Mathematics for Economists
ECON-C1000
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Problem Set 0: Getting Started

These exercises are designed to help you prepare for the course. They will be covered in the first review session, but you do not have to return them.

1. (Vectors):

   (a) Familiarize yourself with the notion of a vector as an ordered list of real numbers so that a k -dimensional vector $x = (x_1, x_2, ..., x_k)$, where each x_i for $i \in \{1, ..., k\}$ is a real number.

      i. For concreteness: write your daily plan of using EUR200 over the coming week as a 7-dimensional vector.

      ii. For labor economics purposes, write the vector summarizing your characteristics, for example: age, years of schooling, parental income, number of siblings as a vector.

      iii. Put all Finns in alphabetical order and consider the vector listing their taxable income last year.

   (b) In high school, the length $\|x\|$ of the vector x is calculated using Pythagorean theorem

   $$\|x\| = \sqrt{x_1^2 + x_2^2} \text{ or } \|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

   How would you define the length of a k -dimensional vector?

   (c) The distance between two vectors x and y is given by $\|x - y\|$. Write this in full for 2-dimensional and 3-dimensional vectors. You could also define a distance in other ways. For example, you could define the distance $d(x, y)$ as follows for 2 and 3 dimensions respectively:

   $$d(x, y) = |x_1 - y_1| + |x_2 - y_2| \text{ or } d(x, y) = |x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3|.$$

      i. Draw a picture to see what this distance looks like. Why is it called the Manhattan distance in 2 dimensions?

      ii. Is this a sensible distance? What do you require from a sensible distance?

2. (Matrices):

   (a) As in the previous exercise, familiarize yourself with the notion of a matrix.

      i. How would you imagine the labor economics list of charcteristics for all Finns as a matrix?

ii. Consider the 30 students taking this class. Write a matrix with the students identified in the alphabetical order as rows and as columns of the matrix. Set the element a_ij equal to 1 if students i and j ever work together on a problem set (so that $a_ii = 1$ trivially for all i). Note that you can model any network of connections in this way.

iii. Think of vectors as special matrices. If the matrix has a single row, we have a row vector. If it has a single column, we have a column vector.

(b) Let $X = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$ ja $Y = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$. Compute $Z = X + Y$.

(c) Compute XY and YX. Notice that the sum of diagonal elements in XY is the same as in YX. Show that this is true for all 2x2 matrices. Can you show this for all matrices?

3. A system of linear equations can be written compactly using matrices and vectors: $Ax = b$, where A is a matrix, x is a vector of endogenous variables (i.e. the variables to be solved) and b is a vector of exogenous variables (i.e. variables that are not determined in the linear model but come from outside). Write the following in matrix notation:

$$\begin{aligned} 2x_1 - 7x_2 + 3x_3 &= 10 \\ 2x_1 - 5x_2 + 10x_3 &= 3 \\ x_1 - 5x_2 + 23x_3 &= 43 \end{aligned}$$

Solve the system using elementary row operations.

4. Can you find an example of a linear system of equations that has exactly two solutions? (Hint: if both $x = (x_1, x_2, ..., x_k)$ and $y = (y_1, y_2, ..., y_k)$ solve the set of equations, what about $x + y$?)

5. Give an example of an economic model where prices are exogenous variables and of a model where prices are endogenous variables.

6. Unconstrained optimization: Draw the graph of $f(x) = 2x - x^3$ and find its local and global minima and maxima. Evaluate the second derivative of f at the local maxima and minima.

7. Recall the firm's problem from Principles of Economics I: The demand at price p is given by $Q(p) = a - bp$, where $a, b > 0$. The cost function of a monopolist is given by $C(q) = kq^2$, where q is the amount produced and $k > 0$ is a constant. Write down the monopolist's problem and solve it.

8. Derivatives: find the derivatives for the following functions:

(a) $f(x) = x^4$, $f(x) = e^x$, $f(x) = \ln(x)$

(b) $f(x) = (3x + 2)^3$, $f(x) = \frac{4}{x^2+1}$, $f(x) = \ln(x^\alpha)$

(c) $f(x) = 4e^{-3x^2}$, $f(x) = x \ln x$, $f(x) = x^x$