1 Static Nonlinear Models

Exercise 1. Consider the following non-linear model:

$$\mathbf{y}_n = \mathbf{g}(\mathbf{x}) + \mathbf{r}_n, \quad n = 1, \ldots, N, \tag{1}$$

where $\mathbf{y}_n \in \mathbb{R}^2$, $\mathbf{x}^\mathsf{T} = [5\ 3.7]$, $\mathbf{r}_n$ is zero mean, Gaussian, with covariance matrix $\mathrm{R} = \mathrm{diag}\begin{bmatrix}\sigma_0^2 & \cdots & \sigma_N^2\end{bmatrix}$, and

$$g_i(\mathbf{x}) = \sqrt{(x_1 - s_{i,x})^2 + (x_2 - s_{i,y})^2} \tag{2}$$

Task 1.1. *Write down the least squares objective for the regression problem in Equation* (1)*, i.e*

$$J(\mathbf{x}) = (\mathbf{y} - \mathbf{g}(\mathbf{x}))^\mathsf{T}\mathrm{R}^{-1}(\mathbf{y} - \mathbf{g}(\mathbf{x})). \tag{3}$$

What is $\mathbf{y}$*, what is the Jacobian* $\mathbf{G}$ *of* $\mathbf{g}$*?*

Task 1.2. *Write down the gradient of the least squares objective in Equation* (3)*.*

Task 1.3. *Let* $N = 4$ *and* σ^2 *for each sensors are* $[1, 0.1, 0.8, 0.5]$*, and*

$$\mathbf{s}_x = \begin{bmatrix}2 & 0 & 10 & 7\end{bmatrix}^\top,$$
$$\mathbf{s}_x = \begin{bmatrix}6 & 0 & 2 & 8\end{bmatrix}^\top.$$

Implement a gradient descent algorithm with fixed step-size and try to estimate $\mathbf{x}$ *with different initial guesses (vary the distance from the true paramater!). Plot the progress of the algorithm on top of a contour plot of the least squares cost function. Note that you might have to put a very small step-size for the algorithm to not crash.*

Exercise 2. Consider the model from Exercise session 1:

Task 2.1. *Implement the Gauss–Newton algorithm without line search (Algorithm 3.2 in the lecture notes) to solve the least squares problem. That is, set* $\gamma^{(i)} = 1$ *for all* i*.*