Microeconomic Theory I: Lecture 1

Juuso Välimäki

Helsinki GSE, Fall 2019

Microeconomic Theory: Setting the Scene

- Actors: Economic agents
  - consumers
  - producers
  - employers, managers, bankers...
- Stage: economic institutions
  - markets
  - organizations, negotiating tables...
- Storyline: Economic agents pursue their interests within the boundaries set by the institutions

Microeconomic Theory: Methodology

- Each economic agent has well-defined objectives
- Each agent pursues her objectives
- In this course:
  - Analytical representation of objectives
  - Behavioral assumptions underlying pursuit of objectives
  - This lecture: first look at alternative ways of approaching this
- In Microeconomic Theory II-IV: Interactions between agents

Economic Decisions

1. Choices, Preferences and Utility Representations

1.1 From choice to preferences

Four elements for describing choices:

1. The choice set X.
2. Feasible set $B \subset X$.
3. Choice Rule
4. Behavioral assumption.

Choice Set X

- X is the universe of alternative choices
- Examples:
  1. Different Ph.D. programs in economics.
  2. Consumption over time.
  3. Speeding or not speeding.
  4. Occupational choice.
  5. All positive cosnumption levels for n distinct goods: $\mathbb{R}^n_+$

Feasible Set B

- Achievable choices
- Examples:
  Budget set $B(p, m) = \{x \in \mathbb{R}^n_+ : p \cdot x \leq m\}$
  In a normal form game, $X = X_1 \times \cdots \times X_N$, where each player i chooses independently an element in X_i. Then we have $B_i(x_{-i}) = \{(x_i, x_{-i}) \,|x_i \in X_i\}$.
- Why do we need to separate B and X.

Choice Rule

- How is choice made when B is given?
- Let $\mathcal{B}$ denote the collection of all possible feasible sets.
- $C(B)$ is the choice correspondence such that $C(B) \subset B$ for all $B \in \mathcal{B}$
- $(\mathcal{B}, C(\cdot))$ is called a choice structure

Behavioral assumptions

- Nonemptyness: For all $B \in \mathcal{B}$,

$C(B) \neq \emptyset$

- Weak Axiom of Reveled Preference (WA):
  If $x, y \in B$ and $x \in C(B)$, then for all B' such that $x, y \in B'$ and $y \in C(B')$, we have $x \in C(B')$.
- This axion is sometimes called choice coherence. Can you see why?
- We could alternatively require that
  i) If $x \in C(A)$ and if $x \in B \subset A$, then $x \in C(B)$.
  ii) If $y \in B \subset A$, then $[y \in C(A)] \Rightarrow [C(B) \subset C(A)]$.
- You are asked in the problem sets to show that requiring i) and ii) is equivalent to WA.

Revealed Preference Relation

- For a given $(\mathcal{B}, C(\cdot))$, we can define a binary relationrelation $\succeq^*$ on $X \times X$ by

$$\begin{aligned} x & \succeq^* & y \\ & \Longleftrightarrow & \\ \left[\text{there is } B \right. & \in & \left. \mathcal{B} \text{ with } x, y \in B \text{ and } x \in C(B)\right]. \end{aligned}$$

- We call this derived binary relation the *Revealed Preference Relation*

1.2 From prefences to choice

Four elements:

1. The choice set X.
2. Feasible set $B \subset X$.
3. Preference relation $\succeq$ on X
4. Behavioral assumption $c(B, \succeq)$.

What is a preference relation?

- A binary relation R is defined by writing $R \subset X \times X$, and $(x, y) \in R$ if the ordered pair (x, y) is in the relation R
- Write
$$(x, y) \in R \Leftrightarrow x \succeq y$$
- We could also define other relations P with other notation
$$(x, y) \in P \Leftrightarrow x \succ y$$
- The interpretation of the relation determines the properties that we impose on the relation

Rational Preference Relation $\succeq$

A binary relation $\succeq$ is said to be rational if it satisfies the following two requirements:

1. Completeness: For all $x, y \in X$ either $x \succeq y$ or $y \succeq x$ or both.
2. Transitivity: For all $x, y, z \in X$, $x \succeq y$ and $y \succeq z$ imply that $x \succeq z$.

Other binary relations derived from $\succeq$:

- Indifference: Write $x \sim y$ if $x \succeq y$ and $y \succeq x$.
- Strict preference: Write $x \succ y$ if $x \succeq y$ and not $y \succeq x$.

Other properties can be derived for rational preferences:

- $\succeq$ is reflexive: For all $x \in X$, $x \succeq x$.
- $\succ$ is asymmetric: For all $x, y \in X$, $(x \succ y) \Rightarrow \neg (y \succ x)$.
- $\succ$ is negatively transitive: For all $x, y, z \in X$, $(x \succ z) \Rightarrow (y \succ z)$ or $(x \succ y)$.

- We could have started with a relation P and assumed it to be asymmetric and negatively transitive. If we assume this and define xRy if and only if it is not true that yPx, then we can show (in the problem set maybe) that R thus defined is compete and transitive.

Behavioral assumption

The choice is induced from preferences according to the following:

$c^*(B, \succeq) = \{x \in B : x \succeq y, \text{for all } y \in B\}$.

Hence for all possible feasible sets B, the decision maker chooses the most preferred alternative according to $\succeq$ in B.

1.3 Connections between the choice- and preference-based approaches

From preferences to WA

Proposition

If $\succeq$ *is a rational preference relation, the strucure* $(\mathcal{B}, c^*(\cdot, \succeq))$ *induced by* $\succeq$ *satisfies WA.*

From WA to preferences

Proposition

Let $\mathcal{B}$ *include all subsets of* X *with two or three elements. Then, if* $(\mathcal{B}, C(\cdot))$ *satisfies WA, the induced revealed preference relation* $\succeq^*$*is rational and the unique preference relation that induces* $C(B) = c(B, \succeq^*)$.

- Why is the restriction on the sets in Proposition 2 important?
- Ex.1: $X = \{x, y, z\}$, $\mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}\}$
- Ex.2: As Ex. 1 but add $\{x, y, z\}$ to $\mathcal{B}$

1.4 Utility representation
Overview

- In most models we work with a utility function for convenience: it can be manipulated using standard math tools
- Is it OK to let a real-valued function to represent potentially complicated preferences over the choice set?
- What are we exactly assuming when taking this approach?
- The objective: analyze the relationship between the axioms and the utility function

Representation for $\succeq$

We are looking for numerical representation of $\succeq$, which is a function $u : X \to \mathbb{R}$ such that

$$u(x) \geq u(y) \iff x \succeq y. \tag{1}$$

Proposition

If there exists utility function representing $\succeq$, then $\succeq$ is rational.

Proposition

If the choice set X is finite and $\succeq$ is rational, then $\succeq$ has a representation.

- If u represents $\succeq$, then so does $f \circ u$ for any increasing $f : \mathbb{R} \to \mathbb{R}$
- Note that u maps into real line on which the binary relation "$\geq$" complete and transtive.
- The restriction to finite X in Proposition above is quite demanding: Rules out e.g. choice from linear budget sets. Extending the result to infinite sets requires more assumptions and will be discussed later in the course.

Challenges for choice based on Binary Comparisons

- Problem 1: Framing
- Problem 2: Judgements that are hard
- Problem 3: Aggregation