

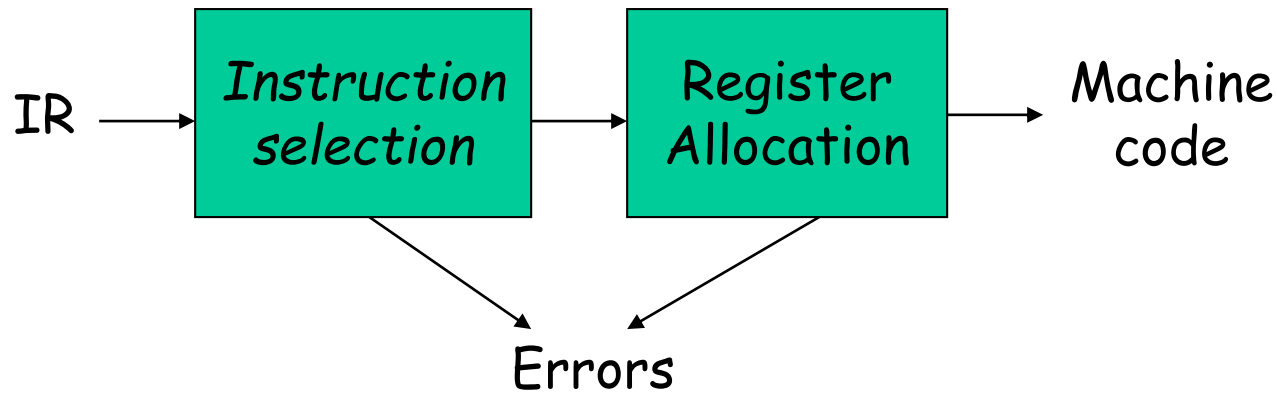
Register Allocation

Lecture 11

Lecture Outline

- Register Allocation
 - Register interference graph
 - Graph coloring heuristics
 - Spilling
- Cache Management

Back-End (Revisited)



Back-End:

- Translate IR into machine code
- Choose instructions for each IR operation
- Decide what to keep in registers at each point

The Register Allocation Problem

- Intermediate code uses unlimited temporaries
 - Simplifies code generation and optimization
 - Complicates final translation to assembly
- Typical intermediate code uses too many temporaries

The Register Allocation Problem (Cont.)

- The problem:

Rewrite the intermediate code to use no more temporaries than there are machine registers

- Method: Many temps to one
→
 - Assign multiple temporaries to each register
 - But without changing the program behavior

An Example

- Consider the program

```
a := c + d
e := a + b
f := e - 1
```

Many to one mapping 

```
r1 := r2 + r3
r1 := r1 + r4
r1 := r1 - 1
```

- Assume **a** and **e** dead after use
 - Temporary **a** can be “reused” after **e := a + b**
 - So can temporary **e**

- A dead temporary is not needed
 - A dead temporary can be reused

History

- Register allocation is as old as compilers
 - Register allocation was used in the original FORTRAN compiler in the '50s
 - Very crude algorithms
- A breakthrough came in 1980
 - Register allocation scheme based on graph coloring
 - Relatively simple, global and works well in practice

The Idea

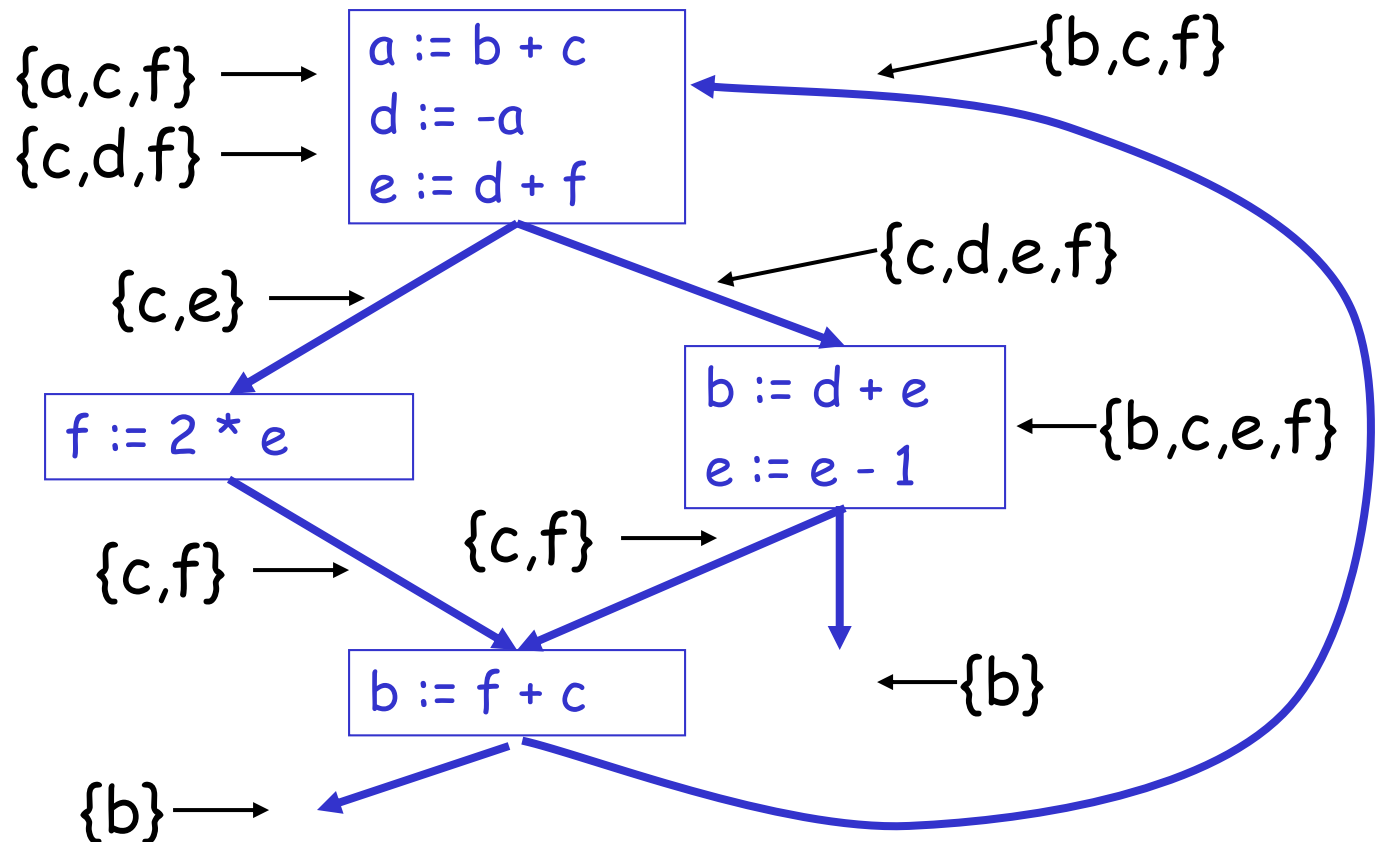
Temporaries t_1 and t_2 can share the same register if at any point in the program at most one of t_1 or t_2 is live .

Or

If t_1 and t_2 are live at the same time, they cannot share a register

Algorithm: Part I

- Compute live variables for each point:

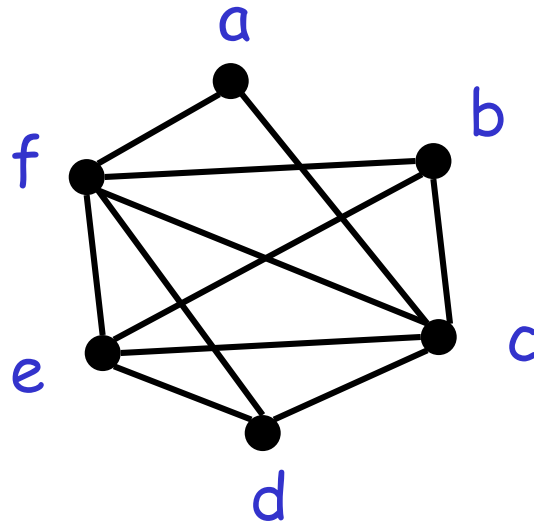


The Register Interference Graph

- Construct an undirected graph
 - A node for each temporary
 - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the *register interference graph (RIG)*
 - Two temporaries can be allocated to the same register if there is no edge connecting them

Example

- For our example:



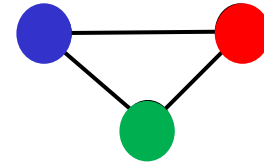
- E.g., **b** and **c** cannot be in the same register
- E.g., **b** and **d** could be in the same register

Notes on Register Interference Graphs

- Extracts exactly the information needed to characterize legal register assignments
- Gives a global (i.e., over the entire flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent
 - It does not depend on any property of the machine except for the number of registers

Definitions

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors



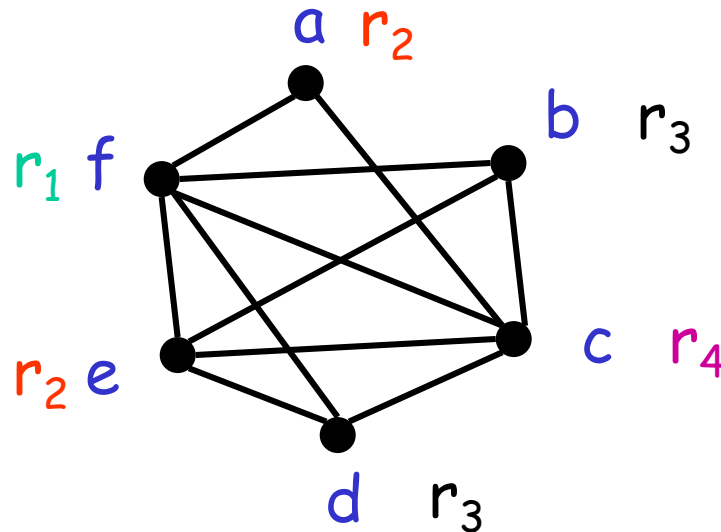
- A graph is k-colorable if it has a coloring with k colors

Register Allocation Through Graph Coloring

- In our problem, colors = registers
 - We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k -colorable then there is a register assignment that uses no more than k registers

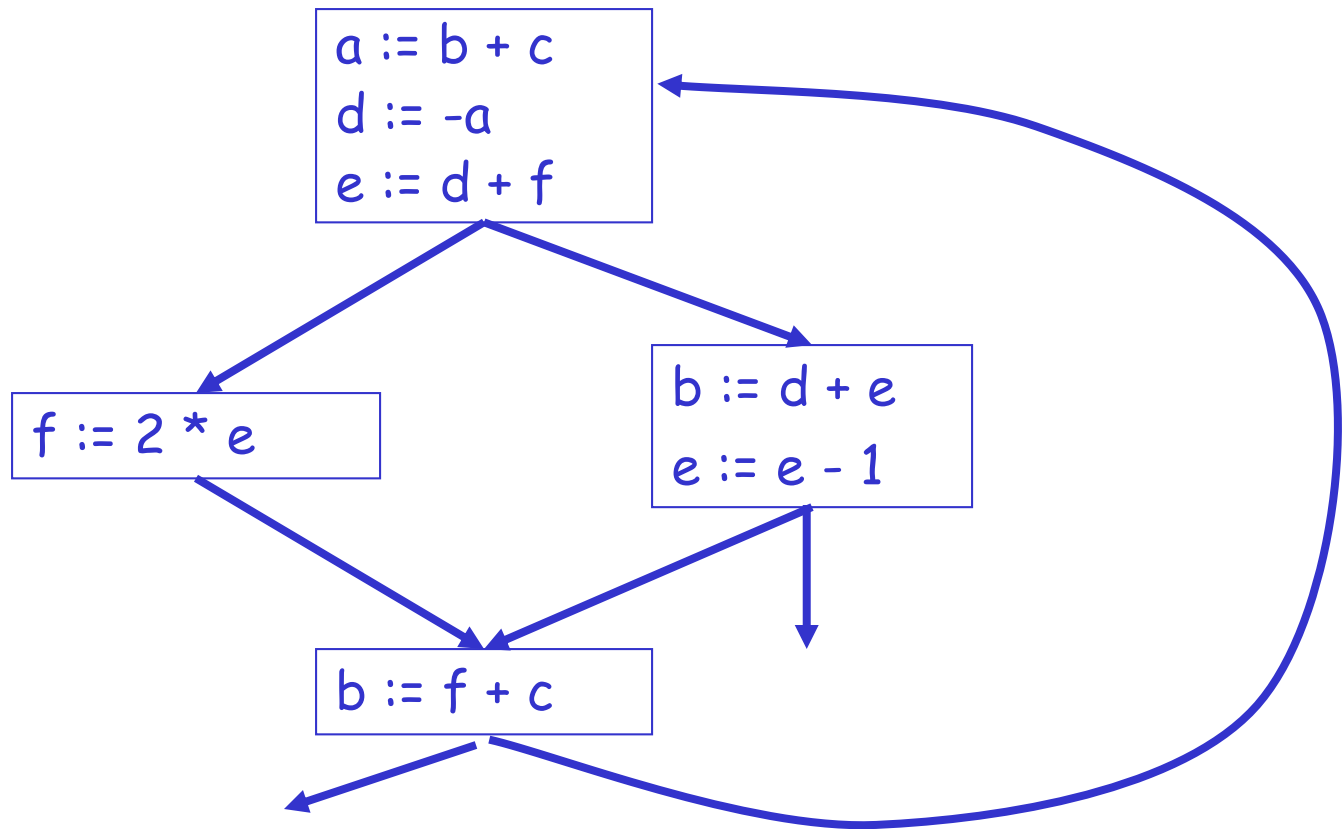
Graph Coloring Example

- Consider the example RIG



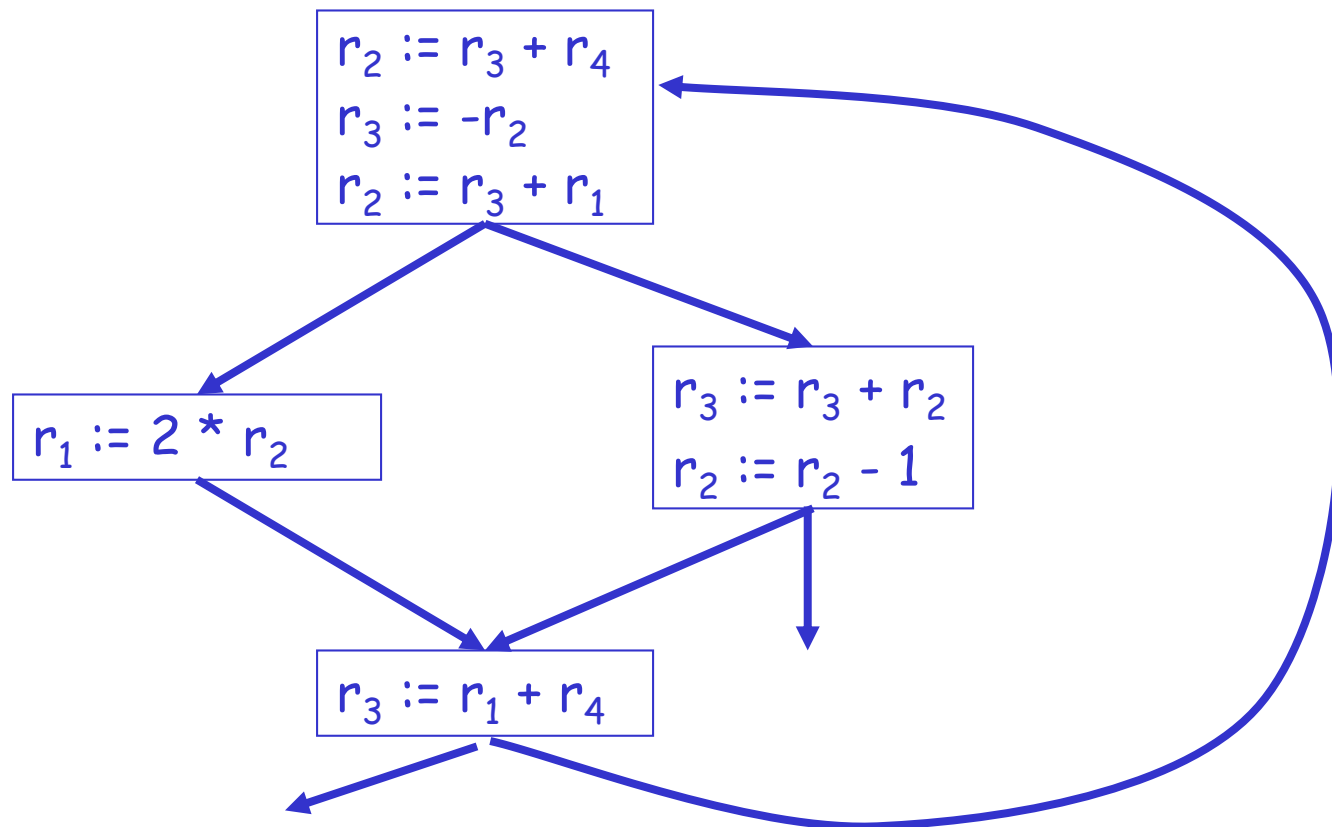
- There is no coloring with less than 4 colors
- There are 4-colorings of this graph

Example Review



Example After Register Allocation

- Under this coloring the code becomes:

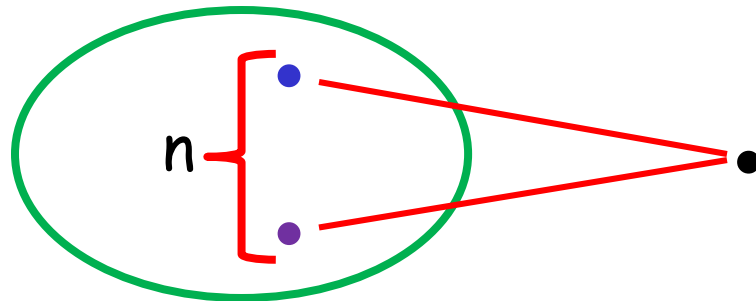


Computing Graph Colorings

- How do we compute graph colorings?
- It isn't easy:
 1. This problem is very hard (NP-hard). No efficient algorithms are known.
 - *Solution: use heuristics*
 2. A coloring might not exist for a given number of registers
 - *Solution: later*

Graph Coloring Heuristic

- Observation:
 - Pick a node t with fewer than k neighbors in RIG
 - Eliminate t and its edges from RIG
 - If resulting graph is k -colorable, then so is the original graph



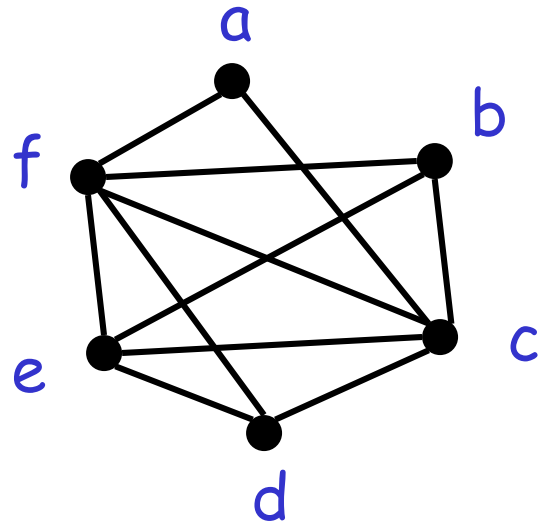
- Why?
 - Let c_1, \dots, c_n be the colors assigned to the neighbors of t in the reduced graph
 - Since $n < k$ we can pick some color for t that is different from those of its neighbors

Graph Coloring Heuristic

1. The following works well in practice:
 - Pick a node t with fewer than k neighbors
 - Put t on a stack and remove it from the RIG
 - Repeat until the graph has one node
2. Assign colors to nodes on the stack
 - Start with the last node added
 - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring Example (1)

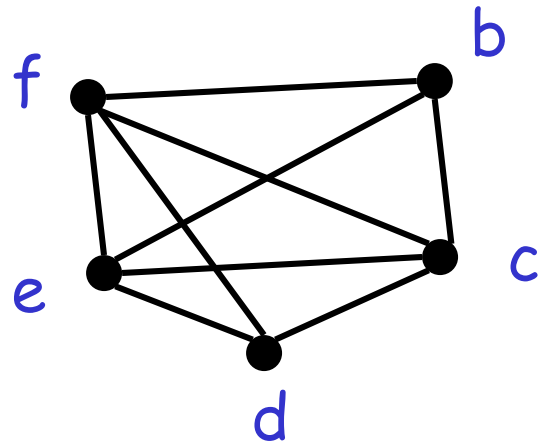
- Start with the RIG and with $k = 4$:



Stack: {}

- Remove a

Graph Coloring Example (2)

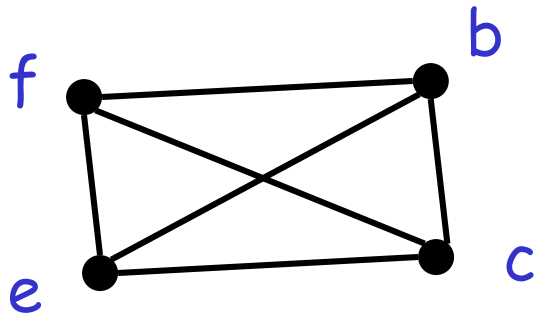


Stack: {a}

- Remove d

Graph Coloring Example (3)

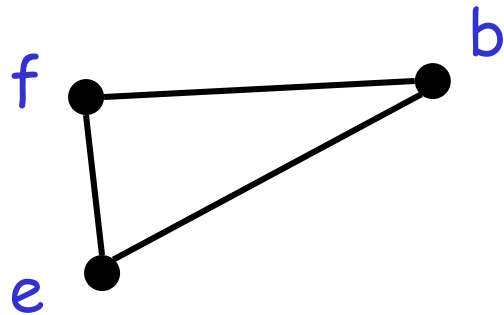
- Note: all nodes now have fewer than 4 neighbors



Stack: {d, a}

- Remove c

Graph Coloring Example (4)



Stack: {c, d, a}

- Remove b

Graph Coloring Example (5)



Stack: {b, c, d, a}

- Remove e

Graph Coloring Example (6)

f ●

Stack: {e, b, c, d, a}

- Remove f

Graph Coloring Example (7)

- Empty graph - done with the first part!

Stack: {f, e, b, c, d, a}

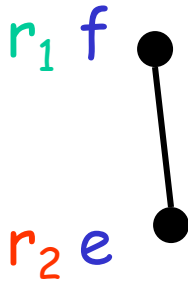
- Now start assigning colors to nodes, starting with the top of the stack

Graph Coloring Example (8)

r_1 f ●

Stack: {e, b, c, d, a}

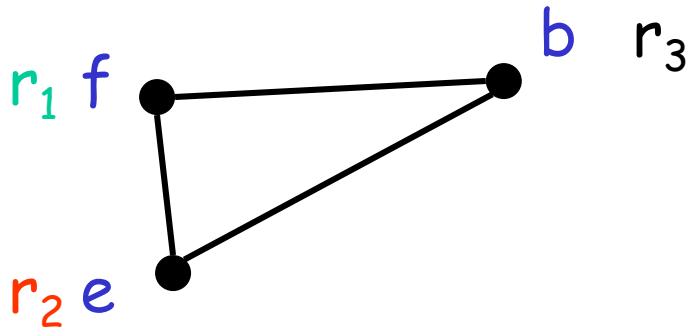
Graph Coloring Example (9)



Stack: {b, c, d, a}

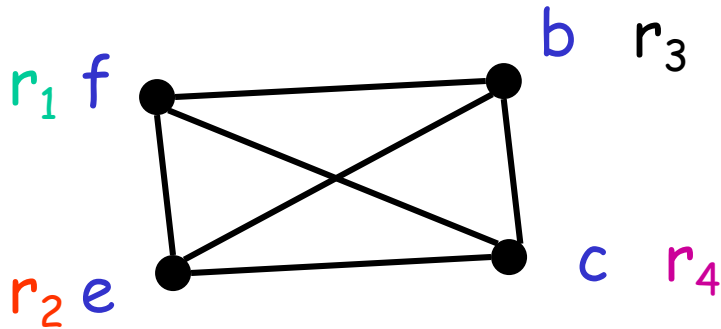
- e must be in a different register from f

Graph Coloring Example (10)



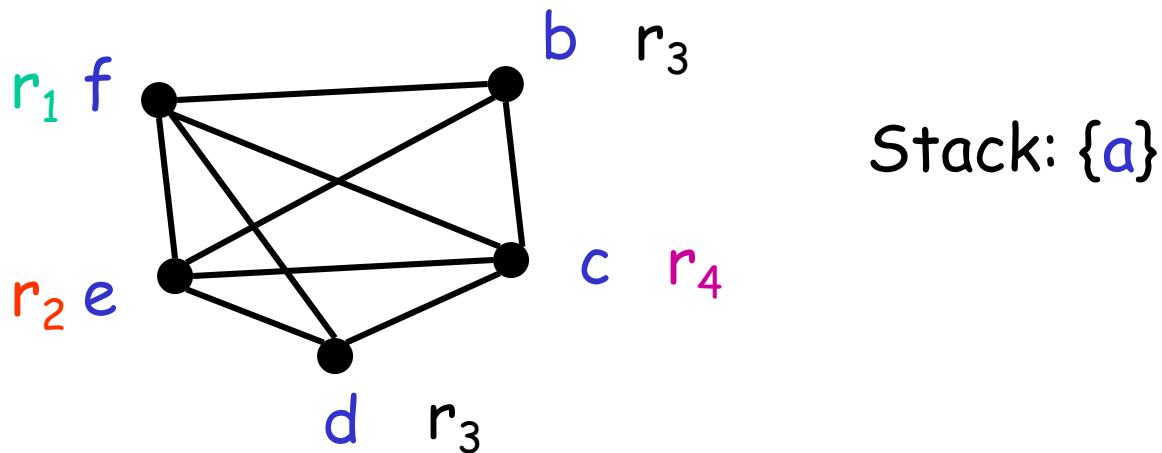
Stack: {c, d, a}

Graph Coloring Example (11)



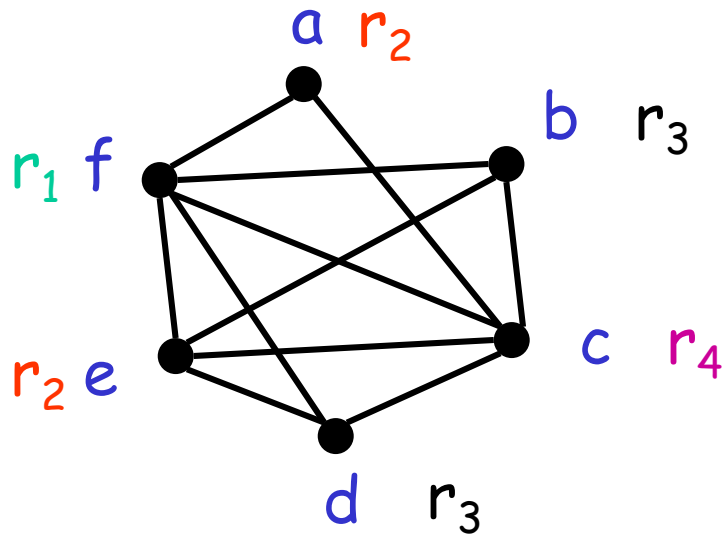
Stack: {d, a}

Graph Coloring Example (12)



- d can be in the same register as b

Graph Coloring Example (13)

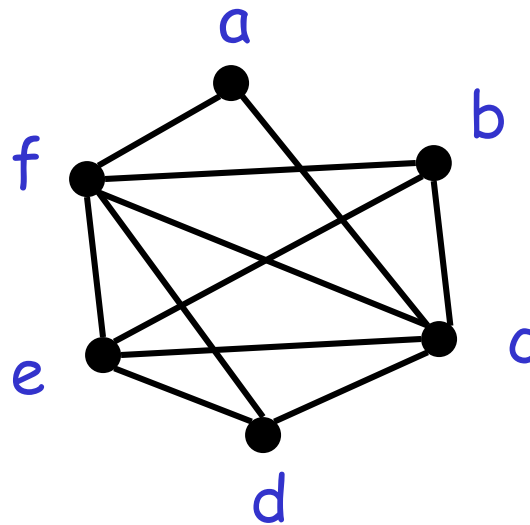


What if the Heuristic Fails?

- *What happens if the graph coloring heuristic fails to find a coloring?*
- *In this case, we can't hold all values in registers.*
 - *Some values are spilled to memory*

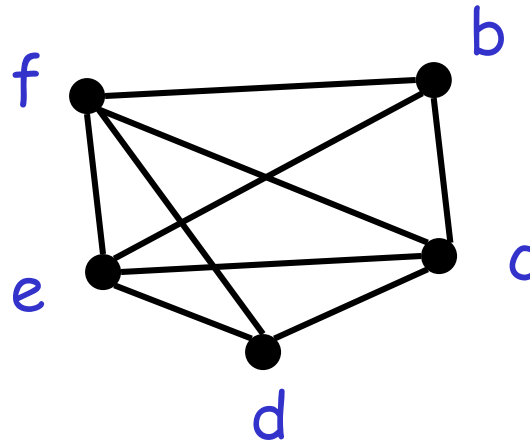
What if the Heuristic Fails?

- What if all nodes have k or more neighbors ?
- Example: Try to find a 3-coloring of the RIG:



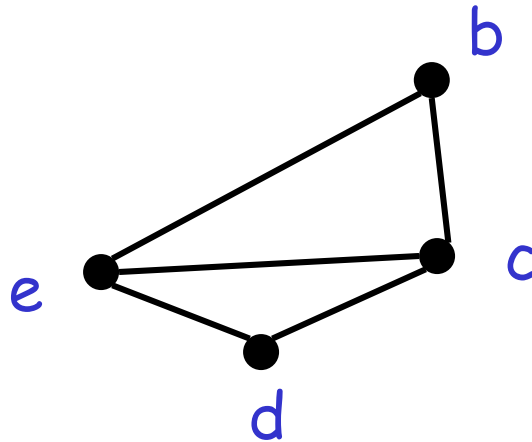
What if the Heuristic Fails?

- Remove **a** and get stuck (as shown below)
 - There is no node with fewer than 3 neighbors
- Pick a node as a candidate for *spilling*
 - A spilled temporary “lives” in memory
 - Assume that **f** is picked as a candidate



What if the Heuristic Fails?

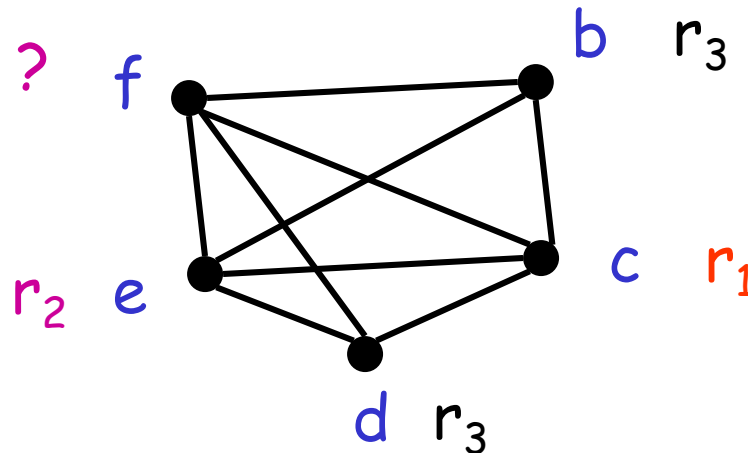
- Remove **f** and continue the simplification
 - Simplification now succeeds: **b, d, e, c**



What if the Heuristic Fails?

- Eventually we must assign a color to **f**
- We hope that among the 4 neighbors of **f** we use less than 3 colors \Rightarrow optimistic coloring

In this ex., it doesn't work

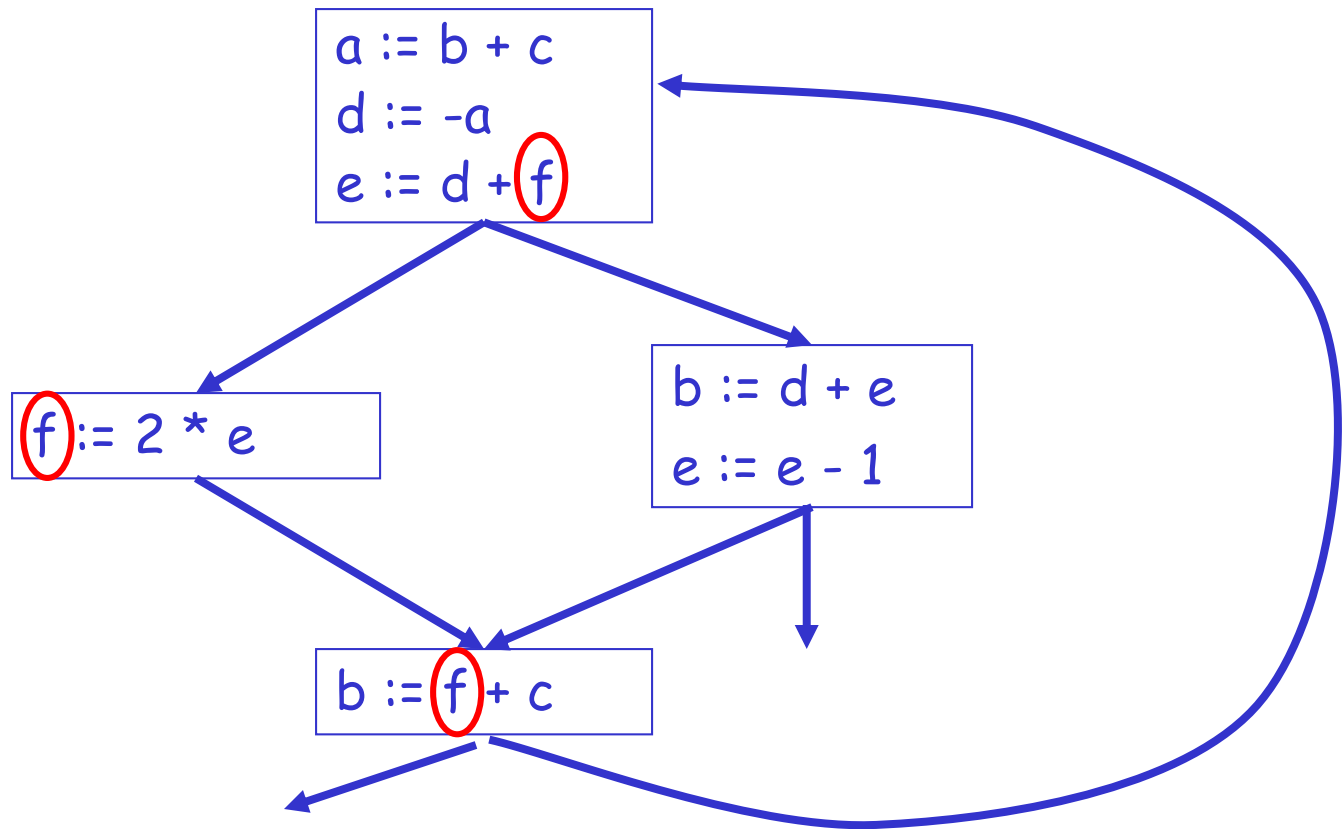


Spilling

- If optimistic coloring fails, we spill f
 - Allocate a memory location for f
 - Typically in the current stack frame
 - Call this address fa
- Before each operation that reads f , insert
 $f := \text{load } fa$
- After each operation that writes f , insert
 $\text{store } f, fa$

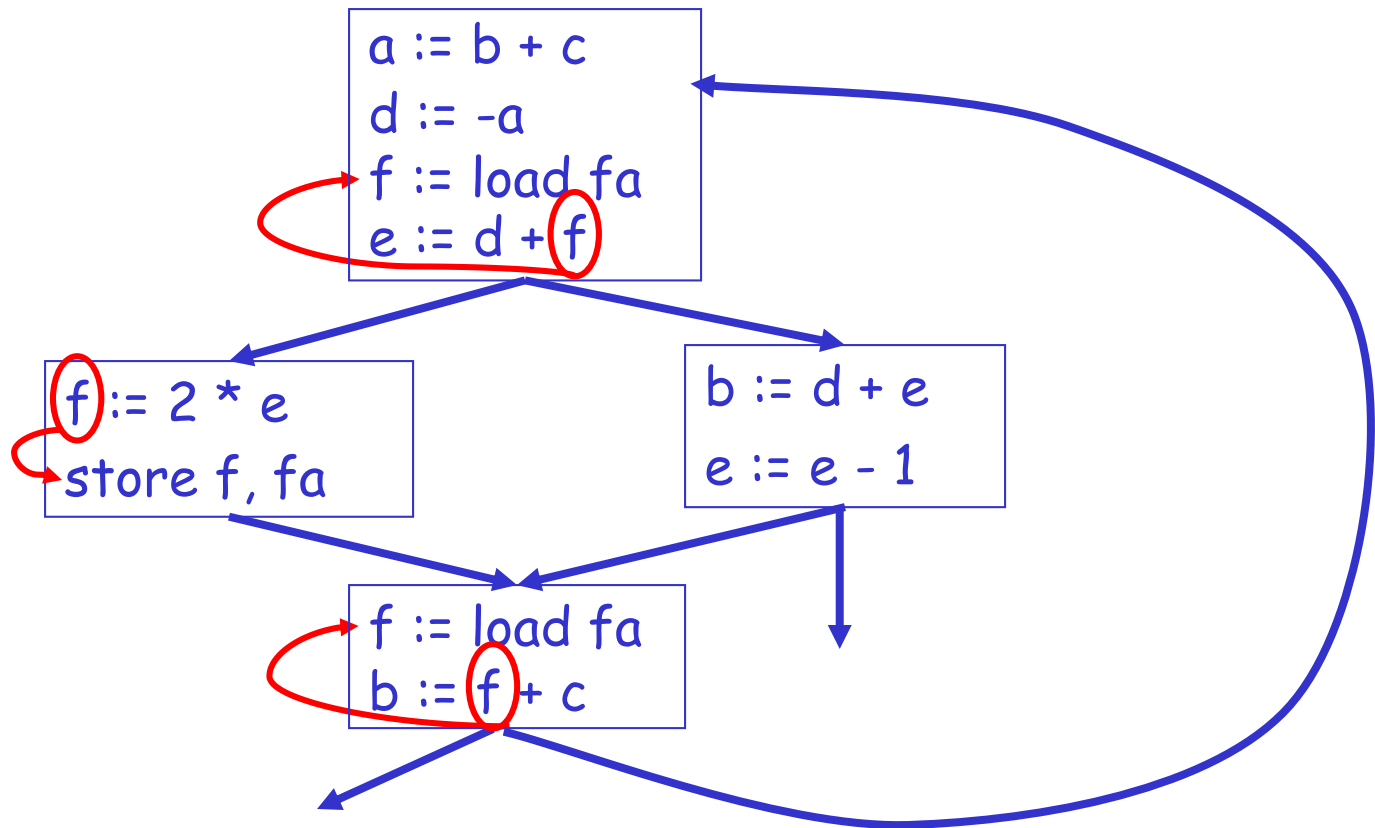
Spilling Example

- Original code



Spilling Example

- This is the new code after spilling f

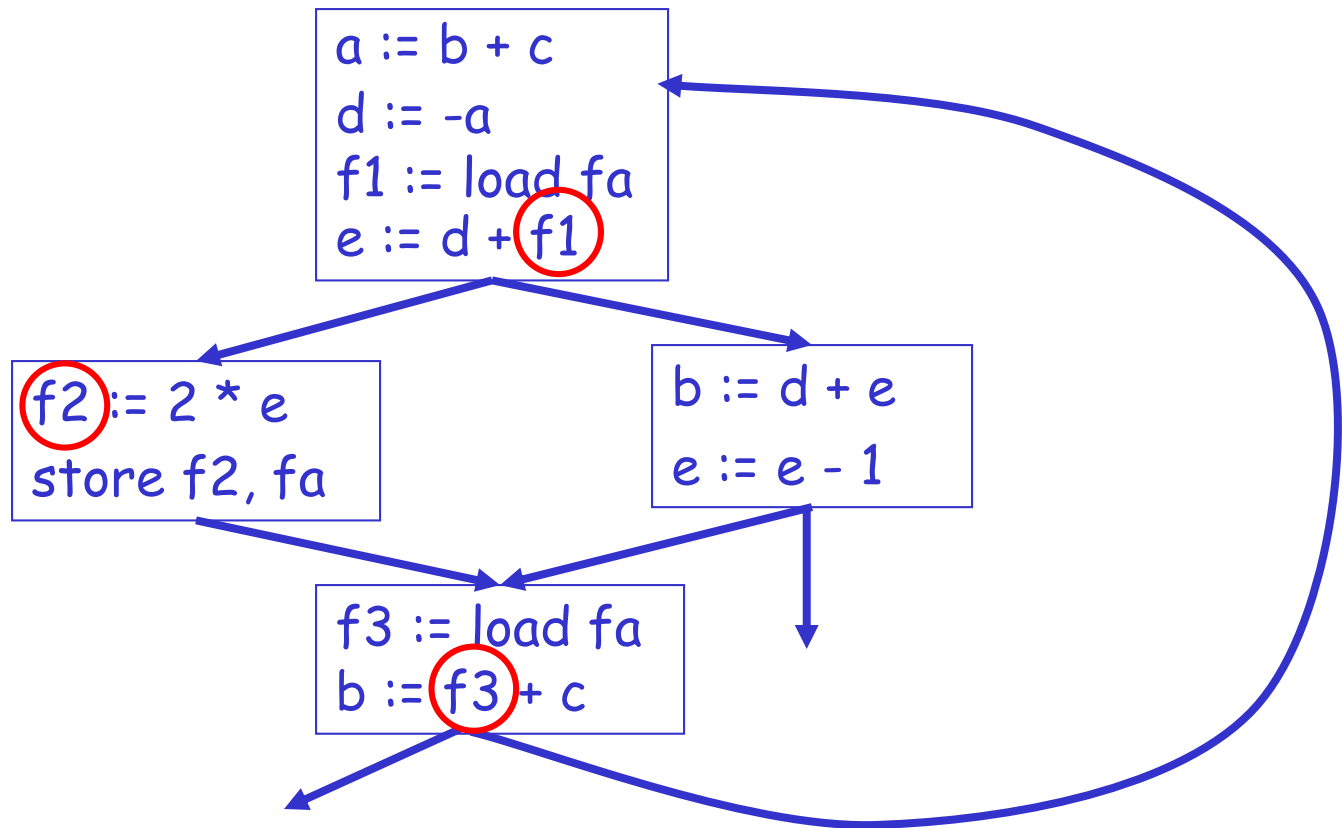


A Problem

- This code reuses the register name f
- Correct, but suboptimal
 - Should use distinct register names whenever possible
 - Allows different uses to have different colors

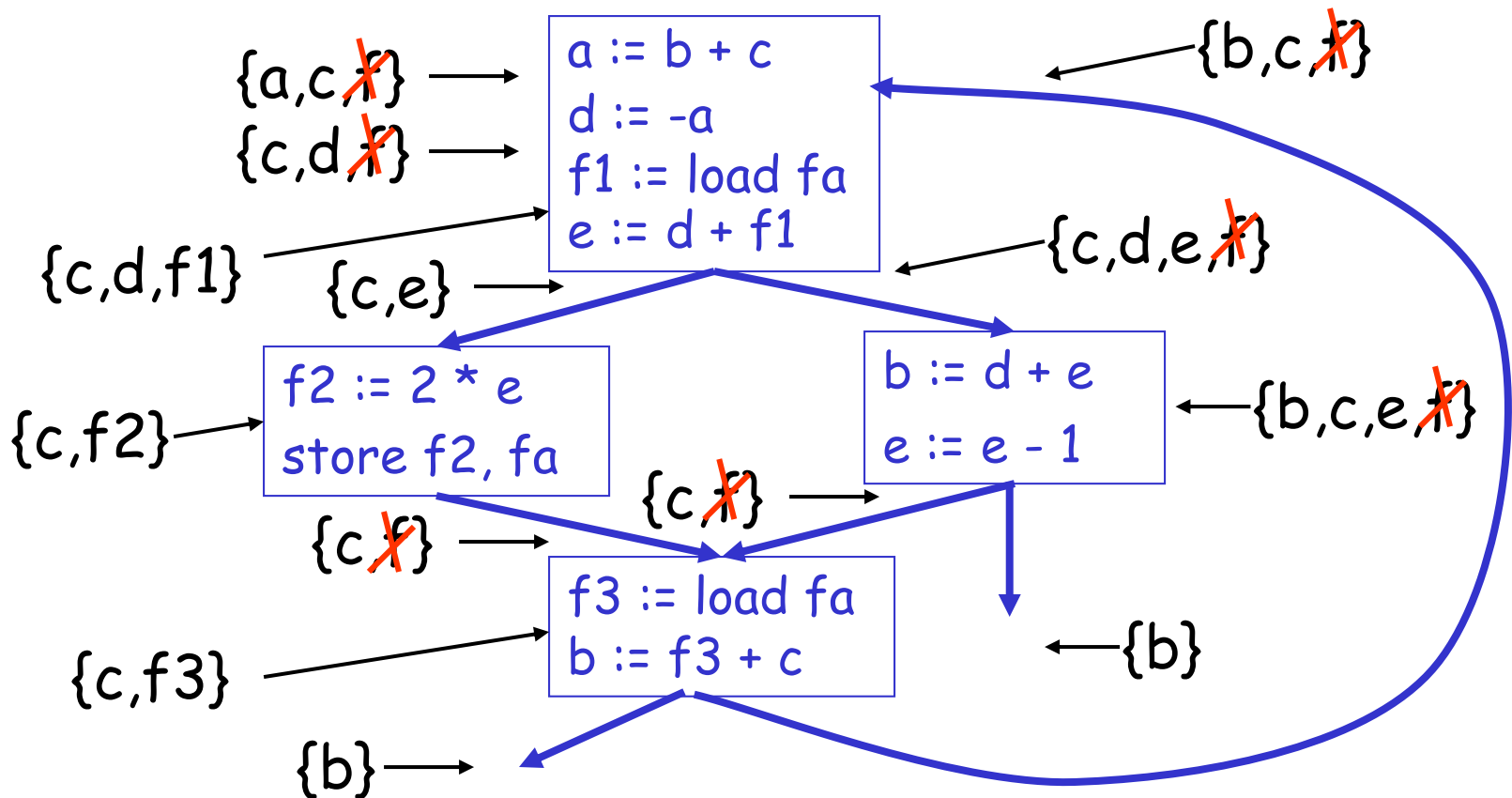
Spilling Example

- This is the new code after spilling f



Recomputing Liveness Information

- The new liveness information after spilling:

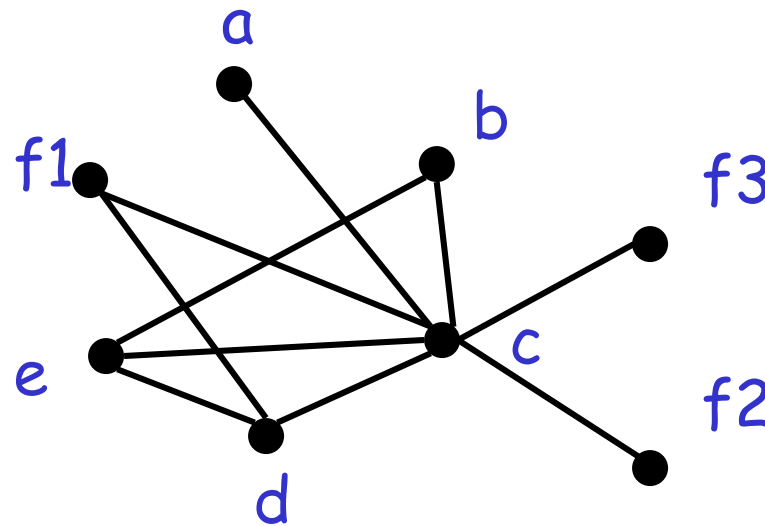


Recomputing Liveness Information

- New liveness information is almost as before
 - Note f has been split into three temporaries
- f_i is live only
 - Between a $f_i := \text{load } f_a$ and the next instruction
 - Between a $\text{store } f_i, f_a$ and the preceding instr.
- Spilling reduces the live range of f
 - And thus reduces its interferences
 - Which results in fewer RIG neighbors

Recompute RIG After Spilling

- Some edges of the spilled node are removed
- In our case f still interferes only with c and d
- And the resulting RIG is 3-colorable



Spilling Notes

- Additional spills might be required before a coloring is found
- The tricky part is deciding what to spill
 - But any choice is correct
- Possible heuristics:
 - Spill temporaries with most conflicts
 - Spill temporaries with few definitions and uses
 - Avoid spilling in inner loops

Conclusions

- Register allocation is a “must have” in compilers:
 - Because intermediate code uses too many temporaries
 - Because it makes a big difference in performance
- Register allocation is more complicated for CISC machines

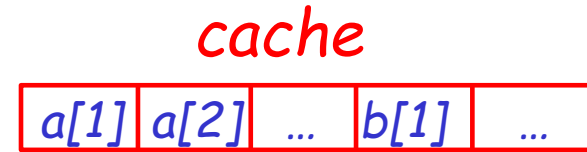
Caches

- Compilers are very good at managing registers
 - Much better than a programmer could be
- Compilers are not good at managing caches
 - This problem is still left to programmers
 - It is still an open question how much a compiler can do to improve cache performance
- Compilers can, and a few do, perform some cache optimizations

Cache Optimization

- Consider the loop

```
for(j := 1; j < 10; j++)  
  for(i=1; i<1000; i++)  
    a[i] *= b[i]
```



```
a[1] *= b[1]  
a[2] *= b[2]  
...
```

- This program has terrible cache performance
 - Because each iteration of the inner loop refers to a new element of arrays (i.e., fresh data) = [a cache miss]

Cache Optimization (Cont.)

- Consider the program:

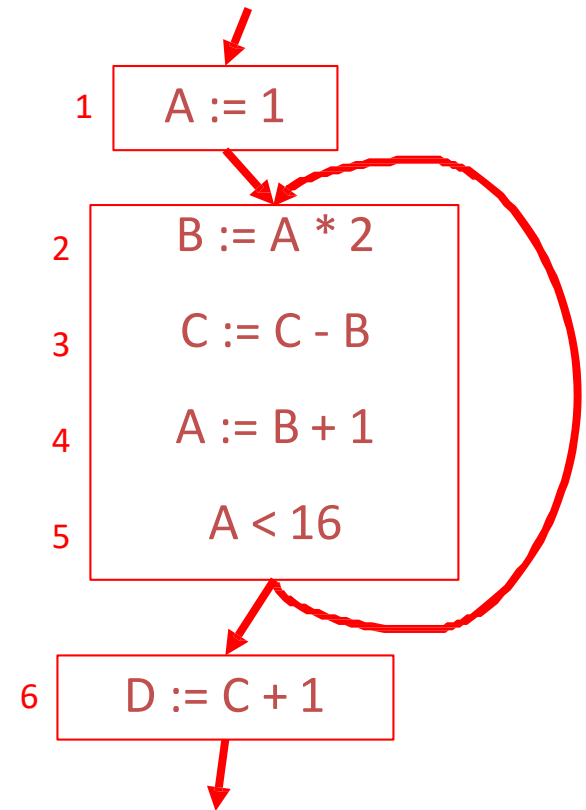
```
for(i=1; i<1000; i++)  
    for(j := 1; j < 10; j++)  
        a[i] *= b[i]
```

- Computes the same thing
 - But with much better cache behavior
 - Might actually be more than 10x faster
- A compiler can perform this optimization
 - called loop interchange

Question?

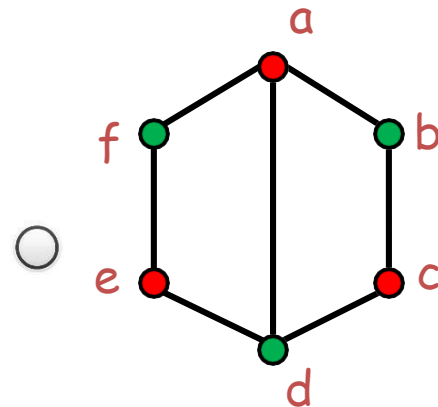
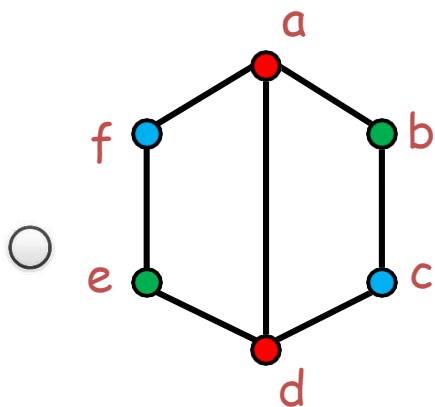
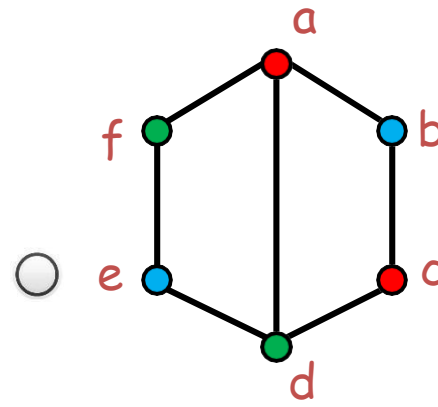
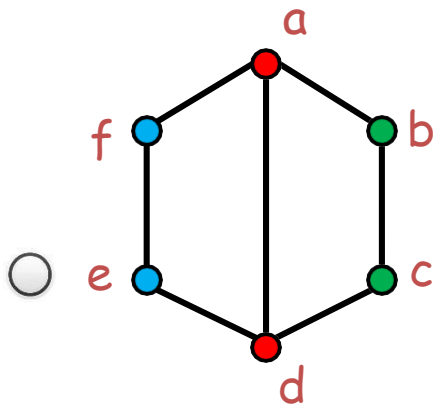
Which of the following pairs of temporaries interfere in the code fragment given at right?

- A and B
- A and C
- B and C
- C and D



Question?

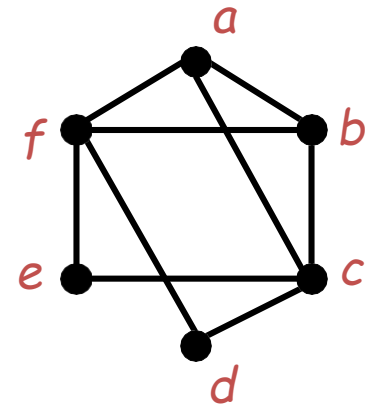
Which of the following colorings is a valid minimal coloring of the given RIG?



Question?

For the given RIG and $k = 3$, which of the following are valid deletion orders for the nodes of the RIG?

- $\{d, e, c, b, a, f\}$
- $\{e, f, a, b, c, d\}$
- $\{d, c, b, a, f, e\}$
- $\{d, e, b, c, a, f\}$



Question?

For the given code fragment and RIG, find the minimum cost spill. In this example, the cost of spilling a node is given by:

of occurrences (use or definition)
- # of conflicts
+ 5 if the node corresponds to a variable used in a loop

- A
- B
- C
- D

