## Exercise 1 solution

1. (Exercise 1.7 from O’Hayre)

Energy is released when hydrogen and oxygen react to produce water. This energy comes from the fact that the final hydrogen-oxygen bonds represent a lower total energy state than the original hydrogen-hydrogen and oxygen-oxygen bonds. Calculate how much energy (in kilojoules per mole of product) is released by the reaction

$$
\begin{equation*}
\mathrm{H}_{2}+\frac{1}{2} \mathrm{O}_{2} \rightleftharpoons \mathrm{H}_{2} \mathrm{O} \tag{1.12}
\end{equation*}
$$

at constant pressure and given the following standard bond enthalpies. Standard bond enthalpies denote the enthalpy absorbed when bonds are broken at standard temperature and pressure ( 298 K and 1 atm ).

$$
\begin{gathered}
\text { Standard Bond Enthalpies } \\
\hline \mathrm{H}-\mathrm{H}=432 \mathrm{~kJ} / \mathrm{mol} \\
\mathrm{O}=\mathrm{O}=494 \mathrm{~kJ} / \mathrm{mol} \\
\mathrm{H}-\mathrm{O}=460 \mathrm{~kJ} / \mathrm{mol}
\end{gathered}
$$

Solution
The reaction can be written also as

$$
2 \mathrm{H}_{2}+\mathrm{O}_{2} \rightarrow 2 \mathrm{H}_{2} \mathrm{O} \text { or } \mathrm{H}-\mathrm{H}+\mathrm{H}-\mathrm{H}+\mathrm{O}=\mathrm{O} \rightarrow \mathrm{H}-\mathrm{O}-\mathrm{H} \quad \mathrm{H}-\mathrm{O}-\mathrm{H}
$$

which is the original reaction multiplied by two for clarity. The overall heat of this reaction can be calculated as follows:

| Bond | Bond energy <br> (kJ/mol) | Number of <br> broken bonds | Energy required <br> (kJ/mol per reaction) | Number of <br> bonds formed | Energy released <br> (kJ/mol per reaction) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| H-O | 460 | 0 | 0 | 4 | 1840 |
| H-H | 432 | 2 | 864 | 0 | 0 |
| $\mathrm{O}=\mathrm{O}$ | 494 | 1 | 494 | 0 | 0 |

Sum of energy required $=1358 \mathrm{~kJ} / \mathrm{mol}$
Sum of energy released $=1840 \mathrm{~kJ} / \mathrm{mol}$
Net difference $=$ Sum of energy required - Sum of energy released $=-482 \mathrm{~kJ} / \mathrm{mol}$.
The negative sign means that energy is released in the reaction. To get the energy per mole of product (water) we need to divide by two which is the stoichiometric coefficient of $\mathrm{H}_{2} \mathrm{O}$ molecules in the calculated reaction.

Answer: Net energy released per mole of $\mathrm{H}_{2} \mathrm{O}=482 \mathrm{~kJ} / \mathrm{mol} / 2=\underline{\mathbf{2 4 1} \mathrm{kJ} / \mathrm{mol}}$.
Note: It would have been equally OK to make the calculation for the original reaction, taking into account that that only a "half" oxygen molecule is consumed per produced $\mathrm{H}_{2} \mathrm{O}$.
2. Let's examine the electrolysis of water. The general reaction is given by:

$$
2 \mathrm{H}_{2} \mathrm{O} \rightarrow 2 \mathrm{H}_{2}+\mathrm{O}_{2}
$$

Is this an endothermic reaction (energy required) or exothermic reaction (energy released)? How much energy ( $\mathrm{kJ} / \mathrm{mol}$ ) is absorbed or released per mole of $\mathrm{H}_{2} \mathrm{O}$ consumed?

## Solution

This reaction is the opposite of to the problem 1, and so, obviously equal amount of energy is required in it (per mole of consumed $\mathrm{H}_{2} \mathrm{O}$ ) than was released in reaction of problem 1. For the sake of practice, let's nevertheless do the corresponding calculation.

The overall heat of reaction can be calculated as follows:

| Bond | Bond energy <br> (kJ/mol) | Number of <br> broken bonds | Energy required <br> (kJ/mol per reaction) | Number of <br> bonds formed | Energy released <br> (kJ/mol per reaction) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| H-O | 460 | 4 | 1840 | 0 | 0 |
| H-H | 432 | 0 | 0 | 2 | 864 |
| $\mathrm{O}=\mathrm{O}$ | 494 | 0 | 0 | 1 | 494 |

Sum of energy required $=1840 \mathrm{~kJ} / \mathrm{mol}$
Sum of energy released $=1358 \mathrm{~kJ} / \mathrm{mol}$
Net difference $=$ Sum of energy required - Sum of energy released $=482 \mathrm{~kJ} / \mathrm{mol}$.
Because the sign of positive, energy is required.
The thermochemical equation is:
$2 \mathrm{H}_{2} \mathrm{O}+482 \mathrm{~kJ} / \mathrm{mol} \rightarrow 2 \mathrm{H}_{2}+\mathrm{O}_{2}+0 \mathrm{~kJ} / \mathrm{mol}$
3. Label the following reactions as oxidation or reduction reactions:

## Solution

a) $\mathrm{Cu} \rightarrow \mathrm{Cu}^{2+}+2 \mathrm{e}^{-} \quad$ (Oxidation)
b) $2 \mathrm{H}^{+}+2 \mathrm{e}^{-} \rightarrow \mathrm{H}_{2} \quad$ (Reduction)
c) $\mathrm{O}^{2-} \rightarrow 1 / 2 \mathrm{O}_{2}+2 \mathrm{e}^{-}$(Oxidation)
d) $\mathrm{CH}_{4}+4 \mathrm{O}^{2-} \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}+8 \mathrm{e}^{-}$(Oxidation)
e) $\mathrm{O}^{2-}+\mathrm{CO} \rightarrow \mathrm{CO}_{2}+2 \mathrm{e}^{-}$(Oxidation)
f) $1 / 2 \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{O}+2 \mathrm{e}^{-} \rightarrow 2(\mathrm{OH})^{-}$(Reduction)
g) $\mathrm{H}_{2}+2(\mathrm{OH})^{-} \rightarrow 2 \mathrm{H}_{2} \mathrm{O}+2 \mathrm{e}^{-}$(Oxidation)
4.
a) If a portable electronic device draws 1 A current at a voltage of 2.5 V , what is the power requirement for the device?
b) You have designed a fuel cell that delivers 1 A at 0.5 V . How many of your fuel cells are required to supply the above portable electronic device with its necessary voltage and current requirements?
c) You would like the portable electronic device to have an operating lifetime of 100 h . Assuming $100 \%$ fuel utilization, what is the minimum amount of $\mathrm{H}_{2}$ fuel (in grams) required?
d) If this $\mathrm{H}_{2}$ fuel is stored as a compressed gas at 500 atm, what volume would it occupy (assume ideal gas, room temperature)? If it is stored as a metal hydride at $5 \mathrm{wt} \%$ hydrogen, what volume would it occupy? (Assume the metal hydride has a density of $10 \mathrm{~g} / \mathrm{cm}^{3}$.)

## Solution

a) Power requirement: $P=U I \rightarrow P=(1 \mathrm{~A})(2.5 \mathrm{~V})=2.5 \mathrm{~W}$.
b) Current of a single cell is sufficient so there is no need to connect cells in parallel. Voltage of a single cell on the other hand is not sufficient so $\frac{2.5 \mathrm{~V}}{0.5 \mathrm{~V}}=5$ cells have to be connected in series.
c) Each cell consumes 1 A worth of hydrogen for 100 h . Each hydrogen molecule produces 2 electrons so the amount of consumed hydrogen is $n=\dot{n} t=\frac{5 \times I}{z F} t=\frac{5 \times 1 \frac{C}{s}}{2 * 96485 \frac{\mathrm{C}}{\mathrm{mol}}} \times 100 \times 3600 \mathrm{~s}=$ 9.33 mol . The total amount of supplied hydrogen ( $100 \%$ fuel utilization) is $m=M n=$ $\left(2 \frac{\mathrm{~g}}{\mathrm{~mol}}\right)(9.33 \mathrm{~mol})=18.66 \mathrm{~g}$
d) Compressed gas: $p V=n R T \rightarrow V=\frac{9.33 \operatorname{mol} \times 8.314 \frac{J}{K m o l} \times 300 \mathrm{~K}}{500 \times 1.013 \times 10^{5} \mathrm{~Pa}} \approx 465 \mathrm{~cm}^{3}$

Metal hydride: $\rho_{\text {hydride }}=\frac{m_{\text {hydride }}}{V}=\frac{m_{H_{2}}}{0.05 \times V} \rightarrow V=\frac{18.66 \mathrm{~g}}{0.05 \times 10 \frac{g}{c m^{3}}}=37 \mathrm{~cm}^{3}$

