

## Exercise 5 solutions

### 1. O'Hayre (2016) Example 10.1

#### Problem

**Example 10.1** The fuel cell system shown on the left of Figure 10.1 is an MCFC that produces 200 kW of electric power with an electrical efficiency of 52% based on the higher heating value (HHV) of natural gas fuel it consumes. (1) Calculate the quantity of heat released by the fuel cell. Assume that any energy not produced as electric power from the fuel cell stack is released as heat. (2) You would like to use the heat released by the fuel cell to heat a building. Assume that you can recover 70% of the available heat for this purpose, with 30% of the available heat lost to the surroundings. Calculate the amount of heat recovered and the amount lost to the environment.

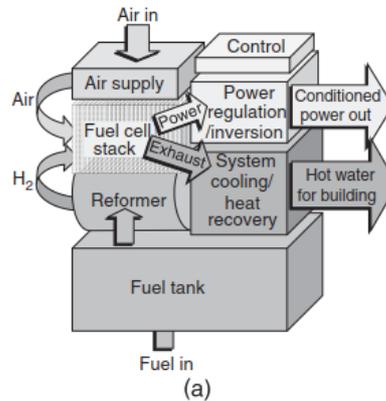


Figure 10.1.

#### Solution

- As discussed in Chapter 2, the real electrical efficiency of the fuel cell stack is described by

$$\epsilon_R = \frac{P_e}{\Delta\dot{H}_{(\text{HHV}),\text{fuel}}} \quad (10.2)$$

where  $P_e$  is the electrical power output of the fuel cell stack. We assume any energy that is not produced as electric power from the stack is produced as heat. This assumes that the parasitic power draw from pumps, compressors, and other components is negligible. The amount of heat released by the fuel cell is the maximum quantity of recoverable heat ( $d\dot{H}_{\text{MAX}}$ ). The maximum heat recovery efficiency ( $\epsilon_{\text{H,MAX}}$ ) is

$$\epsilon_{\text{H,MAX}} = 1 - \epsilon_R = 1 - 0.52 = 0.48 = 48\% \quad (10.3)$$

The amount of heat released by the fuel cell is

$$d\dot{H}_{\text{MAX}} = \frac{(1 - \epsilon_R)P_e}{\epsilon_R} = \frac{(1 - 0.52)200\text{ kW}}{0.52} = 185\text{ kW} \quad (10.4)$$

- The amount of heat recovered is  $0.70 \times 185\text{ kW} = 130\text{ kW}$  and the amount of heat lost to the environment is  $0.30 \times 185\text{ kW} = 55\text{ kW}$ .

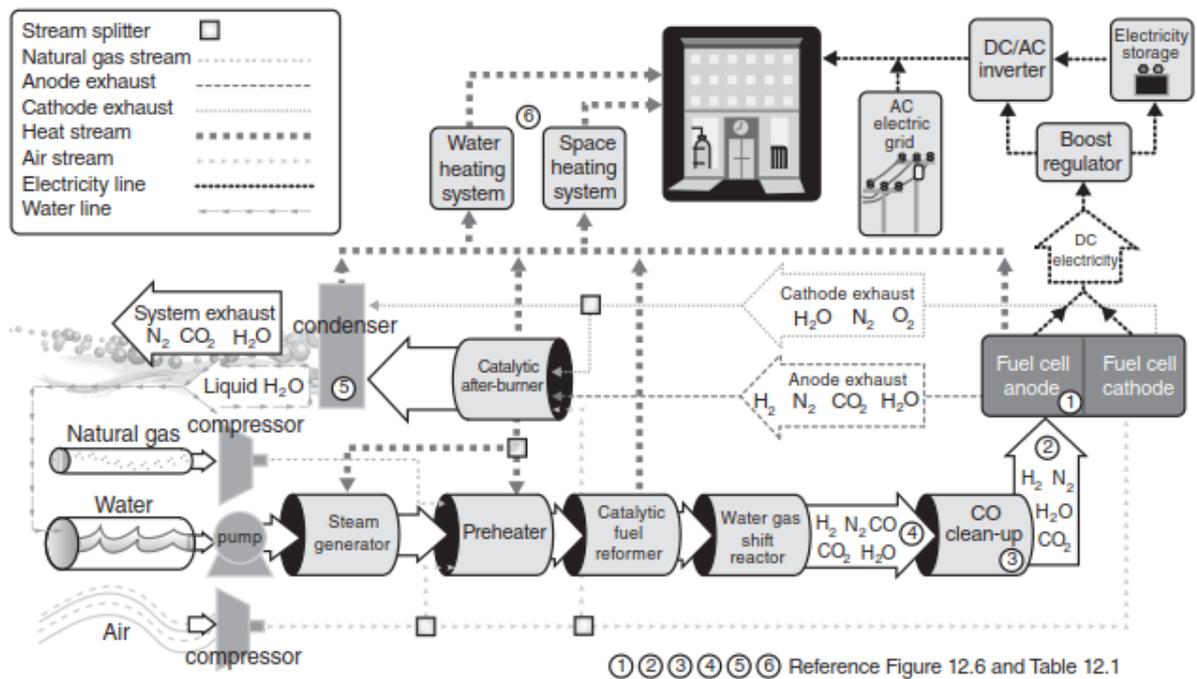
## 2. O'Hayre (2016) Example 10.4

### Problem

**Example 10.4** The Table below gives realistic efficiency values for the various subsystems of the stationary fuel cell system shown in Figure 10.14. Based on these efficiencies, calculate (1) the fuel cell system's electrical efficiency, (2) the system's heat recovery efficiency, and (3) the system's overall efficiency and (4) report the  $H/P$ .

**TABLE 10.5. Electrical Efficiency and Heat Recovery Efficiency for Four Main Subsystems**

	Fuel Processing Subsystem	Fuel Cell Subsystem	Power Electronics Subsystem	Thermal Management Subsystem	Overall System
Electrical efficiency	85%	42%	92%	NA	
Heat recovery efficiency			NA	80%	



**Figure 10.14.** Process diagram of CHP fuel cell system.

Solution

1. For the four subsystems discussed above, Table 10.5 summarizes the efficiencies for the four individual subsystems, along with the system's net electrical efficiency ( $\epsilon_R$ ), calculated as

$$\epsilon_R = \epsilon_{FP} \times \epsilon_{R,SUB} \times \epsilon_{R,PE} = 0.85 \times 0.42 \times 0.92 = 0.328 \quad (10.31)$$

$$= 33\% \quad (10.32)$$

2. Table 10.5 also summarizes the thermal recovery efficiencies for subsystems along with the overall system heat recovery efficiency ( $\epsilon_H$ ). A thermal management system that is 80% efficient can recover 80% of available heat from the fuel processor subsystem and the fuel cell subsystem, according to

$$\epsilon_{FP,H} = \epsilon_{TM} \times (1 - \epsilon_{FP}) = 0.80(1 - 0.85) = 0.12$$

$$\epsilon_{SUB,H} = \epsilon_{TM} \times (1 - \epsilon_{R,SUB}) = 0.80(1 - 0.42) = 0.46$$

$$\epsilon_{SUB,H,fuel} = \epsilon_{FP} \times \epsilon_{TM} \times (1 - \epsilon_{R,SUB}) = 0.85[0.80(1 - 0.42)] = 0.39$$

$$\epsilon_H = \epsilon_{SUB,H,fuel} + \epsilon_{FP,H} = 0.12 + 0.39 = 0.51$$

$$= 51\%$$

(10.33)

3.  $\epsilon_O = \epsilon_R + \epsilon_H = 0.33 + 0.51 = 84\%$ .

4.  $H/P = \epsilon_H/\epsilon_R = 0.51/0.33 = 1.55$ .

**TABLE 10.5. Electrical Efficiency and Heat Recovery Efficiency for Four Main Subsystems**

	Fuel Processing Subsystem	Fuel Cell Subsystem	Power Electronics Subsystem	Thermal Management Subsystem	Overall System
Electrical efficiency	85%	42%	92%	NA	33%
Heat recovery efficiency	12%	46%	NA	80%	51%

### 3. O'Hayre (2016) Calculation 10.8

#### Problem

- (a) Assuming STP conditions, what is the rate of heat generation from a 1000-W hydrogen/air-fueled PEM running at 0.7 V (assume  $\epsilon_{\text{fuel}} = 1$ )?
- (b) The fuel cell in part (a) is equipped with a cooling system that has an effectiveness rating of 25. To maintain a steady-state operating temperature, assuming no other sources of cooling, what is the parasitic power consumption of the cooling system?

#### Solution

a) The thermodynamic efficiency of a H<sub>2</sub>-O<sub>2</sub> fuel cell at STP is 0.83 and in our case with 0.7 V voltage, the voltage efficiency is  $\epsilon_v = 0.7 \text{ V} / 1.23 \text{ V} = 0.57$ . The total efficiency is  $\epsilon_{\text{total}} = 0.83 * 0.57 = 0.47$ , so the input fuel flow rate must be  $1000 \text{ W} / 0.47 = 2128 \text{ W}$ . Heat generation is then  $2128 \text{ W} * (1 - 0.47) = 1128 \text{ W}$ .

b) The effectiveness of a cooling system is calculated by dividing heat removal rate by the parasitic electric consumption. In this case we know the effectiveness and the heat removal rate, so the parasitic power consumption is simply *heat generation rate/effectiveness*:  $1128 \text{ W} / 25 = 45 \text{ W}$ .

### 4. O'Hayre (2016) Calculation 10.12

#### Problem

We would like to compute the carrier system effectiveness of a fuel cell operating on reformed natural gas. Since the reforming process is not perfectly efficient, in this example, we assume that the enthalpy content of H<sub>2</sub> provided to the fuel cell amounts to only 75% of the original enthalpy content of the natural gas. Furthermore, we recognize that the H<sub>2</sub> supplied by the reformer will be diluted with CO<sub>2</sub>, other inert gases, and perhaps even some poisons. We assume that these diluents lower the efficiency of the fuel cell by 20% compared to operation on pure H<sub>2</sub>. What is the total net effectiveness of this reformed natural gas system?

#### Solution

The carrier system effectiveness is defined as the % conversion of carrier to electricity divided by the % conversion of H<sub>2</sub> to electricity. This can be written as *% conversion of carrier to diluted H<sub>2</sub> \* (% conversion of diluted H<sub>2</sub> to electricity) / % conversion of pure H<sub>2</sub> to electricity*. Therefore, the total net effectiveness here is  $0.75 * 0.8 = 0.60$ .