Practice questions for the exam

1. Consider the following systems. of linear equations:

   (a) For which values of (b_1, b_2, b_3, b_4) does the following system have a solution?

$$\begin{pmatrix} 1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 2 \\ 2 & 4 & 2 & 2 \\ 0 & 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$

   (b) Solve by using Cramer's rule the following system:

$$\begin{pmatrix} 3 & 5 & 9 \\ 1 & 2 & 8 \\ 0 & 0 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

   (c) Let A be an n x n matrix and b an n-dimensional column vector. Show that if $x^{(1)}$ and $x^{(2)}$ solve the system

$$Ax = b,$$

   then $x^{\lambda} = \lambda x^{(1)} + (1 - \lambda)\, x^{(2)}$ is also a solution to this system.

2. Consider the following optimization problem:

$$\max_{x,y} x^{\frac{1}{3}} y^{\frac{1}{2}} - \alpha x - y.$$

   for $x, y \geq 0$.

   (a) What are the first-order conditions for the optimal solutions (x^*, y^*) .

   (b) Is the critical point a minimum or a maximum?

   (c) Consider (x^*, y^*) :t as a function of α around $\hat{\alpha} = 1$. Show that the conditions for the implicit function theorem are satisfied in a neighborhood of $\hat{\alpha}$.

(d) Find the signs for $\frac{dx^*}{d\alpha}$ and $\frac{dy^*}{d\alpha}$ by using Cramer's rule.

3. Solve the following optimization problem:

$$\max_{x_1,x_2} \frac{1}{2}\ln x_1 + \ln x_2$$
$$\text{s.e. } x_1 + 2x_2 = 2,\ \ x_1, x_2 > 0.$$

   (a) Formulate the Kuhn-Tucker first-order conditions for this problem.

   (b) Is the feasible set closed and bounded? How do you know that a solution exists?

   (c) Solve the problem.

   (d) Give an interpretation for the Lagrange multiplier of the binding constraint.

4. Solve the following system of difference equations by diagonalizing the system.

$$x_{t+1} = x_t + 2y_t,$$
$$y_{t+1} = -x_t + 4y_t$$

   for an arbitrary initial value (x_0, y_0). Is the origin a stable steady state for the system?