Microeconomic Theory 1
Helsinki GSE
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Problem Set 3 (Due Oct 7), 2019

1. A firm's workers have identical and well-behaved preferences over leisure and income. They are paid \$10/hour for the first 40 hours and \$15/hour for each additional hour. They choose to work 50 hours per week . Management proposes to replace the current 50% overtime bonus pay schedule with a constant wage rate of \$11/hour. The workers claim that this will reduce their income, management claims that this will make workers better off. Who is right?

2. Fred's preferences betweeen pairs of shoes (S), and all other goods, (D), are represented by the utility function

$$u(S, D) = SD.$$

   Because of government regulations, left shoes L and right shoes R are *not* sold together. They are sold at prices p_L and p_R respectively. Fred has w dollars of money and the price of D is 1.

   a) Set up Fred's optimization problem and derive expressions for his optimal purchases and his maximal level of utility as functions of the exogenous variables.

   b) Fred's one-legged uncle has died and left him all his shoes. They wear the same size. Thus Fred now enters the market with w dollars and L_0 left shoes. Do the analysis of part a) for this case.

3. Consider the normalized price vector, $\mathbf{q} = \frac{1}{w}\ \mathbf{p} \in R^L$ and write the indirect utility function as a function of $\mathbf{q}$ only, i.e. $v(\mathbf{p}, w) = v^*(\mathbf{q})$. Preferences are said to display indirect additivity if

$$v^*(\mathbf{q}) = f(\Sigma_l v_l(q_l)).$$

Show that indirect additivity implies that for all distinct i, j, k, the price elasticity of good i with respect to price k is the same as the price elasticity of good j w.r.t. the price of good k.

4. A real valued function $f : \mathbb{R}^L_+ \to \mathbb{R}$ is called superadditive if for all z^1, z^2,

$$f\left(z^1 + z^2\right) \geq f\left(z^1\right) + f\left(z^2\right).$$

a) Show that every cost function is superadditive in input prices.

b) Using this fact, show that the cost function is nondecreasing in input prices.

5. Consider a firm which has n independent divisions. Each division uses a common factor x and an individual factor z_i.The question deals with the issue of allocating the cost of the common factor to divisions. The production function of division i is given by

$$q_i = f_i\left(z_i, x\right).$$

Assume that this function can be inverted to yield the common factor requirement

$$x_i = g_i\left(q_i, z_i\right).$$

In words, division i requires x_i units of the common factor to convert z_i units of private factor into q_i units of its output. Let w be the vector of private input prices, v the price of the common input and p the vector of output prices. The cost function of the firm is obtained from

$$\begin{aligned} c\left(q, w, v\right) &= \min_{z,x}\; w \cdot z + vx \\ \text{s.t. } x_i &\geq g_i\left(q_i, z_i\right) \text{ for all } i. \end{aligned}$$

The proposal is to allocate the price of the common resource according to the Lagrange multipliers in the above minimization problem, i.e. each division is to buy the common resource at price λ_i.

(a) Show that these prices cover the cost, i.e.

$$\Sigma_{i=1}^{n} \lambda_i = v.$$

(b) Based on these prices, consider the individual cost minimization problems of the divisions and show that the individual cost functions sum to the common cost function.

(c) Suppose that

$$f_i\left(z_i, x\right) = \min\{z_i, x\} \text{ for all } i.$$

Calculate the total cost function and determine the cost allocation.

6. (by John Panzar)

Joe is an empire builder. That is, his goal is to produce and sell as much output as possible. However, his stockholders impose the constraint that he not lose money. He operates using the production function $q = f(z)$, and faces parametric prices p and w for his output and (vector of) inputs. The production function is with positive marginal products.

(a) Set up Joe's problem and state the first order necessary conditions

(b) Is the nonnegativity constraint on profit binding? Why or why not?

(c) Interpret the Lagrangian multiplier. What is its sign?

(d) Show that Joe's supply curve slopes up.

(e) Show that Joe's output is decreasing function of all input prices.