

Probability generating functions cheat sheet

The 7 properties of probability generating functions

Random variable X where $P(X = k) = p(k)$. Further, $S = \sum_{i=1}^N X_i$ and X_i are identically independently distributed random variables.

$$g(z) = p(0) + p(1)z + p(2)z^2 \dots = \sum_{k=0}^{\infty} p(k)z^k \quad (1)$$

$$p(k) = \left[\frac{1}{k!} \frac{d^k}{dz^k} g(z) \right]_{z=0} \quad (2)$$

$$\langle X^m \rangle = \left[\left(z \frac{d}{dz} \right)^m g(z) \right]_{z=1} \quad (3)$$

$$g_{X_1+X_2}(z) = g_{X_1}(z) * g_{X_2}(z) \quad (4)$$

$$g_S(z) = [g_{X_i}(z)]^N \quad (5)$$

$$g_{X_1+c}(z) = g_{X_1}(z) * z^c \quad (6)$$

$$g_S(z) = g_N(g_{X_i}(z)), \quad (7)$$

where N is a constant in (5) and independent random variable in (7).

Newman's notation

g_0 : PGF for the degree distribution

g_1 : PGF for the excess degree distribution

h_0 : PGF for the small component size distribution (select a random node, probability that it in a component of size s that is not the giant)

h_1 : PGF for the "excess small component size distribution" (select a random node, follow a link and delete it, probability that you reach a component of size s' that is not the giant)

S : The probability that a randomly selected node belongs to the giant component

$u = h_1(1)$: Probability that following a link does not lead to the giant component

Solutions

Assumptions: The network is created with the configuration model and is sparse enough such that there are no loops

$$h_0(z) = z g_0(h_1(z)) \quad (8)$$

$$h_1(z) = z g_1(h_1(z)) \quad (9)$$

$$S = 1 - g_0(u) \quad (10)$$

$$u = g_1(u) \quad (11)$$