Microeconomic Theory I: Lecture 6

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Special Classes of Preferences: Homothetic

A rational preference relation $\succeq$ on $\mathbb{R}^L_+$ is said to be *homothetic* if $x \succeq y$ if and only if $tx \succeq ty$ for all $t > 0$.

Proposition

A continuous $\succeq$ is homothetic if and only if it admits a utility representation with $u(\alpha x) = \alpha u(x)$, i.e. a utility representation that is homogenous of degree 1 in x

Proposition

If preferences are homothetic, then then $x(p, \alpha w) = \alpha x(p, w)$ and therefore for some $x(p, w) = b(p)w$. for some $b(p)$.

What do we know about $b(p)$ in this case?

A function $v(p, w)$ is of Gorman form if $v(p, w) = a(p) + b(p)w$. When is such a function a legitimate indirect utility function? Can you allow more general forms if you restrict the range of w?

Special Classes of Preferences: Quasilinear

A rational preference relation $\succeq$ on $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ is said to be *quasilinear with respect to good 1* if $x \succeq y$ if and only if $(x_1 + t, x_2, ..., x_L) \succeq (y_1 + t, y_2, ..., y_L)$ for all t.

Proposition

$\succeq$ is quasilinear with respect to good 1 if and only if it has a utility representation of the form $u(x) = x_1 + v(x_2, ..., x_L)$.

This special case is used a lot in partial equilibrium analysis. The consumption of goods $x_2, ..., x_L$ is independent of x_1. The first good is often thought of as income and hence this case implies no income effects for the demands of $x_2, ..., x_L$.

Easy consequences of quasilinearity:

1. $v(p, w) = p_1 w + \theta(p)$.
2. $x_l(p, w) = \xi_l(p)$ for $l \in \{2, ..., L\}$.
3. $e(p, u) = \frac{u}{p_1} - \zeta(p)$.

Special Classes of Preferences: Separability

A rational preference relation $\succeq$ on $\mathbb{R}^L_+$ is said to be *separable* if it admits a utility representation $u(x) = v(u_1(x_1, .., x_{g_1}), u_2(x_{g_1+1}., ..., u_{g_2})..u_G(x_{g_{G-1}+1}, ..., x_L))$. In other words, we can group the goods in G groups so that for l, k within the same group, $MRTS_{l,k}(x)$ depends only on goods in that group. (Prove this).

This allows us to use a two-stage budgeting procedure: For each group g, compute the optimal demands within that group at income w_g.

A very strong form of separability is the following. A rational preference relation $\succeq$ on $\mathbb{R}^L_+$ is said to be *additively separable* if it admits a utility representation $u(x) = \sum_{l=1}^{L} u_l(x_l)$.

Special Classes of Preferences: Additive Separability

Additive separability has strong implications.

1. If u is strictly quasiconcave, then either all goods are normal or one is normal and all others are inferior.
2. If $u_l(x_l) = u_k(x_k)$ and $u''(x_l) < 0$, then all goods are normal.
3. If $u_l(x_l) = u_k(x_k)$ and $-\frac{x_l u''(x_l)}{u'(x_l)} < 1$, then $\frac{\partial x_l(p,w)}{\partial p_k} > 0$ for all $k \neq l$.
4. For all separable u, $\frac{\partial x_i(p,w)/\partial p_k}{\partial x_j(p,w)/\partial p_k} = \frac{\partial x_i(p,w)/\partial w}{\partial x_j(p,w)/\partial w}$.
5. Element s_{ij} of the slutsky matrix is of the form

$$-\frac{\lambda(p,w)}{D_w\lambda(p,w)}\frac{\partial x_i}{\partial w}\frac{\partial x_j}{\partial w} \text{ for } i \neq j,$$

   and for s_{ii} it is of the form

$$\frac{\lambda(p,w)}{p_i D_w\lambda(p,w)}\frac{\partial x_i}{\partial w}\left(1 - p_i\frac{\partial x_i}{\partial w}\right).$$

Evaluating economic welfare

Consider two price vectors, p, p'. How can we say which price vector is better for the consumer? How about using indirect utility function?

Can be done, but unfortunately the measurement is in utils rather than anything cardinal.

Can we use the expenditure function to measure welfare changes in monetary terms?

Obviously it is not a great idea to compare $e(p, v(p, w))$ and $e(p', v(p', w))$. (Why?)

Consider $e(p, u)$ where u is the maximum utility reachable in the UMP at prices p versus $e(p', u)$. Similarly consider $e(p, u')$ and $e(p', u')$ where u' is the maximal utility in UMP with prices p'. We can measure the welfare either with equivalent variation EV or compensating variation CV:

$$EV := e(p, u') - e(p', u') = e(p, u') - w,$$

or

$$CV := e(p, u) - e(p', u) = w - e(p', u).$$

If only one price, say p_l changes from p_l to p_l' while the other prices remain fixed at p_{-l}, we can write the two measures using the fact that

$$\frac{\partial e(p,u)}{\partial p_l} = h_l(p,u)$$

and the fundamental theorem of differential and integral calculus:

$$EV = \int_{p_l}^{p_l'} h_l(s, p_{-l}, u')\,ds, \text{ and } CV = \int_{p_l}^{p_l'} h_l(s, p_{-l}, u)\,ds.$$

With quasilinear preferences with respect to good 1, we have for all $l \geq 2$ the following: $x_l(p, w) = h_l(p, u)$ for all w, u. This follows from the property noted above that for quasilinear preferences, there are no income effects. As a result, we can compute the equivalent and compensating variation using the Marshallian consumer surplus.

How far the welfare measures are from consumer surplus depends on the size of the income effects. For small changes, the difference is obviously small since at the original prices, Hicksian demand coincides with the Walrasian demand.

Application: Labour Supply

- Define: $l =$ leisure, $T =$ total time, $q =$ wage, $c =$ consumption of 1 good, $p =$ price of good
- Choose (c, l) to maximise $u(c, l)$ s.t. $pc = q(T - l)$
- Rewrite constraint: $pc + ql = qT$
- Solving gives demands $c(p, q, qT)$, $l(p, q, qT)$
  - $qT := w$ plays role of income
  - wage affects demand as a price and as a component of income

- Consider the total effect from changing wages:

$$\frac{dl(p,q,qT)}{dq} = \frac{\partial l}{\partial q} + T\frac{\partial l}{\partial} w$$

- Slutsky:

$$\frac{\partial l}{\partial q} = \frac{\partial h_l}{\partial q} - l\frac{\partial l}{\partial w}$$

- Therefore

$$\frac{dl}{dq} = \frac{\partial h_l}{\partial q} + (T - l)\frac{\partial l}{\partial w}$$

- when leisure is a normal good: $\partial l / \partial w \geq 0$, dl/dq can be positive (i.e., $d(T-l)/dw$ is negative)

Application: Two-Period Model of Saving

- Income in period i is y_i for $i \in \{1, 2\}$
- Consumption in period i is x_i
- δ is the discount factor
- Maximization problem

$$\max_{x_1, x_2} \; u_1(x_1) + \delta u_2(x_2)$$
$$s.t. \;\; p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$

- Savings if $x_1 < y_1$, borrowing if $x_1 > y_1$
- Assume increasing utility functions u_i and

$$\frac{\partial u_i}{\partial x_i} \rightarrow \infty \;\; \text{as} \;\; x_i \rightarrow 0$$

- Lagrangean

$$\mathcal{L}(x_1, x_2, \lambda) = u_1(x_1) + u_2(x_2) \\ - \lambda(p_1 x_1 + p_2 x_2 - p_1 y_1 + p_2 y_2)$$

- First-order conditions

$$u_1'(x_1) = \lambda p_1, \qquad \delta u_2'(x_2) = \lambda p_2, \\ p_1 x_1 + p_2 x_2 = p_1 y_1 + p_2 y_2$$

- First two lines imply

$$u_1'(x_1) = \frac{p_1}{p_2} \delta u_2'(x_2)$$

  Euler equation of macroeconomics.
- If $u_1'(x_1) = u_2'(x_2)$, then $p_2 / p_1 = \delta$: relative prices determined by time preference (rate of interest)
- Can assume e.g., $u_1(x_1) = \ln x_1$ and $u_2(x_2) = \ln x_2$ to calculate demand functions