Evolutionary dynamics exercises

Topics in Complex Systems

May 2020

Return your solutions as a pdf (they don't need to be very detailed) to the exercise return box on MyCourses.

1 Replicator equation

Consider the replicator equation: $\frac{d}{dt}x_i = x_i[f_i(\boldsymbol{x}) - \phi(\boldsymbol{x})]$, where $\phi(\boldsymbol{x}) = \sum_{j=1}^{N} x_j f_j(\boldsymbol{x})$ and $x_i \in [0,1]$ are the relative abundances. A simple case is when the fitness function of species i is a constant that does not depend on the abundances of the species in the system, $f_i(\boldsymbol{x}) = c_i$. Now, let $c_1 > c_2 > c_3 > c_4 > 0$ in a four-species system. What happens to the system as time progresses? How does the weighted average fitness $\phi(\boldsymbol{x})$ evolve over time? What is the end state at $t \to \infty$?

2 Jain-Krishna model

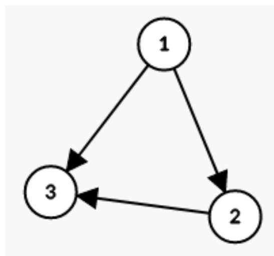
Go to `https://csh.ac.at/vis/Jain-Krishna/`. There is a simulator for the Jain-Krishna model. You can see a visualization of the network defined by the interaction matrix M, its Perron-Frobenius eigenvalue λ_1, and the sizes of the components of the Perron-Frobenius eigenvector $\boldsymbol{v}_1$ (when $\lambda_1 > 0$, when $\lambda_1 = 0$ the eigenvector should still be nonzero by definition, but the simulator does not show that).

The simulator has three parameters: average/expected degrees $\langle k_{in} \rangle = \langle k_{out} \rangle$ are related to the probability p of forming new links ($\langle k_{in} \rangle = \langle k_{out} \rangle = (N-1)p$). Pool adjusts the set among which the node to be removed is chosen from (in the lecture slides version only the least-fitness node is removed, so this parameter is 1). Rate defines the speed of the simulation. You might want to set rate to a lower number; to do this, you might need to zoom the web page. To reset the model, reload the page.

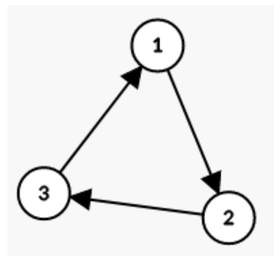
Play around with the parameters and see what happens. Why is $\langle k_{in} \rangle = \langle k_{out} \rangle = 1$ a relevant threshold, and why do we want to initialize the network such that it is below 1? What happens when the value is set to 0? What happens when the value is increased from close to 0 to close to 1? Is the relationship between mass extinctions and the parameter linear or nonlinear?

3 Perron-Frobenius eigenvalue of an autocatalytic cycle

Let 1, 2, and 3 be our species. Let M_1 be the adjacency matrix of the interaction network in Figure 1a (which doesn't contain an autocatalytic cycle), and let M_2 be the adjacency matrix of the interaction network in Figure 1b (which contains an autocatalytic cycle). Show that $\lambda_1 = 0$ for M_1 and $\lambda_1 = 1$ for M_2.



(a)



(b)

Figure 1