1 Static Nonlinear Models-Gauss Newton with Line Search and Levenberg-Marquardt algorithms

Exercise 1. Consider the following non-linear model

$$y_i = g_i(\mathbf{x}) + r_i, \tag{1}$$

The noise r has a Gaussian distribution with zero mean and covariance σ_i^2. The nonlinear function $\mathbf{g}$ is given by

$$g_i(\mathbf{x}) = \sqrt{(x_1 - s_{i,x})^2 + (x_1 - s_{i,y})^2}. \tag{2}$$

Suppose $\mathbf{x}^\top = \begin{bmatrix} 5 & 3.7 \end{bmatrix}^\top$ and let $N = 4$ and the variances of each sensor are given by $\begin{bmatrix} 1 & 0.1 & 0.8 & 0.5 \end{bmatrix}$. Let also

$$\mathbf{s}_x = \begin{bmatrix} 2 & 0 & 10 & 7 \end{bmatrix}^\top,$$
$$\mathbf{s}_y = \begin{bmatrix} 6 & 0 & 2 & 8 \end{bmatrix}^\top.$$

Task 1.1. *Write down the measurement model in a vector form:*

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}.$$

Task 1.2. *Write down the Jacobian* $\mathbf{G}$ *of* $\mathbf{g}$.

Task 1.3. *Assume that sensor noises are uncorrelated, and has a covariance mnatrix* $\mathbf{R}$*. Write down the weighted least square cost function with the weighting matrix is* $\mathbf{R}^{-1}$*.*

Task 1.4. *Implement the Gauss-Newton algorithm with line-search.*

Task 1.5. *Implement the Levenberg-Marquardt algorithm, where the parameter* λ *is scale invariant.*