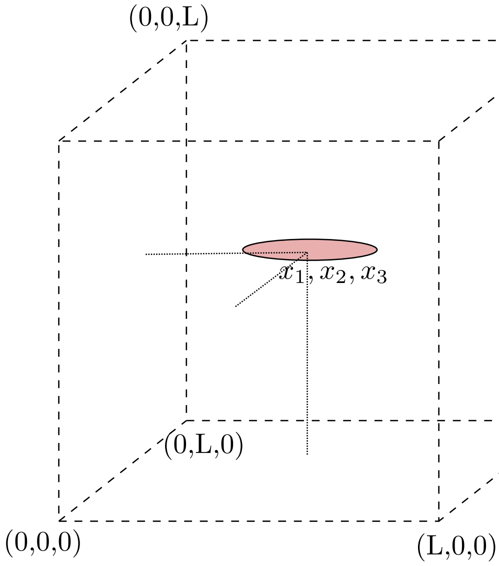
1 Introduction

Exercise 1. Suppose we wish to determine a drone position in a closed three dimensional



box (x_1, x_2, x_3). In this scenario, suppose the drone is equipped with sensors that measure the distance between the drone and the surrounding walls. The measurements from these sensors are given by

$$
\begin{aligned}
y_1 &= x_1 + r_1, \\
y_2 &= x_2 + r_2, \\
y_3 &= x_3 + r_3, \\
y_4 &= L - x_1 + r_4, \\
y_5 &= L - x_2 + r_5, \\
y_6 &= L - x_3 + r_6
\end{aligned}
$$

Task 1.1. *Rewrite the measurement model above in vector notation, with the following form:*

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{b} + \mathbf{r}$$

2 Sensors, Models, and Least Square Criterion

Exercise 1. A resistor with unknown resistance x was found at Sensor Fusion lab. A student tries to estimate what is the value of x from a set of k noisy multimeter measurements.

Task 1.1. *Assuming that the measurement noise is affine, construct the measurement model for this.*

Task 1.2. *Construct the quadratic cost function this problem.*

Exercise 2. Suppose there are other $N-1$ students involved during inspection of the unknown resistance in the previous exercise. Each students perform one measurement with his/her own multimeter, where the measurement of i-th student is given by

$$y_i = x + r_i.$$

The variance of r_i is σ_i^2.

Task 2.1. *The students come with idea to weight an individual measurement according to one over the variance of his/her multimeter. Construct the weighted the quadratic cost function for this problem, and what is the weighting matrix?*

Exercise 3. Suppose we want to estimate unknown variables $\mathbf{x} \in \mathbb{R}^d$ with a set of measurements $\mathbf{y}_n \in \mathbb{R}^m$ that are taken at time multiple time samples n.

$$\mathbf{y}_n = \mathbf{g}_n(\mathbf{x}) + \mathbf{r}_n \tag{1}$$

Task 3.1. *Suppose that the measurement noise vector $\mathbf{r}_n$ is not zero-mean, but has some static bias. How would you convert measurement model* (1) *so that it will work with zero-mean measurement noise assumption?*

Task 3.2. *Suppose that you have obtained an equivalent measurement equation to* (1) *where the measurement noise are now zero-mean. Compare the covariance matrix of $\mathbf{r}_n$ from* (1) *and the covariance matrix of measurement noises you obtained from Task 3.1.*

Task 3.3. *Let the measurement noise element r_i in* (1) *has a Gaussian probability density function with variance σ_i^2. Write down the probability densities of $i-th$ sensor measurement y_i. Using this probability densities, assuming that each sensor measurement noises are independent, construct the joint probability density of the measurement vector $\mathbf{y}_n$.*

Exercise 4. Suppose the student collected k samples from measurement in Exercise 1. Assume that the most recent measurement has more valuable information to the old one. For some $\lambda \in (0, 1]$, construct a square weighting matrix $\mathbf{W}$ so that the contribution of the masurement noise at time index i to the quadratic cost function J is weighted λ times the contribution of the measurement error at time index $i + 1$.