Microeconomic Theory 1
Helsinki GSE
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Problem Set 4 (Due Oct 14), 2019

1. MWG 6.C.20.

2. Prove that if a risk averse decision maker rejects a fixed favorable bet at all levels of wealth, then the Bernoulli utility of the decision maker is bounded from above.

3. A rational preference relation $\succeq$ satisfies betweenness if for all $p, q \in \mathcal{L}$ and all $\alpha \in (0,1)$, we have

$$p \succ q \;\Rightarrow\; p \succ \alpha p + (1-\alpha)\, q \succ q.$$

Show that betweenness is equivalent to the following condition. For all $p, q \in \mathcal{L}$ and all $\alpha \in (0,1)$, we have

$$p \sim q \;\Rightarrow\; p \sim \alpha p + (1-\alpha)\, q \sim q.$$

In other words, betweenness is equivalent to linearity of indifference curves in the simplex.

4. Consider the savings and consumption model analyzed in lectures. There are two periods, $t = 0, 1$. The decision maker has a strictly concave separable Bernoulli utility function

$$u(c_0, c_1) = u_0(c_0) + \delta u_1(c_1),$$

where c_t denotes consumption in period t. Assume that the consumer receives a certain income w_0 in period 0 and a random income $\widetilde{w_1}$ in period 1. The only means for transferring wealth between periods for the consumer is by either borrowing or lending at a risk free rate r.

(a) Set up the consumer's intertemporal budget constraint and characterize the solution to the savings problem through first order conditions (are these also sufficient conditions?).

(b) Consider the changes in optimal savings resulting from changes in interest rate r. Can you find an income and a substitution effect in your expression for $\frac{ds}{dr}$?

(c) Show that when the Arrow-Pratt coefficient of relative risk aversion is less than unity, savings increase in interest rate.

5. Consider the model of the previous exercise. Assume that $u_i(c_i) = \alpha + \beta c_i + (\gamma - c_i)^2$.

   (a) What is the range for possible consumptions where utitlity is increasing in consumption?

   (b) Assume that all the possible realizations from $\widetilde{w_1}$ lie in the range found in part a. Does the demand for savings depend on the riskiness of the distribution of $\widetilde{w_1}$?

6. Consider the following model of criminal behavior due to Becker. There is a continuum population of individuals considering whether to commit a crime. The resulting benefit from crime to an individual i is b_i. Assume that b is distributed in population according to the continuously differentiable strictly positive density function (on the entire real line) $g(b)$ and denote the corresponding cdf. by $G(b)$. If the individual commits a crime, then she will be caught with probability π and in this case she must pay a fine F.

   (a) Show first that there is a unique cutoff level b^* such that individual i commits the crime if and only if $b_i > b^*$.

   (b) Show next that b^* is increasing in π and F.

   (c) Suppose next that the individual has labor income w in case that no crime is committed. If caught in a crime, the individual must go to jail for fraction f of her total labor time. Then we have

$F = fw$. Show that if the coefficient of relative risk aversion is larger than 1, then b^* is increasing in w.

7. Consider an economy where all agents face an independent risk to lose 100 with probability p. N agents decide to create a mutual agreement where the aggregate loss in the pool is equally split among its members.

   (a) Describe the change in the lotteries facing individuals in the pool when N is changed from 2 to 3.

   (b) Show that the risk with $N = 3$ is smaller in the sense of second order stochastic dominance that the risk with $N = 2$.

8. (Harder) In models of oligopolistic competition, it is typical that the profit of a firm lagging behind the leader in the industry in terms of the quality of its product has a profit function that is first convex and then concave in any improvements to its own quality. R&D investments within a firm result normally in random improvements in the quality A possible way of modelling the R&D activity is by considering the choice of various types of projects, i.e. various distributions over quality improvements. With this as a motivation, consider the following decision model. The decision maker has a Bernoulli utility function $u(x)$ defined for $x \geq 0$, and there is an x_0 such that $u''(x) \geq 0$ for all $x \leq x_0$, and $u''(x) \leq 0$ for all $x \geq x_0$, and furthermore $u'(x) > 0$ for all x and $\lim_{x \to \infty} u'(x) = 0$. The decision maker chooses amongst all possible random distributions on $\mathbb{R}_+$.

   (a) For each possible expected value μ of the random variable $\widetilde{x}$, show that the expected utility maximizing distribution has at most two points in its support, and characterize the optimal distributions.

   (b) Suppose that here is a cost of increasing μ. More specifically, let $c(\mu)$ denote the cost, and assume that $c, c', c'' \geq 0$. Find the optimal distribution .