Exercise 1.

Task 1.1.

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & L & L & L \end{bmatrix}^\top, \qquad \mathbf{r} = \begin{bmatrix} r_1 & \cdots & r_6 \end{bmatrix}^\top$$

Exercise 2.

$$\begin{aligned} \mathbf{y} =& \mathbf{G}x + \mathbf{r}, \\ \mathbf{G} =& \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^\top, \\ \mathbf{r} =& \begin{bmatrix} r_1 & \cdots & r_k \end{bmatrix}. \end{aligned}$$

$$J(x) = \left(\mathbf{y} - \mathbf{G}x\right)^\top \left(\mathbf{y} - \mathbf{G}x\right).$$

Exercise 3.

$$\begin{aligned} \mathbf{y} =& \mathbf{G}x + \mathbf{r}, \\ \mathbf{G} =& \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^\top, \\ \mathbf{r} =& \begin{bmatrix} r_1 & \cdots & r_N \end{bmatrix}, \\ \mathbf{R} =& \mathrm{diag}\begin{bmatrix} \sigma_1^2 & \cdots & \sigma_N^2 \end{bmatrix}^\top, \\ \mathbf{W} =& \mathbf{R}^{-1}. \end{aligned}$$

Exercise 4. Let us write $\mathbf{r}_n = \mathbf{r}'_n + \bar{\mathbf{r}}_n$, where $\bar{\mathbf{r}}_n = \mathbb{E}\mathbf{r}_n$. Then we clearly see that the bias $\mathbf{b}$ is equal to $\bar{\mathbf{r}}_n$. The covariance matrix of $\mathbf{r}_n$ is equal to $\mathbf{r}'_n$.

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}'_n + \mathbf{b},$$

Now if $\mathbf{r}_n$ is zero mean, with variance matrix $\mathbf{R}$ then we can write the joint probability density of $\mathbf{y}_n$ as

$$p(\mathbf{y}_n) = C\exp\left(\frac{-1}{2}\left(\mathbf{y} - \mathbf{g}(\mathbf{x})\right)^\top \mathbf{R}^{-1}\left(\mathbf{y} - \mathbf{g}(\mathbf{x})\right)\right).$$

where C is the normalization constant.

Exercise 5. The weighting matrix $\mathbf{W}$ is diagonal with the diagonal element given by

$$\mathbf{W}_{i,i} = \lambda^{i-k}.$$