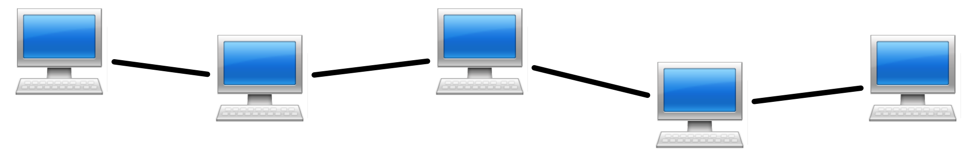
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
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  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
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- **Week 12: recap**

# Week 11

– Applications of Ramsey’s theorem

# Ramsey's theorem

- **For all $c$, $k$, $n$ there are numbers $R_c(n; k)$ s.t.: if we have $N \geq R_c(n; k)$ elements and we label each $k$-subset with one of $c$ colours, there is a monochromatic subset of size $n$**

## $\bar{R}_c(n; 2)$?

$\bar{R}_c(2; 2) = 2$

$M = \bar{R}_c(2; 2)$
$\bar{R}_c(3; 2) \leq 1 + R_c(M; 1)$

$M = \bar{R}_c(3; 2)$
$\bar{R}_c(4; 2) \leq 1 + R_c(M; 1)$

$\ldots$

## $\bar{R}_c(n; 3)$?

$\bar{R}_c(3; 3) = 3$

$M = \bar{R}_c(3; 3)$
$\bar{R}_c(4; 3) \leq 1 + R_c(M; 2)$

$M = \bar{R}_c(4; 3)$
$\bar{R}_c(5; 3) \leq 1 + R_c(M; 2)$

$\ldots$

## $R_c(n; 1)$?

$R_c(n; 1) \leq c \cdot (n-1)+1$

## $R_c(n; 2)$?

$M = R_c(n; 1)$
$R_c(n; 2) \leq \bar{R}_c(M; 2)$

## $R_c(n; 3)$?

$M = R_c(n; 1)$
$R_c(n; 3) \leq \bar{R}_c(M; 3)$

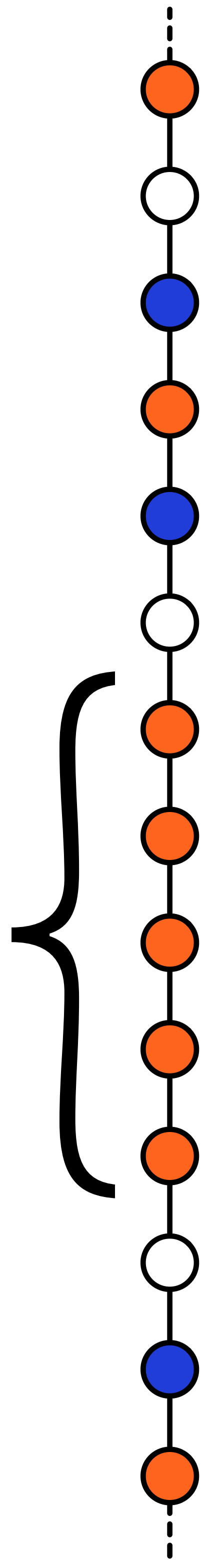
# Applications of Ramsey's theorem

- **Application for $k = 2$, $c = 2$:**
  **any graph with $N$ nodes contains**
  **an independent set or a clique of size $n$**

# Applications of Ramsey's theorem

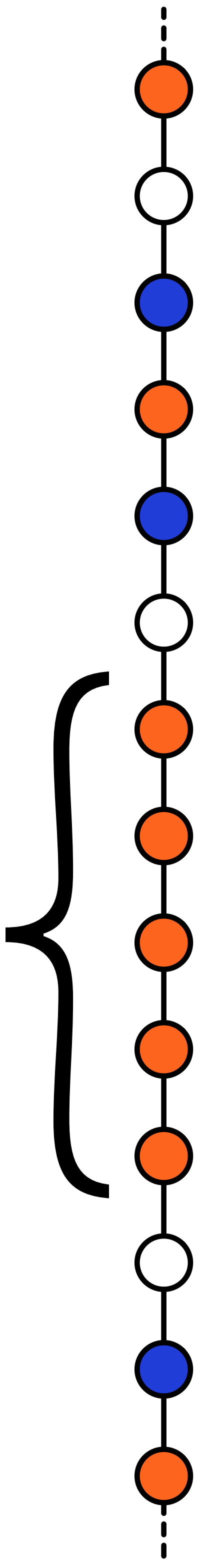
- **Application: negative results for the LOCAL model**

- **For any constant-time algorithm *A*, we can construct a bad input *G* such that there is a large region of nodes with the same output**



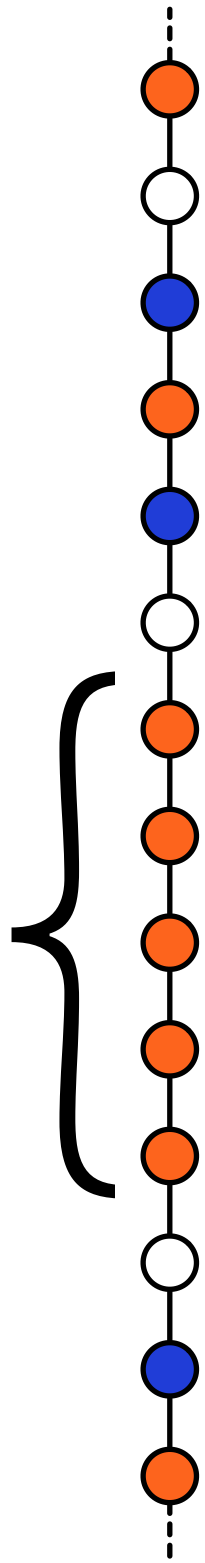
# Applications of Ramsey's theorem

- **For any constant-time algorithm *A*, we can construct a bad input *G* such that there is a large region of nodes with the same output**
  - some technical assumptions, see exercises for details…

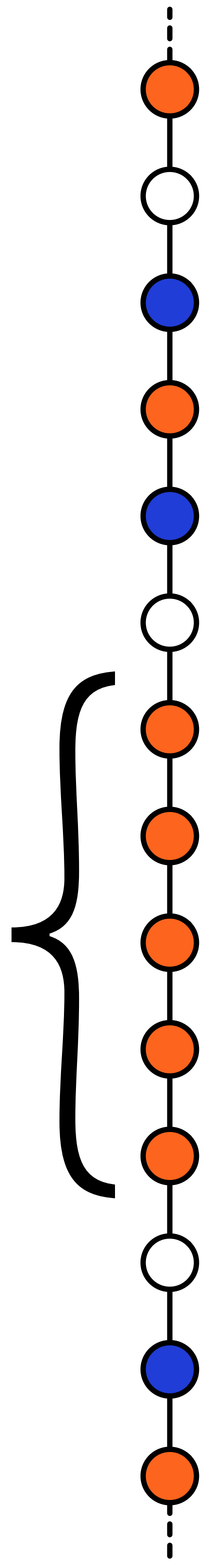


# Applications of Ramsey's theorem

- **For any constant-time algorithm *A*, we can construct a bad input *G* such that there is a large region of nodes with the same output**
  - no constant-time algorithms for vertex colouring, edge colouring, maximal independent sets, …



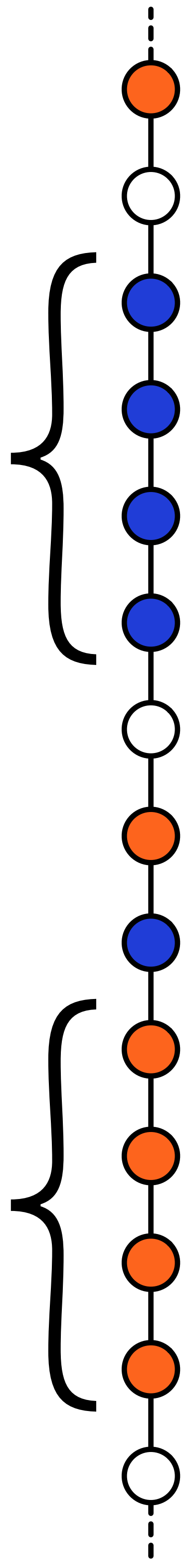
# Applications of Ramsey's theorem



- **We already know all (?) this from week 2**
- **However, Ramsey's theorem has further applications!**

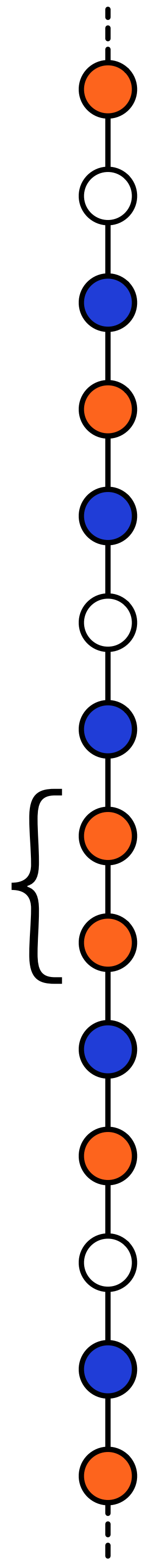
# Applications of Ramsey's theorem

- **For any constant-time algorithm *A*, we can construct a bad input *G* such that there are lots of regions of nodes with the same output**
  - no constant-time algorithms for large independent sets, large matchings, …



# Applications of Ramsey's theorem

- Generalisations in exercises…
- We will now just prove a simple special case: vertex colouring not possible in the LOCAL model with constant-time algorithms



# Vertex colouring and Ramsey's theorem

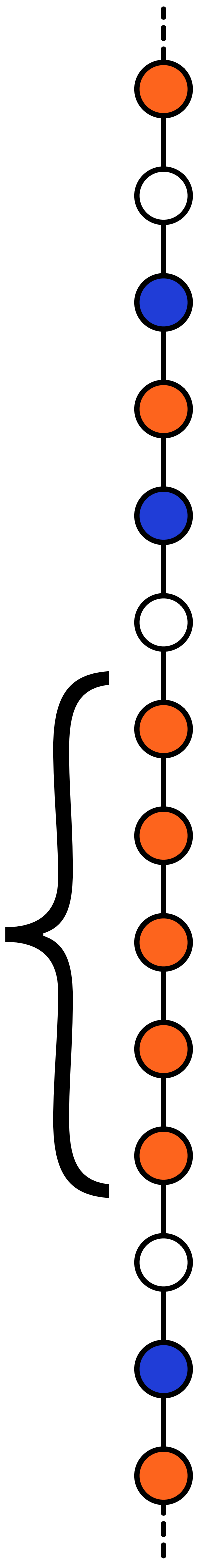
- **Assume:** algorithm $A$ runs in time $T = O(1)$ and outputs values 1, 2, 3
- **Claim:** there is a cycle $G$ with unique identifiers such that $A$ does not find a vertex colouring

# Vertex colouring and Ramsey's theorem

- **Assume: algorithm $A$ runs in time $T = O(1)$ and outputs values 1, 2, 3**
- **Let: $n = 2T + 2$, $k = 2T + 1$, $c = 3$, $N = R_c(n; k)$**
- **Use $A$ to label $k$-subsets of $\{1, 2, \ldots, N\}$**
- **Monochromatic subset → bad output**

# Applications of Ramsey's theorem

- ***O*(1)-time algorithms cannot do much**
  - even if we have unique identifiers
- ***O*(log* *n*)-time algorithms much more powerful:**
  - can find colourings, break symmetry, find large independent sets, …



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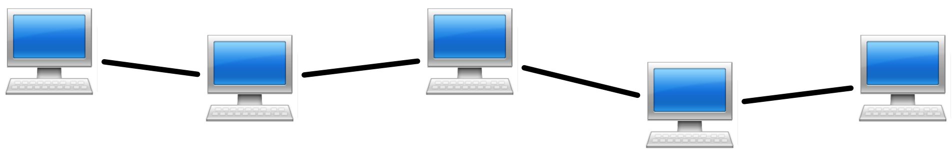
    - network = path 
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