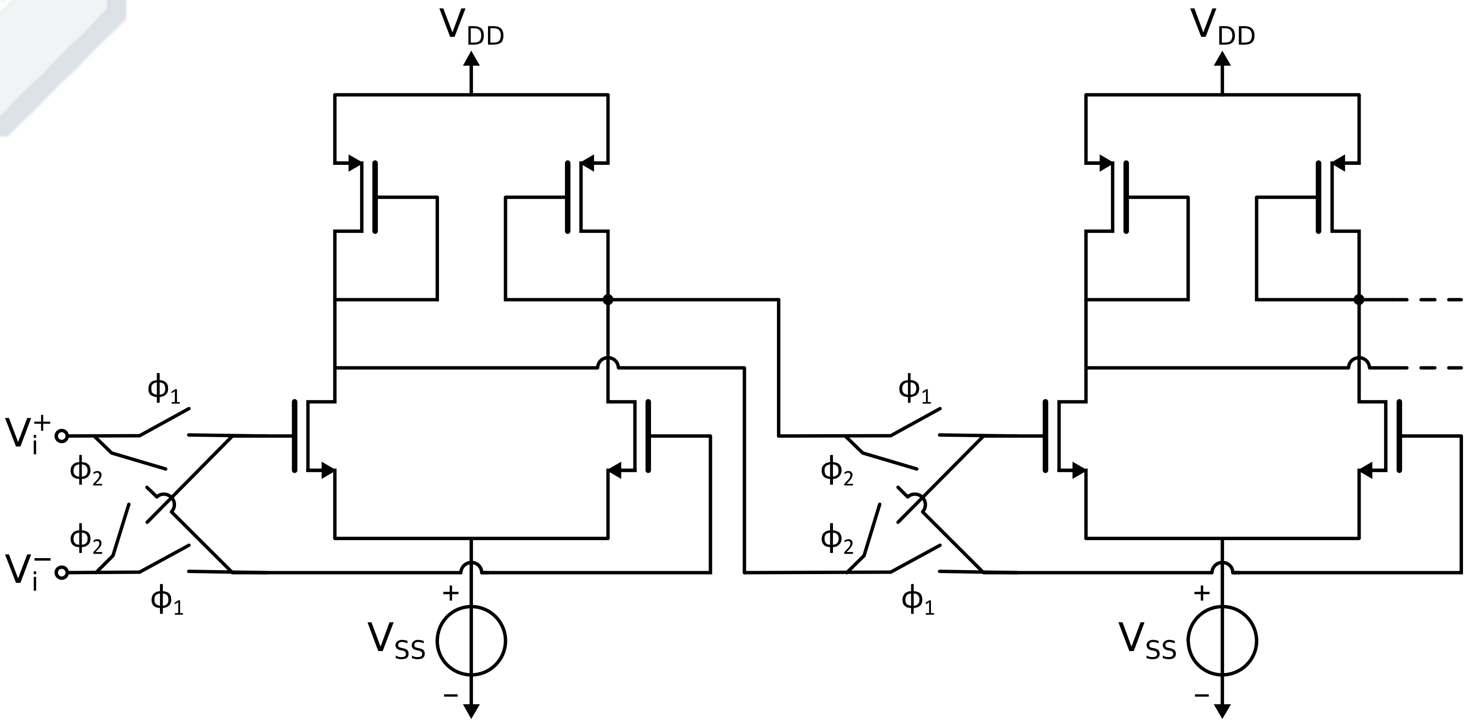
$$\left[A_0+1+s\left(\frac{1}{p_1}+\frac{1}{p_2}\right)+\frac{s^2}{p_1p_2}\right]\left[1+\frac{s}{p_3}\right]$$

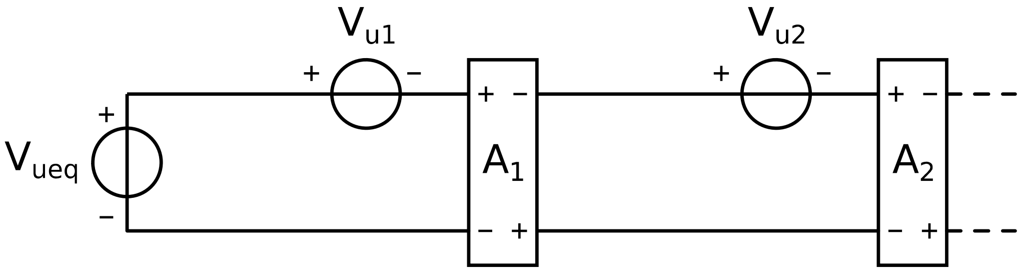
Assume $A_0 >> 1$, $p_1 << p_2$

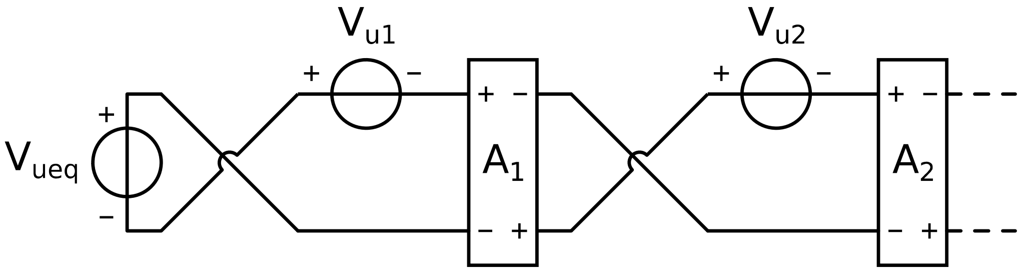
$$A_0\left[1+s\frac{1}{p_1A_0}+\frac{s^2}{A_0p_1p_2}\right]\left[1+\frac{s}{p_3}\right]$$

Assume $GBW = A_1p_1 << p_2$

$$A_0\left[1+s\frac{1}{p_1A_0}\right]\left[1+\frac{s}{p_2}\right]\left[1+\frac{s}{p_3}\right]$$







$V_{ueq} = V_{u1} + V_{u2}/A_1$

$V_{ueq} = V_{u1} + V_{u2}/A_1$

$V_{ueq}(\textit{Average}) = V_{u2}/A_1$

PSRR – Power Supply Rejection Ratio

Definition:

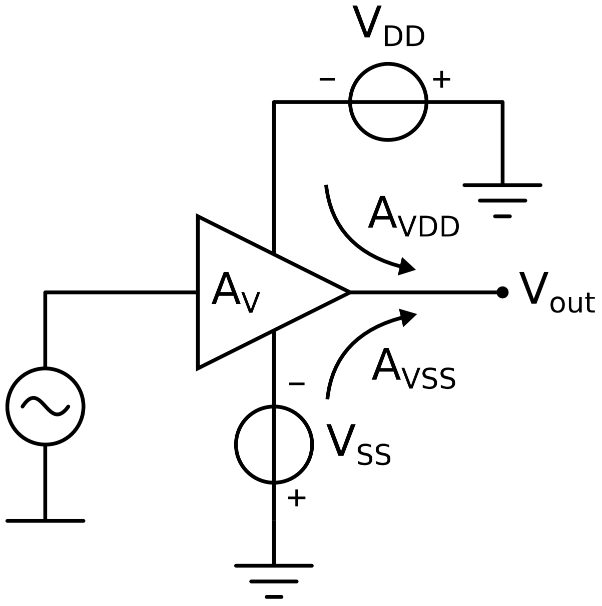
$$PSRR^{+} = 20\log\left|\frac{A_V}{A_{V_{DD}}}\right|$$

$$PSRR^{-} = 20\log\left|\frac{A_V}{A_{V_{SS}}}\right|$$

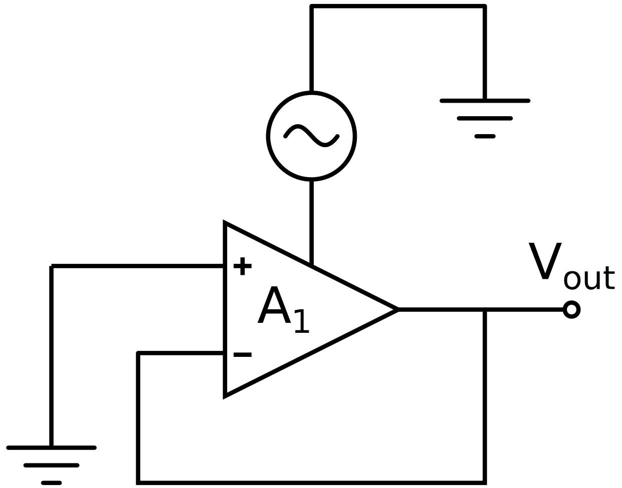
$$A_V = \frac{V_{out}}{V_{in}}$$

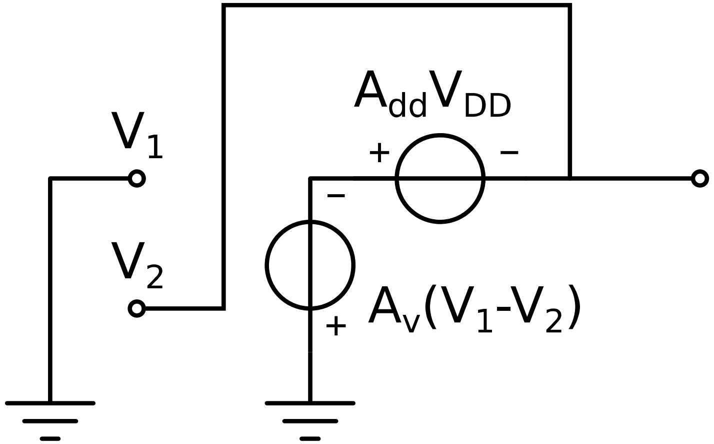
$$A_{V_{DD}} = \frac{V_{out}}{V_{DD}}$$

$$A_{V_{SS}} = \frac{V_{out}}{V_{SS}}$$



Feedback amplifier

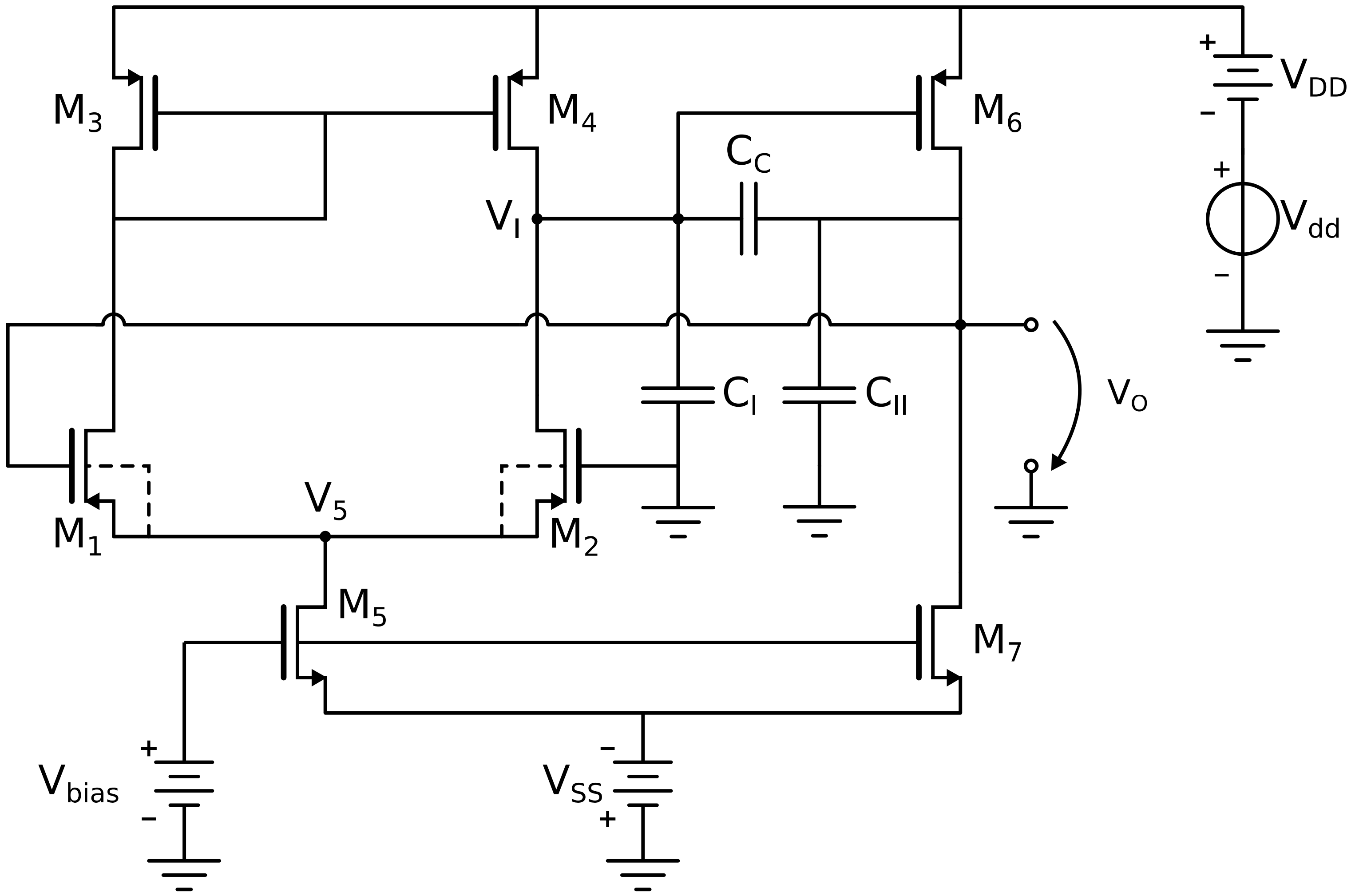


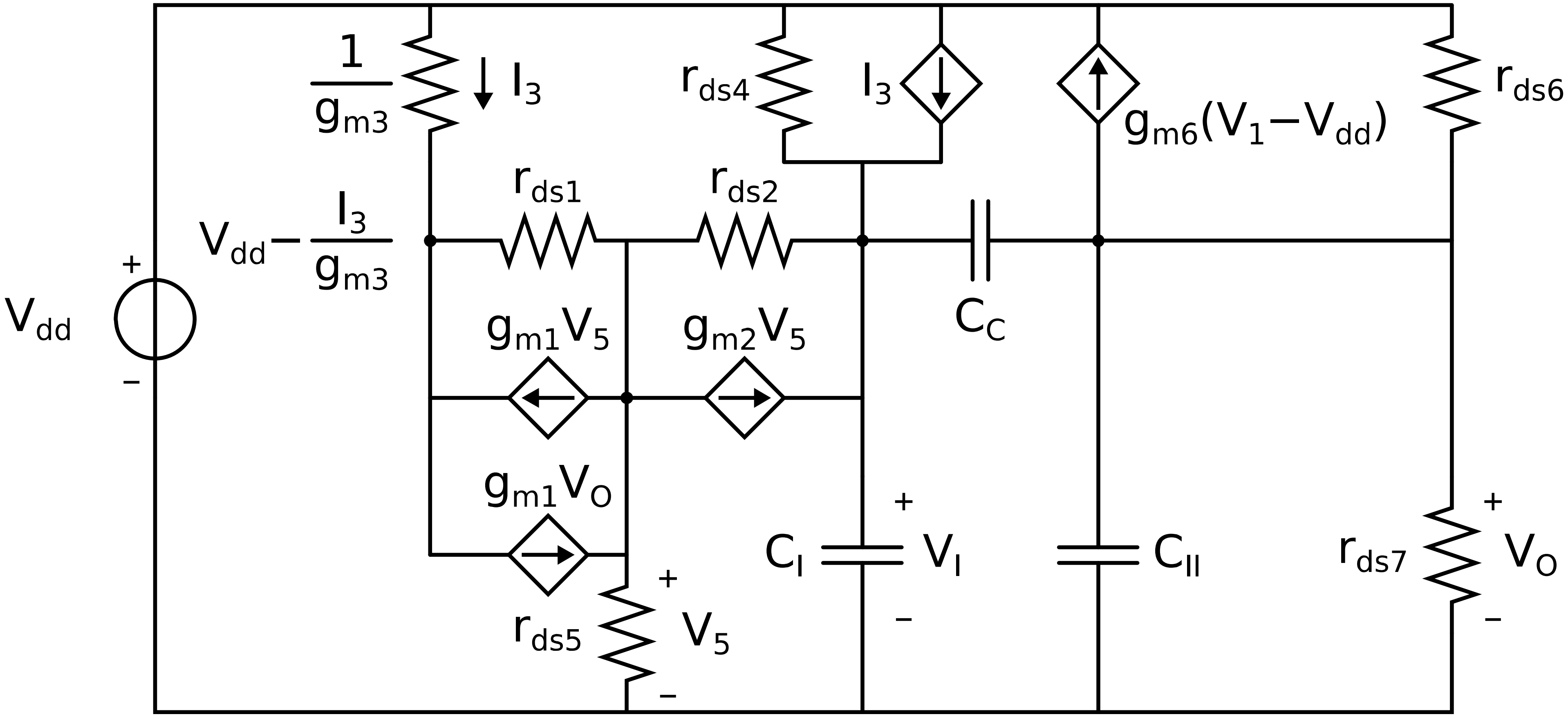


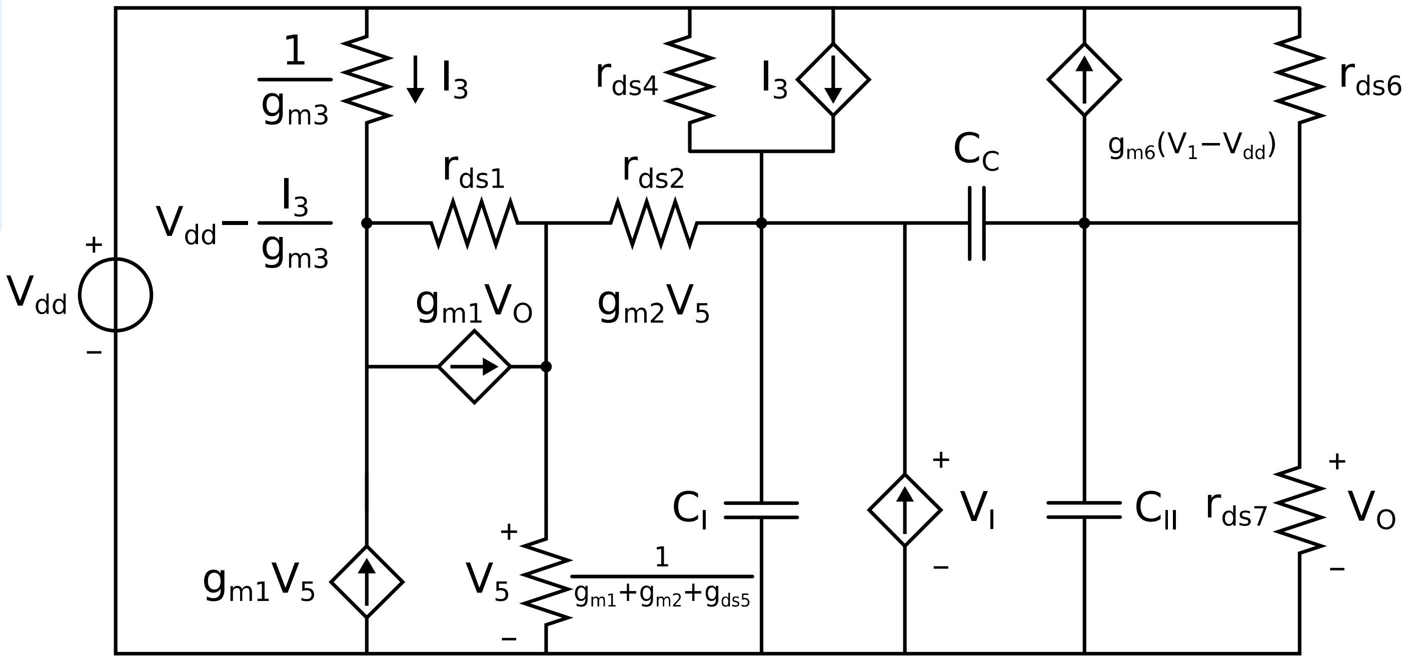
$$V_{out} = \frac{A_{dd}}{1+A_v} V_{dd} \approx \frac{A_{dd}}{A_v} V_{dd} = \frac{1}{PSRR} V_{dd}$$

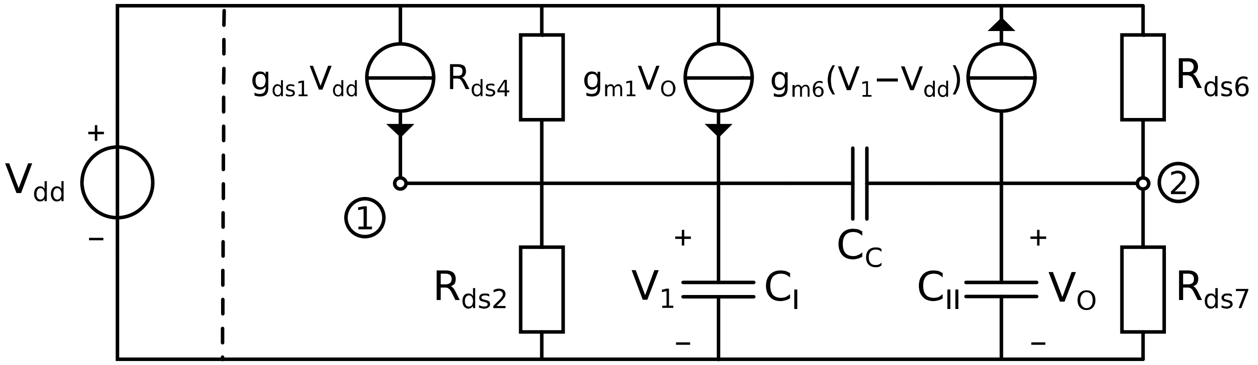
Definition: $\boxed{PSRR = \frac{A_v}{A_{vdd}}}$ $\Rightarrow$ $\boxed{PSRR = \frac{V_{dd}}{V_{out}}}$

PSRR of V_{DD} in 2-stage amplifier




1
gm3
I3
rds4
I3
gm6(V1−Vdd)
rds6
rds1
rds2
Vdd−
I3
gm3
+
Vdd
−
CC
gm1V5
gm2V5
gm1VO
+
CI
VI
−
CII
+
rds7
VO
−
+
rds5
V5
−



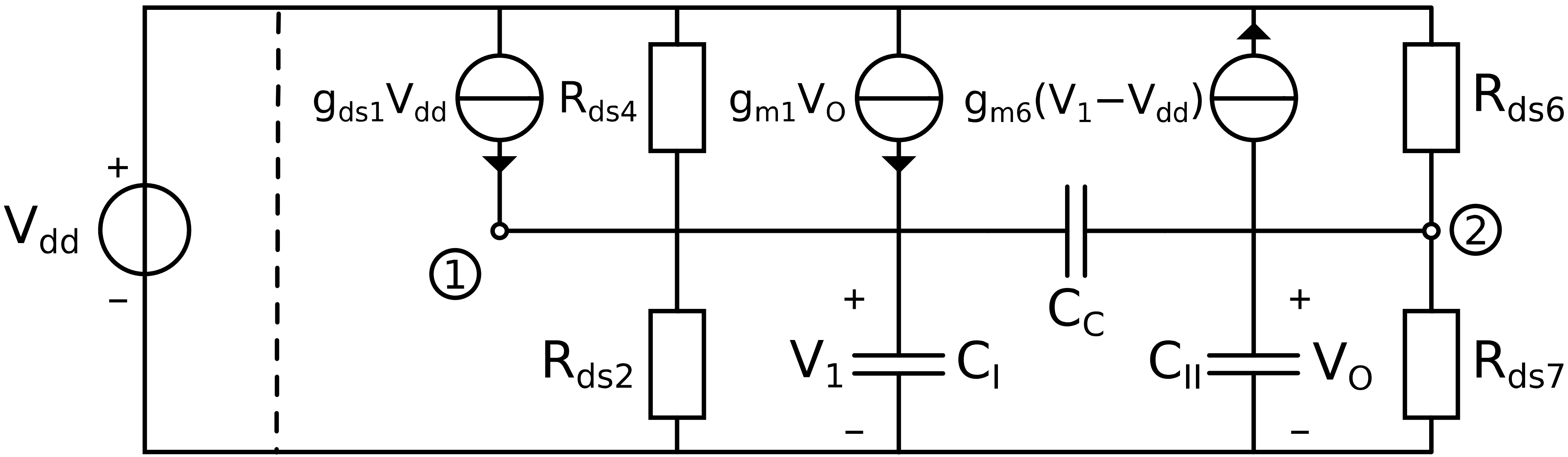


$$I_3 = g_{m1}V_O + g_{ds1}\left(V_{DD} - \frac{I_3}{g_{m3}}\right)$$

$$\approx g_{m1}V_O + g_{ds1}V_{dd}$$

$$I_3 + \frac{g_{DS1}}{g_{m3}}\cdot I_3 = \left(1+\frac{g_{DS1}}{g_{m3}}\right)I_3 \approx I_3$$

$$g_{DS1} << g_{m3}$$



V_{dd}
+
−
$g_{ds1}V_{dd}$
R_{ds4}
$g_{m1}V_O$
$g_{m6}(V_1-V_{dd})$
R_{ds6}
①
②
C_C
R_{ds2}
+
V_1
−
C_I
C_{II}
+
V_O
−
R_{ds7}

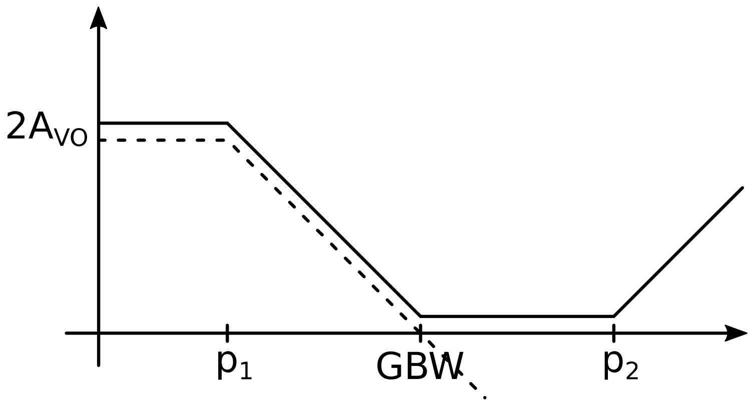
Current equations for nodes 1 and 2

$$
\begin{aligned}
&(1)\quad (g_{DS1}+g_{DS4})V_{DD} = (g_{DS2}+g_{DS4}+sC_c+sC_I)V_1 - (g_{m6}+sC_c)V_O \\
&(2)\quad (g_{m6}+g_{DS6})V_{DD} = (g_{m6}-sC_c)V_1 + (g_{DS6}+g_{DS7}+sC_c+sC_{II})V_O
\end{aligned}
$$

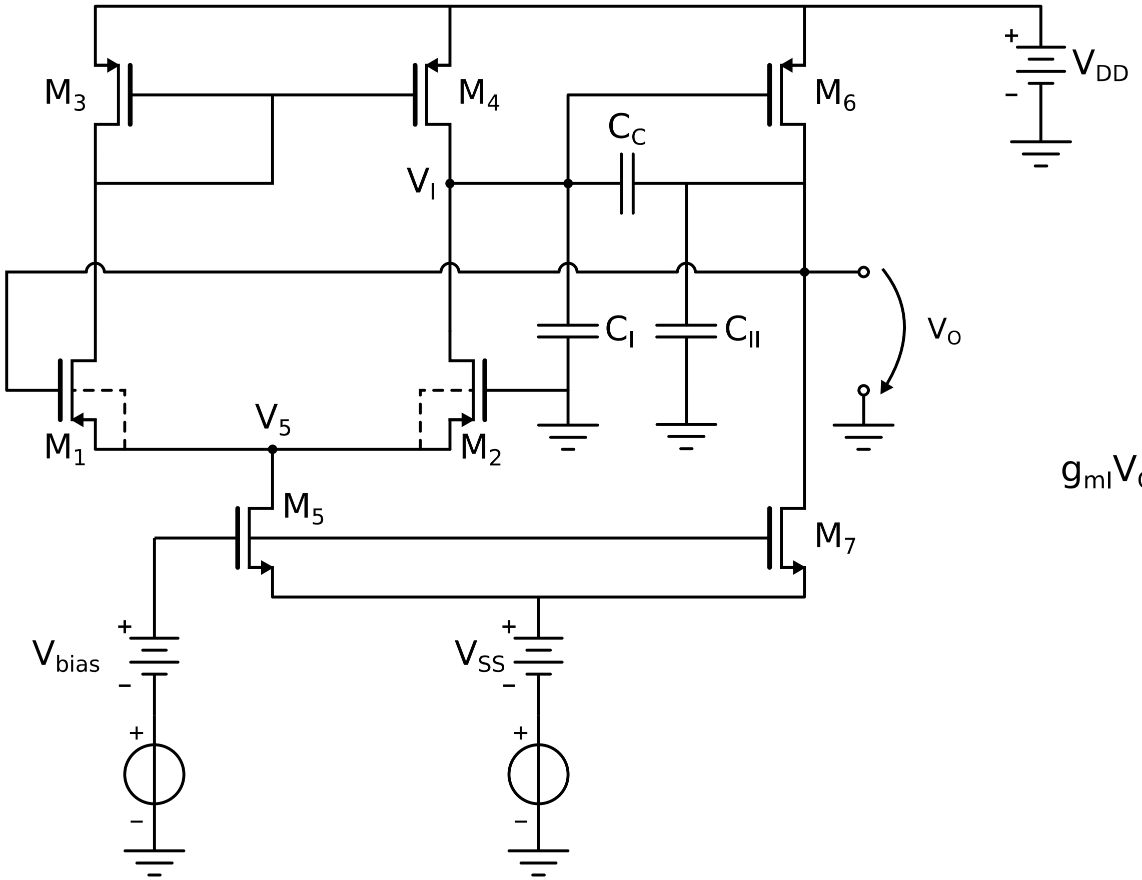
$$
\Rightarrow \frac{V_{DD}}{V_O} = \frac{s^2(C_cC_I + C_IC_{II} + C_{II}C_c) + s\left[(g_{DS2}+g_{DS4})(C_c+C_{II}) + (g_{DS6}+g_{DS7})(C_c+C_I) + C_c(g_{m6}-g_{m1})\right] + (g_{DS2}+g_{DS4})(g_{DS6}+g_{DS7}) + g_{m1}g_{m6}}{s\left[C_c(g_{m6}+g_{DS2}+g_{DS4}+g_{DS6}) + C_I(g_{m6}+g_{DS6})\right] + (g_{DS2}+g_{DS4})g_{DS6}}
$$

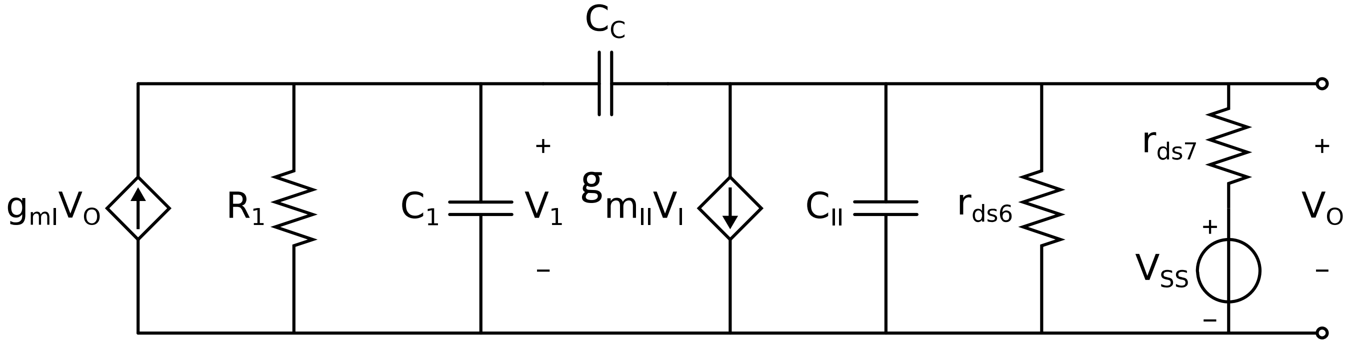
Let us solve the zeros by using the dominant pole method:

$$
PSRR^{+} = \frac{V_{DD}}{V_O} \approx \frac{g_{m1}g_{m6}}{\underbrace{(g_{DS2}+g_{DS4})g_{DS6}}_{\approx 2A_0}} \left[\frac{\left(\frac{sC_c}{g_{m1}}+1\right)\left(\frac{s(C_cC_I + C_IC_{II} + C_cC_{II})}{g_{m6}C_c}+1\right)}{\frac{sg_{m6}C_c}{(g_{DS2}+g_{DS4})g_{DS6}}+1} \right] \approx 2A_{V_O} \left[\frac{\left(\frac{s}{GBW}+1\right)\left(\frac{s}{p_2}+1\right)}{\underbrace{\frac{2sA_O}{GBW}}_{p_1}+1} \right]
$$



PSRR of V_{SS} in 2-stage amplifier





$$\frac{1}{g_{DS2}+g_{DS4}}=R_1$$

$$g_{mII}=g_{m6}$$

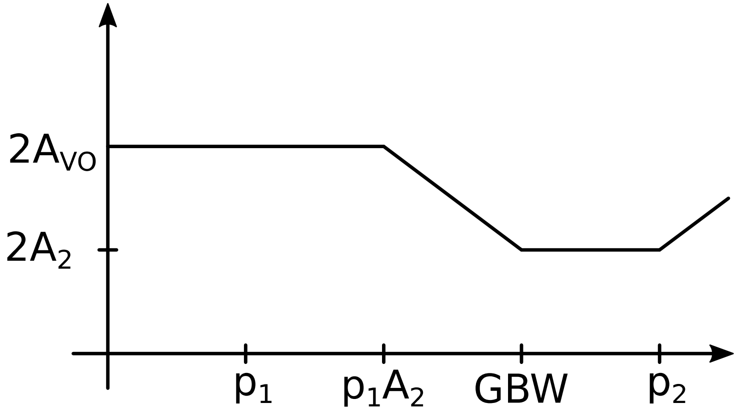
Current equations for nodes 1 and 2

$$1 \quad (g_{DS2}+g_{DS4}+sC_c+sC_1)V_1-(g_{m1}+sC_c)V_O=0$$
$$2 \quad (g_{m6}-sC_c)V_1+(g_{DS6}+g_{DS7}+sC_c+sC_L)V_O=g_{DS7}V_{SS}$$

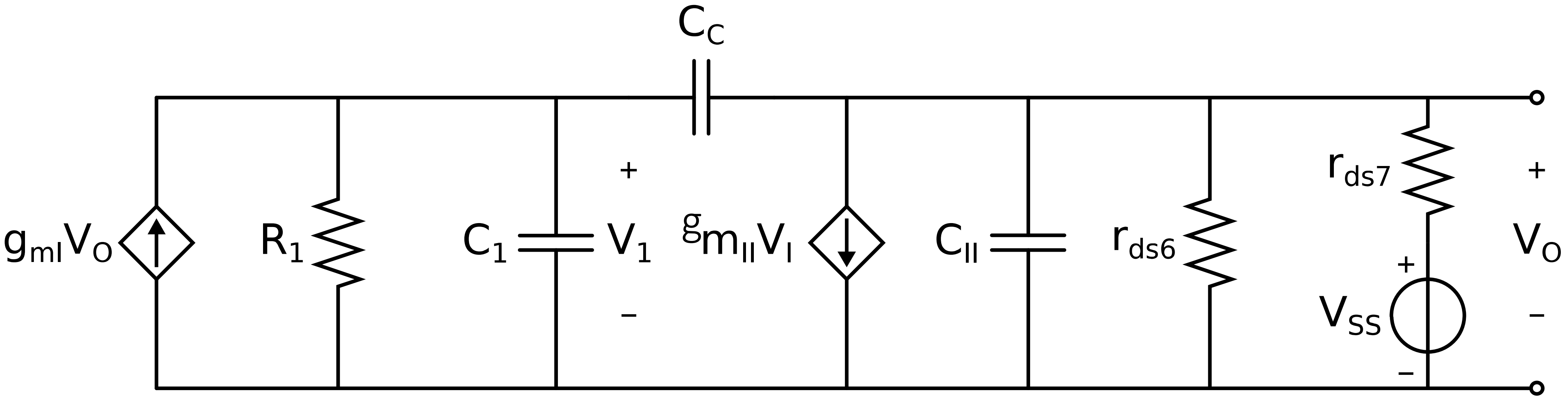
$$\Rightarrow \frac{V_{SS}}{V_O}=\frac{s^2(C_cC_1+C_1C_L+C_LC_c)+s\left[(g_{DS2}+g_{DS4})(C_c+C_L)+(g_{DS6}+g_{DS7})(C_c+C_1)+C_c(g_{m6}-g_{m1})\right]+(g_{DS2}+g_{DS4})(g_{DS6}+g_{DS7})+g_{m1}g_{m6}}{s\left[(C_c+C_1)+(g_{DS2}+g_{DS4})\right]g_{DS7}}$$

Let us solve the zeroes by using the dominant pole method:

$$PSRR=\frac{V_{SS}}{V_O}\approx\frac{g_{m1}g_{m6}}{(g_{DS2}+g_{DS4})g_{DS7}}\left[\frac{\left(\frac{sC_c}{g_{m1}}+1\right)\left(\frac{s(C_cC_1+C_1C_L+C_cC_L)}{g_{m6}C_c}\right)}{\frac{s(C_c+C_1)}{g_{DS2}+g_{DS4}}+1}\right]\approx 2A_{V_O}\left[\frac{\left(\frac{s}{GBW}+1\right)\left(\frac{s}{p_2}+1\right)}{\frac{s}{\left(\frac{GBW}{A_1}\right)}+1}\right]$$



Note: $$\frac{GBW}{A_1}=A_2\cdot p_1$$



$$\frac{1}{g_{DS2}+g_{DS4}} = R_1$$

$$g_{mII} = g_{m6}$$