

Problem Set 1 (Due Sep 9), 2019

1. Start with a strict preference relation \succ on $X \times X$ and assume that \succ satisfies
 - i) Asymmetry: For all $x, y \in X$, if $x \succ y$, then it is not true that $y \succ x$.
 - ii) Negative transitivity: For all $x, y, z \in X$,

$$[x \succ y] \Rightarrow [x \succ z] \text{ or } [z \succ y]. \quad (1)$$

Define another binary relation \succeq by

$$[x \succeq y] \iff \neg[y \succ x].$$

Show that \succeq is complete and transitive.

2. In the next two problems, consider choice rules that satisfy non-emptiness. Recall the definition of WA from class:

For $x, y \in B \cap B'$ with $x \in C(B)$ and $y \in C(B')$, we have $x \in C(B')$.

Show that this definition is logically equivalent to the requirement that for $x, y \in B \cap B'$ we have

$$[x \in C(B) \text{ and } y \notin C(B)] \Rightarrow [y \notin C(B')].$$

3. Consider a choice rule $C(B)$ for $A, B \in \mathcal{B}$ satisfying
 - i) If $x \in C(A)$ and $x \in B \subset A$, then $x \in C(B)$,
 - ii) If $y \in B \subset A$, then $[y \in C(A)] \Rightarrow [C(B) \subset C(A)]$.Can you give an interpretation to these properties?

Show that a choice rule satisfying WA satisfies i) and ii). Show also that a choice rule satisfying i) and ii) satisfies WA. Can you find examples that satisfy i) but not ii) and rules that satisfy ii) but not i)?

4. Let \succeq be a preference relation defined on a finite choice set X with N elements.

(a) Consider $u(x) = \#\{y|x \succeq y\}$, i.e. the number of y that are no better than x . Is this a utility function representing \succeq ? How would you compute it?

(b) Suppose you order the elements of X in an arbitrary order $X = \{x_1, x_2, \dots, x_N\}$. Assign $u(x_1) = \frac{1}{2}$.

For each x_k , set $u(x_k) = u(x_l)$ if there is an $l < k$ such that $x_k \sim x_l$.

If $x_k \succ x_l$ for all $l < k$, set $u(x_k) = \frac{1 + \max_{l < k} u(x_l)}{2}$.

If $x_l \succ x_k$ for all $l < k$, set $u(x_k) = \frac{0 + \min_{l < k} u(x_l)}{2}$.

Finally, in all remaining cases, let x^o and x_o be defined by the requirement that for $l < k$, $x_l \succ x_k$ implies that $x_l \succeq x^o$ (i.e. for $l < k$, all x_l strictly better than x_k are at least as good as x^o) and similarly for $l < k$, $x_k \succ x_l$ implies that $x_o \succeq x_l$. If $x^o \succ x_k \succ x_o$.

In this case, set

$$u(x_k) = \frac{u(x^o) + u(x_o)}{2}.$$

Does this procedure give you a valid utility representation? Is the utility function the same as in the previous part? Is the utility function independent of the ordering of the elements x_i ?

(c) Bonus question: can you think of a generalization to part a) that would give you a utility representation for infinite sets if the indifference curves are not thick?

5. Let's draw some budget sets to see that they do not have to be linear in all relevant cases.

(a) $X = (x, y) \in \mathbb{R}_+^2$, where y is the number of GB that you can use on your phone per month and x is all other consumption. You

can choose from a menu $\{(f_1, c_1), \dots, (f_K, c_K)\}$ where f_i is your fixed cost of plan i and c_i is its variable cost.

- (b) You live for two periods. You have income (w_1, w_2) in the two periods. You can borrow and lend at rates r_B and r_L respectively. What are reasonable assumptions for r_L, r_B ? Draw the budget set for consumptions in the two periods for cases that you consider particularly interesting.
 - (c) Do you have any idea of what the budget set between daily consumption and leisure looks like at different income levels. Consider the unrealistic case where the worker can choose an arbitrary number of hours at a fixed wage rate per hour. Find out and give a qualitative summary of your findings.
6. Suppose someone argued: ?The weak axiom of revealed preference has no empirical content. The WARP requires multiple observations governed by a single preference relation. But we can never expect such observations. Each successive choice a person makes is made with at least a slightly different history, which can affect their preferences.? The WARP thus has empirical content only if one is willing to assume that the various choices one observes are independent, in the sense that previous choices do not affect current preferences. Describe the conditions, preferably using examples of the sort of data an economist might reasonably hope to work with, under which you would be willing to assume such independence, and under which you would not.