Microeconomic Theory I: Lectures 11-12

Juuso Välimäki

Helsinki GSE, Fall 2019

Risky Investment and SOSD

- A firm chooses investment level $a \in \mathbb{R}_+$
- Bernoulli utility $u(a, x)$, where $x \in [0, 1]$ is a random realization of (say random number of potential buyers.)
- Assume that $u_a(a, x) > 0$ and $u_{aa}(a, x) < 0$.
- $F(x; r)$ is the cdf distribution of X and r parametrizes the distributions.
- $F(x; r)$ second order stochastically decreasing in r. (I.e. r SOSD r' if $r < r'$.)
- Firm's problem: The problem is

$$\max_a v(a) := \int_0^1 u(a, x)\, dF(x, r).$$

Risky Investment and SOSD

- Derivative of the expected utility from choice a:

$$v'(a) = \int_0^1 u_a(a, x)\, dF(x, r).$$

- Note that

$$v''(a) < 0 \text{ since } u_{aa}(a, x) < 0.$$

- Comparative statics for optimal investment $a(r)$:
- If $u_a(a, x)$ is concave in u, then

$$\int_0^1 u_a(a(r), x)\, dF(x, r')\, dx \leq 0 \text{ for } r' \geq r.$$

- Since $v''(a)$ is concave, this implies that $a(r') < a(r)$.
- This implies that the optimal action is decreasing in r.
- The opposite conclusion follows if $u_a(a, x)$ is convex in x.

Choice Theory: Extensions and Challenges

Last lecture discussed notions of stochastic dominance.

These are important notions, but unfortunately they induce an very incomplete ordering of the set of monetary lotteries (or risks). (I.e. for many lotteries F and G, neither F dominates G nor G dominates F.)

Is there an alternative that produces a more complete ordering?

Aumann and Serrano: Index of Riskiness

- Aumann and Serrano (2008) propose a real-valued measure of risk for gambles
- Starting point: risk averse decision makers with strictly increasing and strictly concave twice differentiable Bernoulli utility functions.
- Consider a monetary lottery X with distribution G. Decision maker i accepts the gamble at initial wealth w if

$$\mathbb{E}_G(u(w+X)) \geq u(w).$$

- i totally rejects X if she rejects X at all w.
- A decision maker j is uniformly more risk averse than i if whenever j accepts a gamble at some w, i accepts that gamble at all w' or

$$r_A(w, u_i) \leq r_A(w', u_j) \text{ for all } w, w'.$$

Aumann and Serrano: Index of Riskiness

With these preliminaries, one can impose axioms to characterize a good index $\tilde{R}(X)$ for the riskiness of gamble X:

Axiom (Duality)

If i is uniformly more risk averse than j and i accepts X at w and if $\tilde{R}(X) > \tilde{R}(Y)$, then j accepts Y at w.

Axiom (Positive Homogeneity)

$\tilde{R}(tX) = t\tilde{R}(X)$ *for all* $t > 0$.

Let $R(X)$ be defined by $\mathbb{E}_G e^{-\frac{x}{R(X)}} = 1$. Aumann and Serrano prove that:

Theorem

$R(X)$ satisfies duality and positive homogeneity. Any index satisfying these axioms is a positive multiple of $R(X)$.

Aumann and Serrano: Index of Riskiness

How to think about the index?

Claim

Consider CARA utilities with coefficient α. Then $R(X)$ is the reciprocal of the CARA coefficient such that the decision maker is indifferent between accepting the gamble and not.

Sergiu Hart proves that Aumann and Serrano index can be though of in simple order terms:

Definition

X is riskier than Y denoted by $X \succeq_{TR} Y$ if any agent with a monotone utility function that totally rejects Y also totally rejects X.

Theorem

$X \succeq_{TR} Y$ *if and only if* $R(X) \geq R(Y)$.

Famous Paradoxes: Allais' Paradox

- Consider choice between the following two gambles:

$$A : \begin{cases} 2500 \text{ with probability } .33 \\ 2400 \text{ with probability } .66 \\ 0 \text{ with probability } .01 \end{cases}$$

$$B : 2400 \text{ with probability } 1.$$

- Consider next choice between

$$C : \begin{cases} 2500 \text{ with probability } .33 \\ 0 \text{ with probability } .67 \end{cases}$$

$$D : \begin{cases} 2400 \text{ with probability } .34 \\ 0 \text{ with probability } .66 \end{cases} .$$

- Can an expected utility maximizer have preferences: $B \succ A$ and $C \succ D$?
- Verify the answer by:
  1. writing the expected utility formulas for the two comparisons
  2. using the independence axiom directly.

Remedy: Betweenness or Rank-Dependent Utility

- How to account for this? Relax the independence axiom:
- Maybe the indifference lines on the space of lotteries are not parallel:
  1. Betweenness (Dekel, 1986)
  2. If $L \succ L'$, then $L \succ \alpha L + (1-\alpha)L' \succ L'$. If $L \sim L'$, then $L \sim \alpha L + (-\alpha)L' \sim L'$
- Maybe there is something funny with the probabilities?
- Rank Dependent Utility (Quiggin, 1982,1993)
  1. Maybe extreme outcomes get extreme attention as described by a probabilty weighting function w
  2. Suppose there is a function $w : [0,1] \to [0,1]$ with $w(0) = 0, w(1) = 1$ to be applied to the cdf F of the lottery.
  3. Utility of a lottery with probabilities p on outcomes x given by:

$$U(p) = \int u(x) dw(F(x))).$$

  4. Exercise: find a weighting function w that rationalizes the choices in Allais' paradox.

Famous Paradoxes: Ellsberg's Paradox

- An urn contains three balls. One is red, the other two either green or blue.
- A ball is drawn and you are asked to guess its color. You get a prize worth 1 if you guess right.
- What would you guess?
- Consider next a variant of the above problem This time you get a prize if you guess wrong i.e. if the ball does not match your guess.
- What would you guess now?
- Is this consistent with probabilistic beliefs on the colors.
- Behavior of this type is said to show ambiguity aversion

Remedy: Ambiguity Averse Preferences

- How to model ambiguity aversion?
- A set of prior distributions (Gilboa and Schmeidler, 1989)
  - Let Δ denote the set of prior
  - Lotteries are evaluated according to $\min_{p\in\Delta}\sum_x u(x)p(x)$.
- Suppose in the motivating example Δ is set of beliefs where $p_R = 1/3$, p_B and p_G can be anything between 0 and $2/3$ (that add to $2/3$).
- For first question:

$$U(R) = \min_{p\in\Delta} p_R = \frac{1}{3},$$

$$U(B) = \min_{p\in\Delta} p_B = 0 = U(G) = \min_{p\in\Delta} p_G.$$

- For the second:

$$U(R) = \min_{p\in\Delta}(1 - p_R) = \frac{2}{3},$$

$$U(B) = \min_{p\in\Delta}(1 - p_B) = \frac{1}{3} = U(G) = \min_{p\in\Delta}(1 - p_G).$$

Remedy: Ambiguity Averse Preferences

- Smooth ambiguity aversions: Klibanoff,Marinacci and Mukherji, 2005
  - Evaluate utility in a decision situation by the double sum

$$\sum_{\pi} \mu(p) \phi\left(\sum_{x} p(x) u(x)\right),$$

  - where μ is the prior distribution over the probabilities p for the lotteries, each p defines a simple lottery on X, u represents risk attitudes, ϕ represents attitudes towards ambiguity. Linear ϕ takes us back to expected utility framework.

Remedy: Ambiguity Averse Preferences

How to evaluate the model of ambiguity averse preferences?

- If preferences are ambiguity averse, then decision makers may have a strict preference for randomizations
  - To see this, consider state dependent utility in a model with two states ω_0, ω_1.
  - Two investment opportunities $i = 0, 1$ and investment i yields a unit return in state ω_i and zero otherwise.
  - Randomized investment dominates deterministic investments.
- There is an ongoing debate about inherent preference for randomizations when faced with ambiguous decision problems
- What are the effects of ambiguity averse preferences for real market decisions
  - Are the predictions distinct from expected utility theory?
  - Do we have empirical evidence? Not clear.

Rabin's 'Paradox' for Risk Aversion in the Small

- Consider risk aversion as an explanation for declining small favorable risks
- Suppose a decision maker rejects a gamble that wins EUR 11 w.p. $\frac{1}{2}$ and results in a loss of EUR 10 w.p. $\frac{1}{2}$ at all levels of wealth.
- What do we know about this decision maker?

$$u(w+11) - u(w) < u(w) - u(w-10)$$

  for all w.
- Since u is concave,

$$u'(w+11) \leq \frac{u(w+11)-u(w)}{11} < \frac{u(w)-u(w-10)}{10} < u'(w-1$$

- Therefore $\frac{u'(w+11)}{u'(w-10)} < \frac{10}{11}$.
- But this implies that the slope of u decreases quite quickly and in fact also that u is bounded from above.
- How big should the winnings in a 50-50 gamble be for the decision maker to accept it if losses are EUR 200?

Rabin's 'Paradox' for Risk Aversion in the Small

- Is this really surprising?
- Recall that all risk-averse decision makers should accept a little favorable bet
- Yet in experimental situations, subjects turn down small favorable bets
- Maybe this is not because of risk-aversion but avoidance of losses
- Prospect theory of Kahnemann and Tversky assumes that losses are more important than gains: kink at zero gain
  - Their model includes also probability weighting and other features (decreasing sensitivity as losses or gains become larger)

Commitment, Temptation and flexibility

- Is choice over time a decision problem or a game?
- Commodities are (in principle) differentiated by time, location and contingency. Why couldn't we treat decision makers in the same way?
- Will your preferences tomorrow regarding future choices congruent with your preferences today over those same choices?
- Example: Do you want one apple today or two apples tomorrow? Do you want an apple in 30 days or two apples in 31 days?
- Suppose there is some conflict between the preferences over choices at different points in time.
- Then an intertemporal choice problem becomes a game between decision makers at different times.
- How can we capture such preference reversals in a simple model?

Hyperbolic Discounting

- The $\beta - \delta$ -model
- Standard model: Let x_t be the choice in period t.
- Let $\mathbf{x}$ be the sequence of choices and $\mathbf{x_s}$ the sequence of choices from period s onwards.

$$U(\mathbf{x}) = \Sigma_t \delta^t u(x_t).$$

- Hyperbolic discounting (Strotz, REStud 1955):

$$U(\mathbf{x_s}) = u(x_s) + \beta \Sigma_{t>s} \delta^{t-s} u(x_t),$$

  for some $\beta < 1$.
- Notice how this implies a change in perspective as time goes on.
- All other periods apart from the present are discounted by β on top of the usual δ.
- Present bias: in t the MRS between t and $t+1$is $\beta\delta$ whereas between $t+k$ and $t+k+1$ it is δ.

Hyperbolic Discounting

Example

- Take $\beta = \frac{1}{2}$, $\delta = 1$
- You decide sequentially when to harvest a resource that yields utility $u(x_0) = 3$, $u(x_1) = 5$, $u(x_2) = 9$, $u(x_3) = 16$.
- When should you harvest?

Hyperbolic Discounting

- How does this depend on what you know about your future behavior?
- Would you like to commit to a plan of action at $t = 0$?
- How to evaluate welfare? Pareto-efficiency between all multiple or what?
- A big literature on this model. Topics include:
  - Saving for retirement (Laibson).
  - Addiction (O'Donoghue and Rabin)
  - Deadlines in optimal contracts (O'Donoghue and Rabin).

Preference over Menus

- Are there alternative explanations for preference for commitment?
- Temptation and Self-Control by Gul and Pesendorfer, 2001
- Standard neoclassical preference model on an extended domain
- Let $\mathcal{L}$ denote the set of all possible lotteries. $X, Y \subset \mathcal{L}$ are subsets of lotteries called menus. $\mathcal{M}$ denotes the set of all possible menus.
- Preferences are defined on $\mathcal{M}$.
- Preferences are rational, satisfying a form of continuity and independence axiom for singleton menus.

Preference over Menus

In this setting, new cases can be covered:

- Preference for flexibility (Kreps, 1979):

$$\text{If } X \subset Y, \text{ then} Y \succeq X.$$

- Even if $x \succeq z$ for all $z \in Y$ and $z \in X$, we can have $Y \succ X$.
- Temptation (Set Betweenness):

$$\text{If } X \succeq Y, \text{ then } X \succeq X \cup Y \succeq Y.$$

- Why is this axiom related to temptation?

Preference over Menus

Theorem (Gul and Pesendorfer)

The binary relation $\succeq$ on $\mathcal{M}$ satisfies rationality, continuity, independence and set betweenness if and only if there are continuous linear functions U, u, v such that

$$U(X) := max_{x \in X}(u(x) + v(x)) - max_{y \in X} v(y)$$

for all X and U represents $\succeq$.

- This formulation also gives rise for a preference for commitment.
- Interpretation: u is the standard utility from choices, v is the temptation utility.
- $v(x) - \max_{y \in X} v(y)$ is the (utility) cost of temptation
- What about a decision maker that always gives in to temptation (i.e. without self-control)?

Preference over Menus

Note the differences to hyperbolic discouning:

- A single decision maker.
- Welfare comparisons much easier.
- How to think about this in a dynamic framework?
  - Each choice now results in utility and a new decision problem
  - Idea: consider preferences on decision problems and look for a recursive formulation
  - Done in Gul and Pesendorfer, 2004

Stochastic Choice: Model

Setting is similar to choice from menus:

- Set of alternatives X with a typical alternative $x \in X$
- Menu $A \subset X$
- For $x \in A$ the probability of choosing x from A is $\rho(x, A)$
- The new feature is this probabilistic choice rule ρ

A really good source for an overview is Strzalecki's Hotelling Lectures that I have also used here.

Stochastic Choice: Model

Idea: The analyst/econometrician observes an agent/group of agents

- Population-level field data: McFadden (1973)
- Individual-level field data: Rust (1987)
- Between-subjects experiments: Kahneman and Tversky (1979)
- Within-subject experiments: Tversky (1969)

Stylized Fact: Choice can change, even if repeated shortly after

Possible reasons:

- Randomly fluctuating tastes
- Noisy signals
- Attention is random
- People just like to randomize
- Trembling hands
- Experimentation (experience goods)

Stochastic Choice: Model

Goals

1. Better understand the properties of a model. What kind of predictions does it make? What axioms does it satisfy?
   - Ideally, prove a representation theorem (ρ satisfies Axioms A and B if and only if it has a representation R)
2. Identification: Are the parameters pinned down uniquely?
3. Determine whether these axioms are reasonable, either normatively, or descriptively (testing the axioms)
4. Compare properties of different models (axioms can be helpful here, even without testing them on data). Outline the modeling tradeoffs
5. Estimate the model, make a counterfactual prediction, evaluate a policy

General Challenges to Choice Theory: Non-Selfish Behavior

Are decision makers really selfish individualists?

- Social Preferences
  - What do we mean by individualistic approach?
  - Are we assuming and perhaps even endorsing selfishness?
  - What if decision makers care about others?
    - Altruism
    - Envy
    - Preference for equality Are these covered by the standard model of choice?
  - Recall: Preferences are defined over an abstract set X
  - Could be individual consumption levels
  - Could include the social allocations
  - Hence nothing in the neoclassical standard model precludes social preferences.

General Challenges to Choice Theory: Framing Effects

- What you get depends on how you ask: Framing in action
- A new strand of flu is going to hit Finland and kill 600 people for sure if nothing is done. There are two alternative policies to tackle the outbreak:
  - Policy A saves 200 individuals for sure.
  - Policy B has saves nobody with probability $\frac{2}{3}$ and saves 600 with probability $\frac{1}{3}$.
  - which policy is preferable?
- Consider next two other alternatives:
  - Policy C leaves 400 dead for sure
  - Under Policy D, there is a probability $\frac{2}{3}$ that 600 die and a probability $\frac{1}{3}$ that no one dies.
  - Which of the two policies is preferable?
- Unfortunately not much can be done about these effects (just keep in mind when interpreting experiments etc.)