Microeconomic Theory 1
Helsinki GSE
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Problem Set 2 (Due Sep 23), 2019

1. Consider the following statements and determine if they are true or false.

   (a) If $\succsim$ is represented by a continuous function, then $\succsim$ is continuous.

   (b) A continuous preference can be represented by a noncontinuous function.

   (c) Let $X = \mathbb{R}$ and $U(x) =$ [the largest integer n such that $x \geq n$]. The underlying preference relation is continuous.

   (d) If both U and V represent $\succsim$, then there is a strictly monotonic function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x) = f(U(x))$

2. Let $X = \mathbb{R}^2_+$ and assume that preference relation $\succeq$ satisfies three properties:

   (i) Additivity: $(x_1, x_2) \succeq (y_1, y_2) \Longrightarrow (x_1 + t, x_2 + s) \succeq (y_1 + t, y_2 + s)$ for all t, s.

   (ii) Monotonicity: $[x_1 \geq y_1$ and $x_2 \geq y_2] \Longrightarrow (x_1, x_2) \succeq (y_1, y_2)$ and, in addition, if either $x_1 > y_1$ or $x_2 > y_2$, then $(x_1, x_2) \succ (y_1, y_2)$.

   (iii) Continuity.

   (a) Show first that if $\succeq$ has a linear utility representation, then properties (i)-(iii) are all satisfied.

   (b) Now, consider the following pairs of properties: (i,ii), (i,iii), and (ii,iii). For each pair, show that there exists a preference relation that satisfies the properties but not the property dropped.

   (c) Finally, show that properties (i,ii,iii) imply the existence of a linear representation.

3. Consider a standard utility maximizing consumer who consumes three goods per year. The Walrasian consumption bundle in year t is $x^t = (x_1^t, x_2^t, x_3^t)$, prices are $p^t = (p_1^t, p_2^t, p_3^t)$, and income is I^t.

   (a) Consider first three years, $t = 1, 2, 3$, with prices

   $$\begin{aligned} p^1 &= (4, 6, 4) \\ p^2 &= (2, 4, 6) \\ p^3 &= (6, 8, 2). \end{aligned}$$

   Income is constant in all years $I^1 = I^2 = I^3 = I$. The utility in years 2 and 3 is the same. Is the utility level in year 1 below or above the level in years 2 and 3?

   (b) In year $t = 4$, prices and income are the same as in $t = 3$ but the country in which the consumer lives joins a monetary union. As a result all monetary units are multiplied by 1/5,94573. What is the utility level in $t = 4$?

   (c) In year $t = 5$, the consumer has a new wage contract: the income is linked to prices such that whenever prices increase, the consumer's utility remains unchanged. Suppose now that one price has increased as period $t = 5$ arrives. Show that the demand of that good decreases.

4. A consumer in a three-commodity environment (x, y, z) behaves as follows.

   i) When prices are $p_x = 1, p_y = 1$ and $p_z = 1$ the consumer buys $x = 1, y = 2$ and $z = 3$;

   ii) When prices are $p_x = 4, p_y = 6$ and $p_z = 4$ the consumer buys $x = 3, y = 2$ and $z = 1$.

   Does the consumer maximize a strictly quasi-concave utility function?

5. Consider the constant elasticity of substitution utility function (check that you understand the name of this function):

$$u(x_1, ..., x_L) = (\alpha_1 x_1^\rho + ... + \alpha_1 x_1^\rho)^{\frac{1}{\rho}},$$

for $\rho < 1, \rho \neq 0$.

(a) Show that u is strictly increasing in each of its components and that for any p, w, $x(p, w) >> 0$. Conclude that Walras' law holds and $MRS_{i,j}(x) = \frac{p_i}{p_j}$ for all p, w.

(b) Solve for x_j as a function of (x_i, p_i, p_j)

(c) Substitute for x_j for all $j \neq i$ in the budget constraint (that holds with equality by Walras' law). Solve for x_i as a function of p, w.

(d) Specialize to the case $\alpha_i = 1$ for all i. Substitute in the utility function to get the indirect utility function $v(p, w)$. Verify that your calculations are correct using Roy's identity on your indirect utility function.

6. (Becker) Consider the aggregate demand in a model where individual consumers behave in a random manner (and thus do not satisfy any of the rationality criteria that we had for individual choice). To be more specific, assume that a consumer with wealth w facing prices p picks a consumption vector at random from the budget set $B(p, w) = \{x : p \cdot x = w\}$ according to the uniform distribution. Suppose furthermore that there are a continuum of such consumers (and assume that you can apply the law of large numbers for this setting, i.e. the distribution of realized choices in the population coincides with the distribution of a single consumer's choice).
(i) Denote the individual (random) demand by $x^i(p, w)$. Compute the average demand

$$\overline{x}(p, w) = \int x^i(p, w)\, di.$$

(ii) Does this average demand satisfy weak axiom of revealed preference?
(iii) Can you find a utility function such that $x^i(p, w)$ is the walrasian demand function for that utility function?

7. The government finances public expenditure of magnitude g by collecting taxes. In this question, you are invited to think about the optimal ways of collecting taxes.

   (a) Suppose that there are goods x and y. The government can finance g by choosing either a tax on income t_w or by taxing consumption of good x by rate t_x. The government budget constraint for the two cases reads: $t_w w = g$ and $t_x x(p_x, p_y, t_x) = g$. Show that the consumer prefers an income tax in this case.

   (b) Suppose now that there is no exogenous income in the model and good y is now interpreted as leisure. Assume that he consumer has an initial endowment y^e of leisure that she may sell to buy the other good. Hence the budget constraint is now

$$p_x x(p_x, p_y) = p_y (y^e - y(p_x, p_y)), \text{ or}$$

$$p_x x(p) + p_y y(p) = p_y y^e.$$

   This last equation gives a way in which all problems with income resulting from sales of endowments should be thought of. First sell the endowment at market prices and then purchase the desired amounts of the goods wit the proceeds. Compare now the effect of taxes on x and y as in the previous part.

8. Suppose that the expenditure function of a consumer is of Gorman polar form:

$$e(p, u) = a(p) + ub(p).$$

   Derive the demands for each good and calculate also the income shares that each good receives. Can you find an economic interpretation for your results.