

## ELEC-C8201 Control and Automation

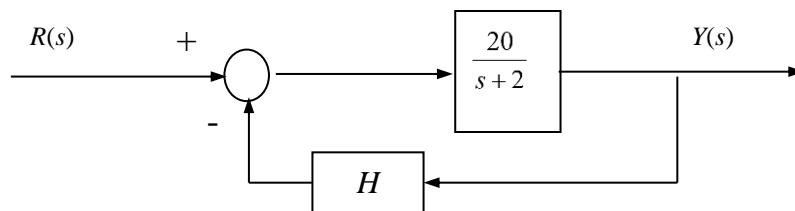
### Exercise 3

- The system is given by the differential equation:  $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 3u(t)$ 
  - Specify the system transfer function  $G(s) = Y(s)/U(s)$
  - Find output  $y(t)$ , when input  $u(t)$  is a unit impulse function.
  - Find output  $y(t)$ , when input  $u(t)$  is a unit step function.
  - Find output  $y(t)$ , when input  $u(t)$  is a unit ramp function.
- Construct the differential equation and transfer function from the following state space representation.

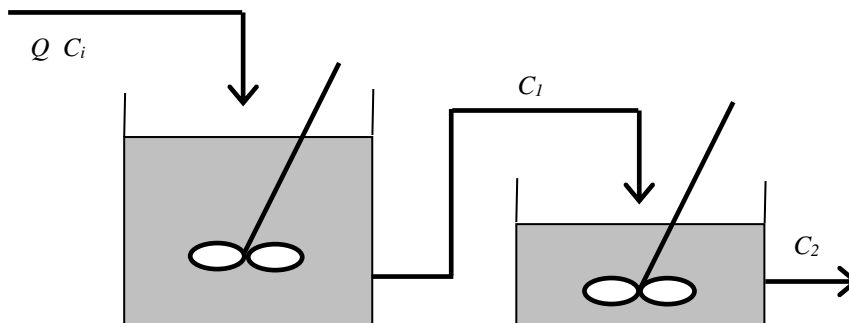
$$\text{a. } \begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 4 \\ 10 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t) \end{cases}$$

$$\text{b. } \begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} \mathbf{x}(t) \end{cases}$$

- Calculate the system transfer function in the image below  $Y(s)/R(s)$ , when  $H = 0.4$  and  $H = 0.9$ . Graphically show the locations of the closed loop poles and calculate the step and impulse responses.



- The picture shows two ideal mixers in series. Concentrations  $C_i(t)$ ,  $C_1(t)$  and  $C_2(t)$  are variables, and mixer volumes  $V_1$ ,  $V_2$  and volume flow  $Q$  are constant parameters ( $V_1 = 0.5 \text{ m}^3$ ,  $V_2 = 0.2 \text{ m}^3$  ja  $Q = 1 \text{ m}^3/\text{s}$ ). The input of the total process is  $C_i(t)$  and output is  $C_2(t)$ .



Construct a differential equation, a state space representation and a transfer function describing the process and calculate:

- Unit impulse response
- Unit step response

- c.** the steady state value of step response
- d.** Static gain
- e.** Weighting function i.e. time domain impulse response