



**ARK-A3001 Design of Structures_Basics
Equilibrium**

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Equilibrium

lat. *aequus* = equal + *libra* = balance

What is it that we need to balance?



Baumschlager Eberle: Rohner Port Building
Fussach, Austria, 1999-2000

Equilibrium

Dead Load

building materials

installations

Live Load

people

furniture

Environmental Load

snow

wind

earthquake

soil pressure

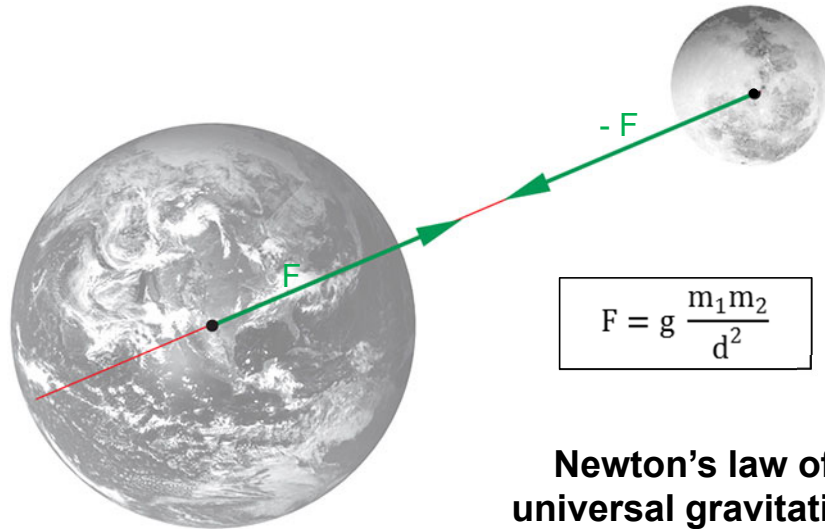
The amount and location of loads is described by **forces**



Baumschlager Eberle: Rohner Port Building
Fussach, Austria, 1999-2000

Isaac Newton

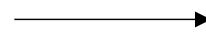
every mass attracts every other mass in the universe with a force



$$F = g \frac{m_1 m_2}{d^2}$$

Newton's law of universal gravitation

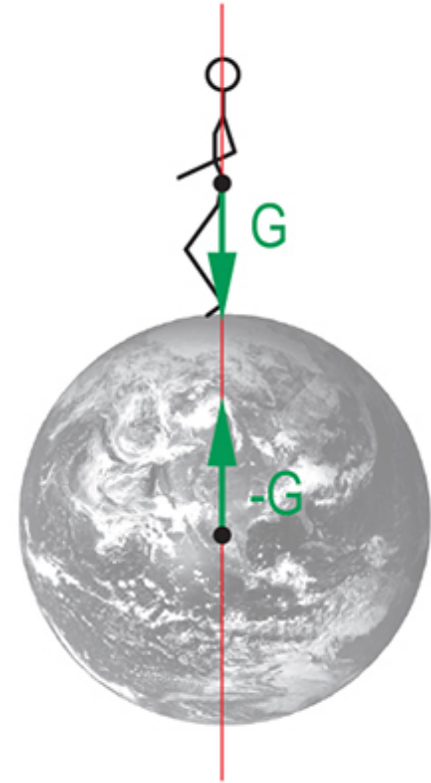
m_1, m_2 ... masses
 d ... distance
 g ... gravitational constant



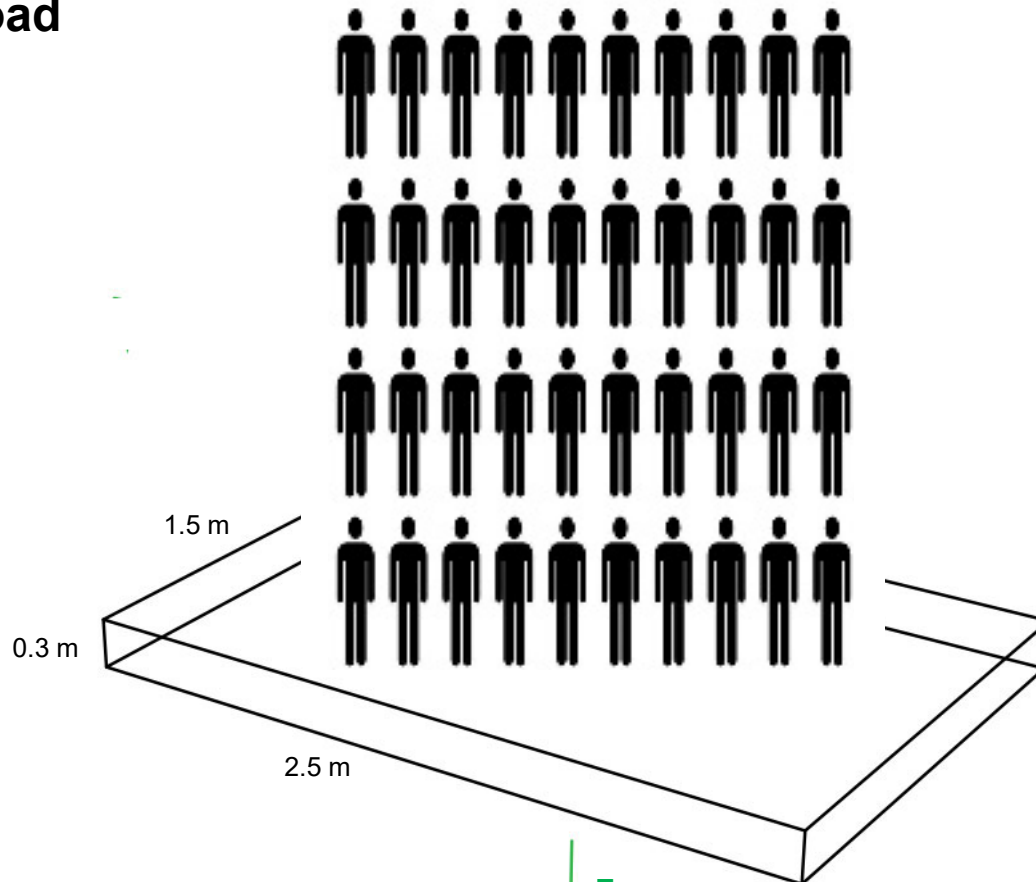
$$G = 9.81 \text{ m}$$
$$1 \text{ [N]} = 1 \left[\text{kg} \frac{\text{m}}{\text{s}^2} \right]$$

gravitational load

m ... mass

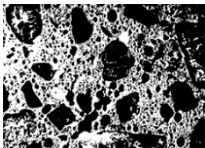


Gravitational Load



40 people
(around 75 kg / person)

$$V = 1.125 \text{ m}^3$$
$$m = V \cdot \rho = 2'812.5 \text{ kg}$$
$$F = g \cdot m = 27'591 \text{ N} = 27.6 \text{ kN}$$

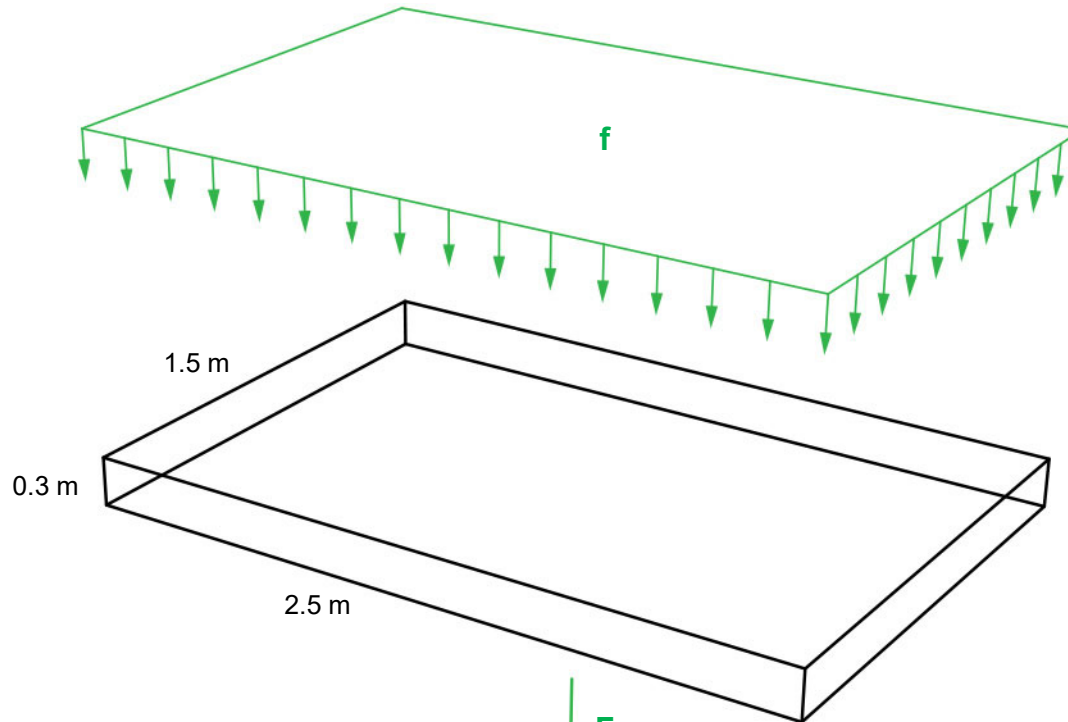


concrete
 $\rho = 2500 \text{ kg/m}^3$



Gravitational Load

distributed load f



$$A = 3.75 \text{ m}^2$$

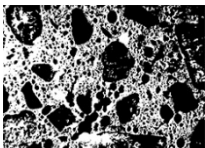
$$f = F/A = 7.36 \text{ kN/m}^2$$

$$V = 1.125 \text{ m}^3$$

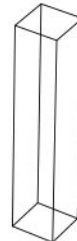
$$m = V \cdot \rho = 2'812.5 \text{ kg}$$

$$F = g \cdot m = 27'591 \text{ N} = 27.6 \text{ kN}$$

point load F



concrete
 $\rho = 2500 \text{ kg/m}^3$



Equilibrium

Dead Load

building materials

distributed load

installations

point load / distributed load

Live Load

people

point load

furniture

point load / distributed load

Environmental Load

snow

distributed load

wind

distributed load

earthquake

point load

soil pressure

distributed load

The amount and location of loads is described by **forces**



Baumschlager Eberle: Rohner Port Building
Fussach, Austria, 1999-2000



Equilibrium

Dead Load

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point load / distributed load

Live Load

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Environmental Load

snow

distributed load

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distributed load

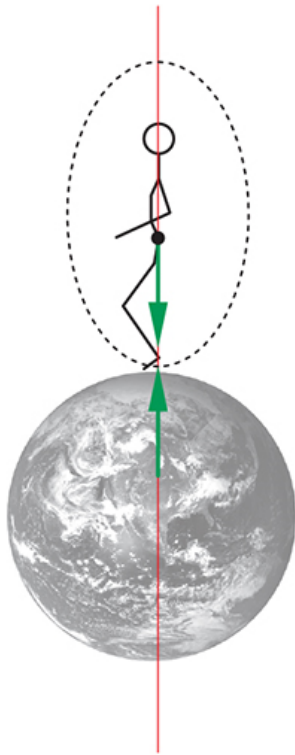
The amount and location of loads is described by **forces**

Baumschlager Eberle: Rohner Port Building
Fussach, Austria, 1999-2000

Isaac Newton

all forces between two objects exist in equal magnitude and opposite direction

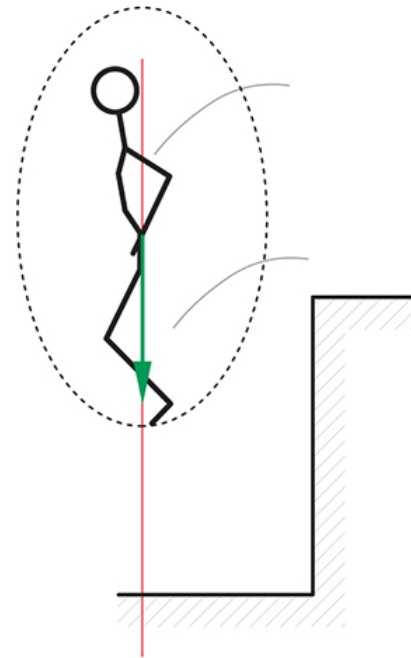
"actio est reactio"



equilibrium

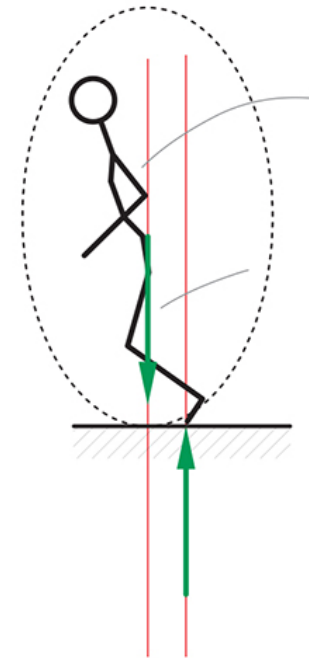
two potential sources of violation of rule for equilibrium

not equal magnitude



translation

not opposite direction



rotation

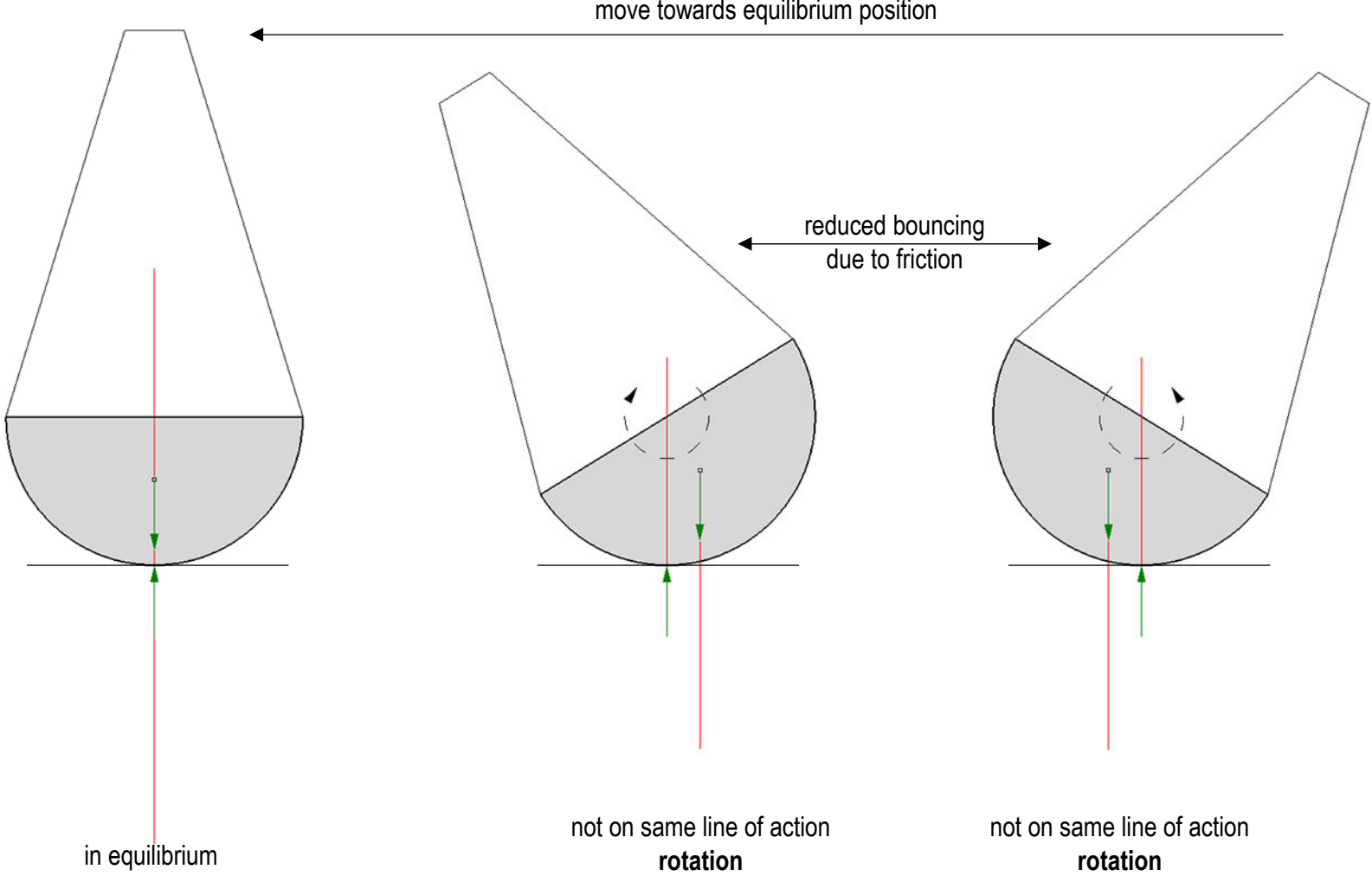


Equilibrium



Why is the figure always bouncing back?
Why does the figure always come to rest?

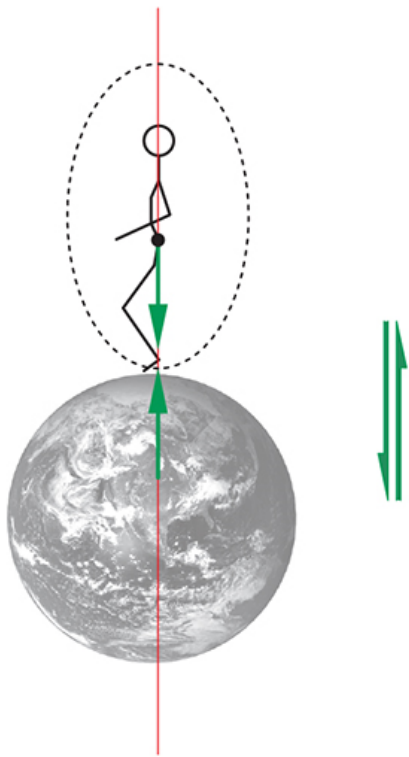
Equilibrium



Isaac Newton

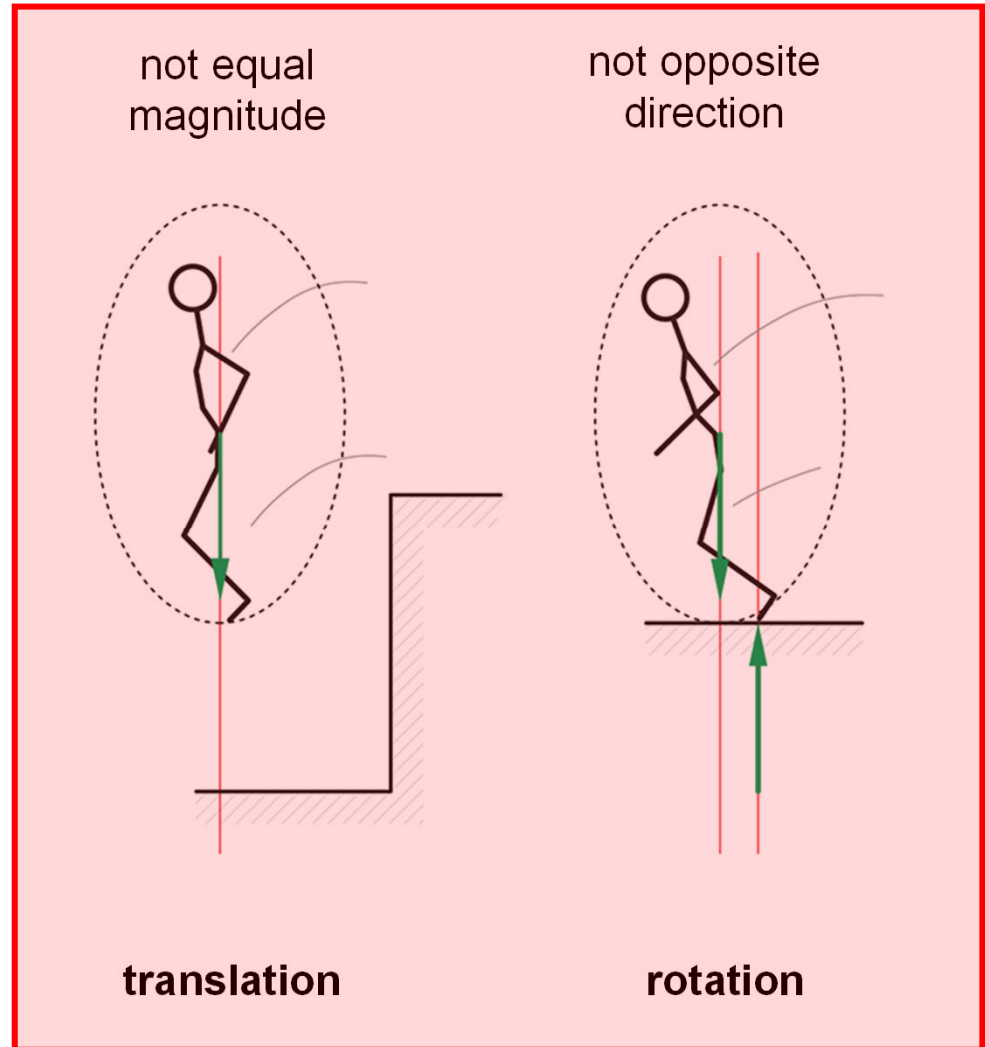
all forces between two objects exist in equal magnitude and opposite direction

"actio est reactio"



equilibrium

two potential sources of violation of rule for equilibrium



translation

rotation

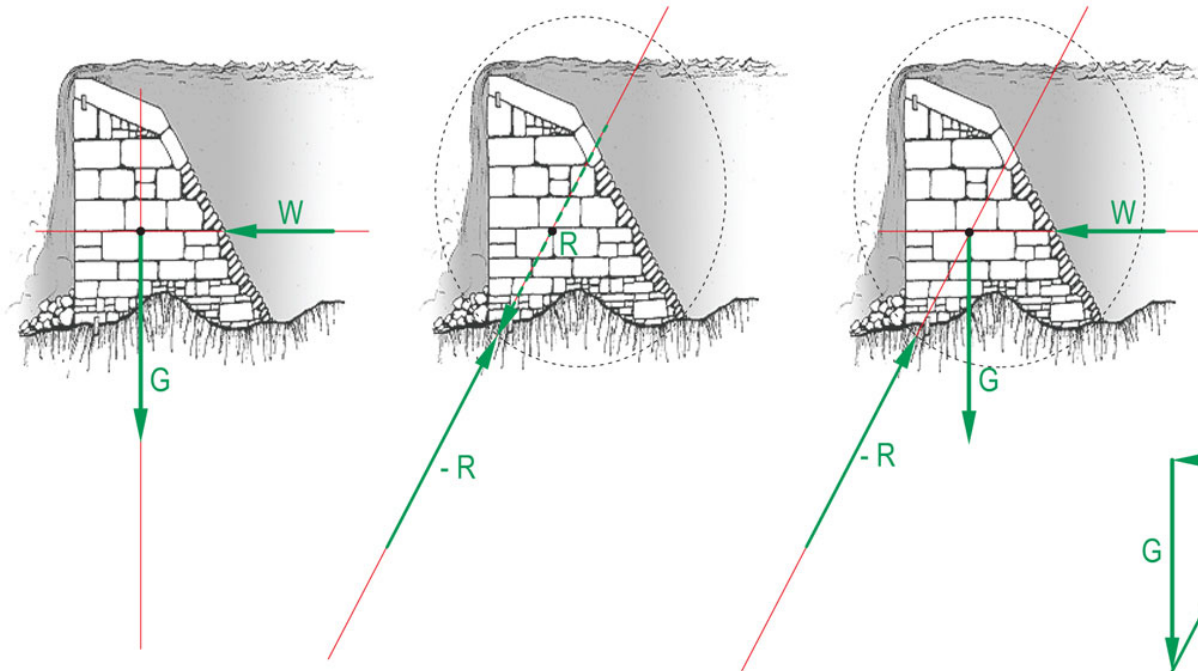
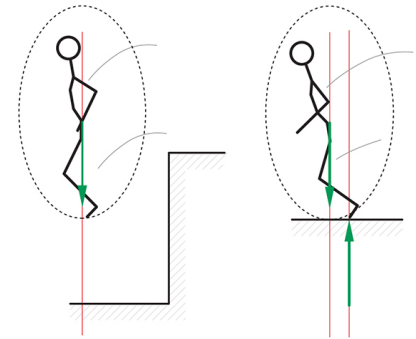
Equilibrium Condition

geometric description

the forces add up to zero
the line of action of the forces intersect in one point

both conditions are checked geometrically with

- location plan (line of action)
- force plan (forces add up)



location plan

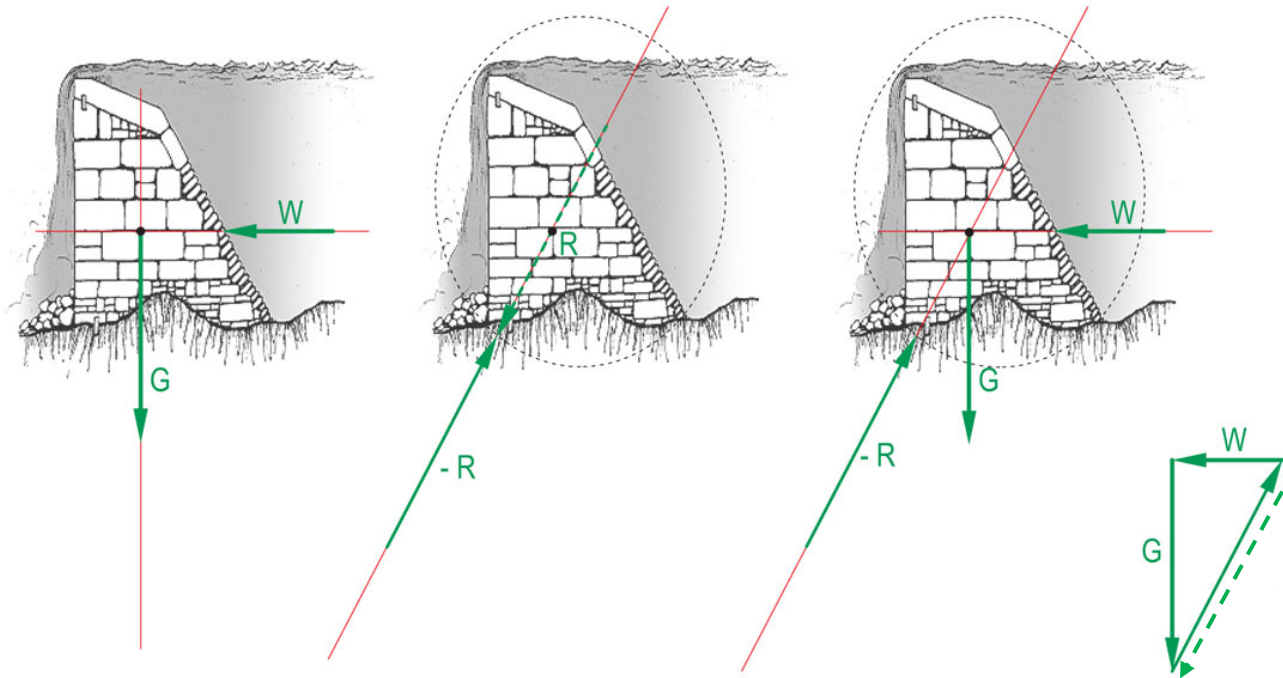
force plan

Resultant

R is the resultant of **G** & **W**

R has the same effect
as **G** & **W** together

$$\mathbf{R} = \mathbf{G} + \mathbf{W}$$

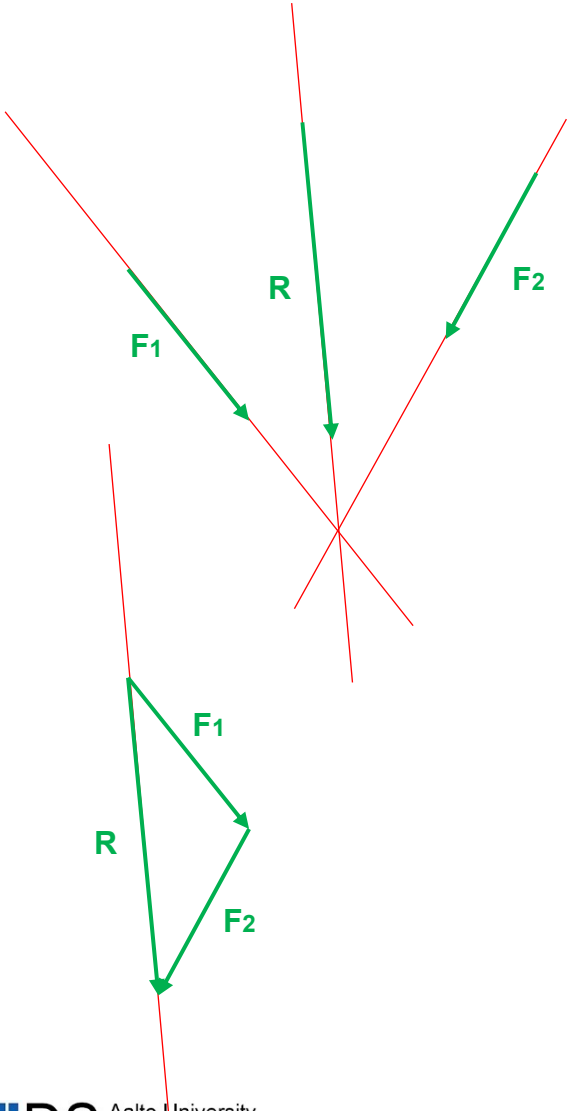


location plan

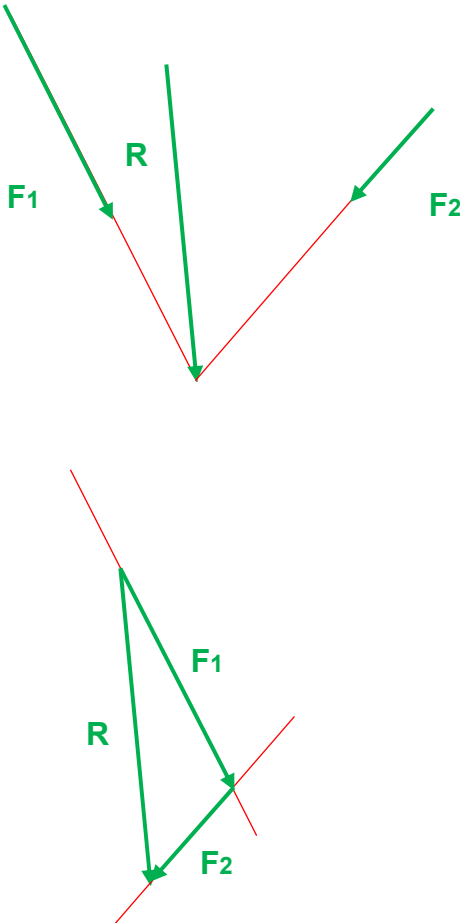
force plan

Basic Operation with Forces

two forces can be added into one resulting force



one force can be separated into two force



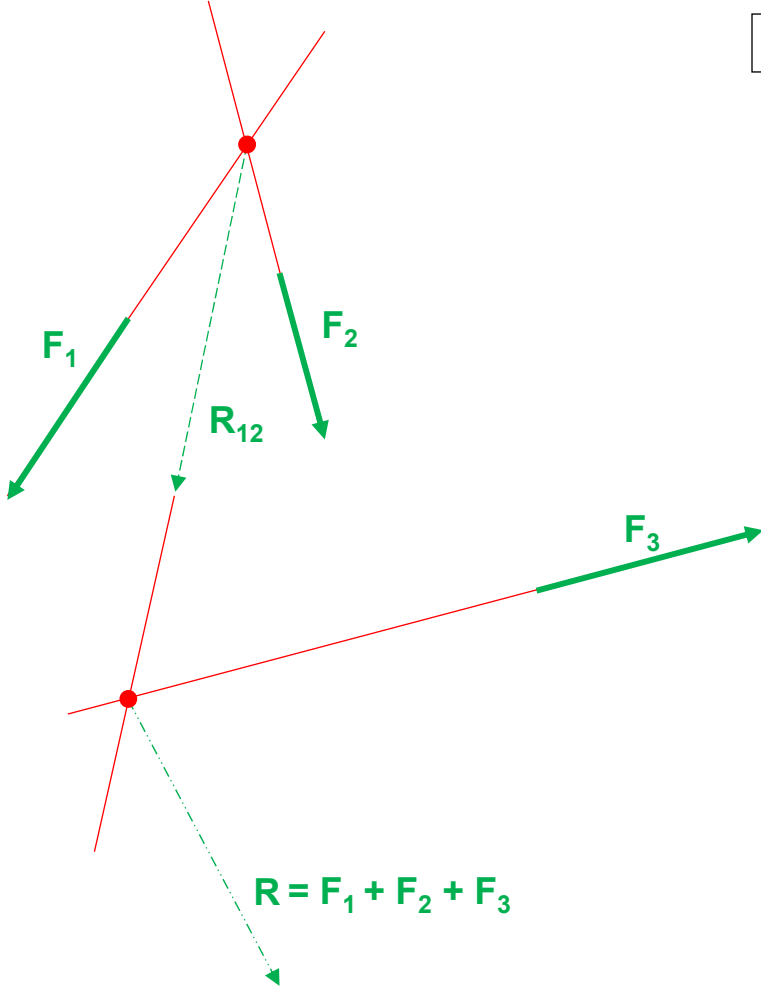
location plan

force plan

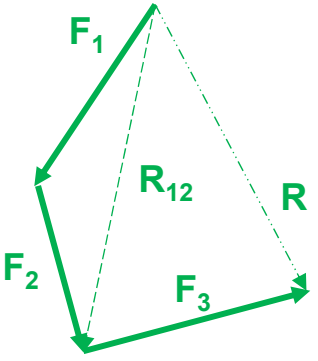
Basic Operation with Forces

construction of resultant

location plan

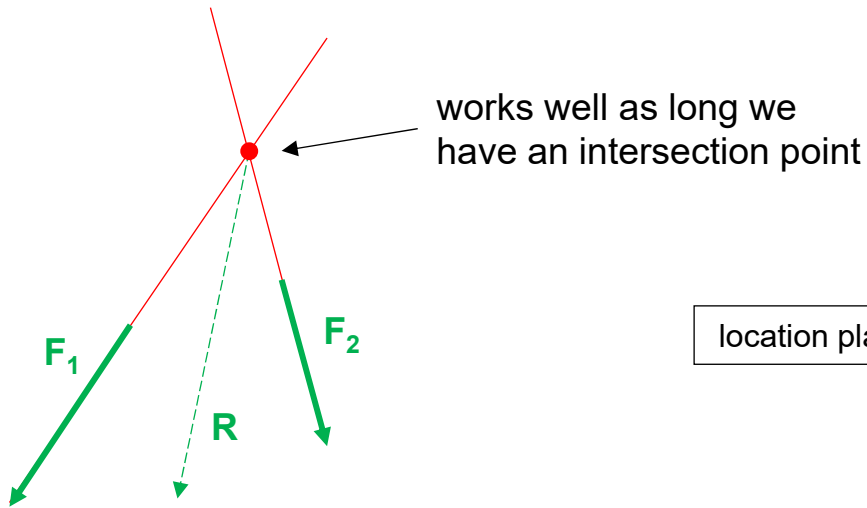


force plan

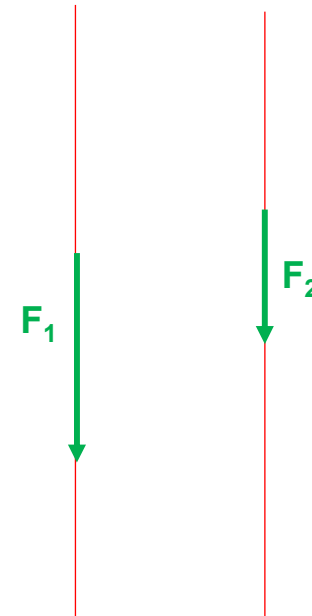


Basic Operation with Forces

construction of resultant

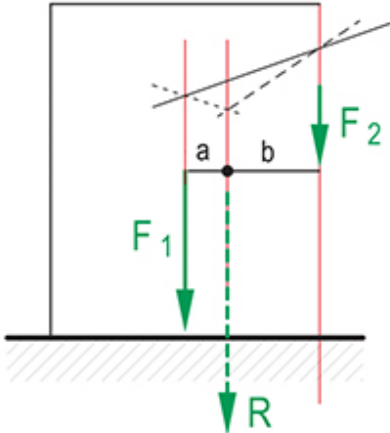
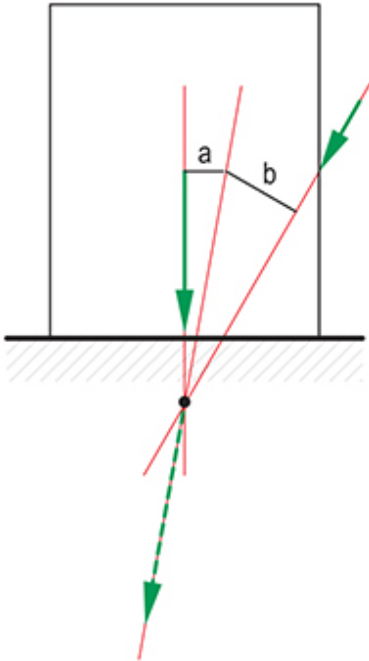
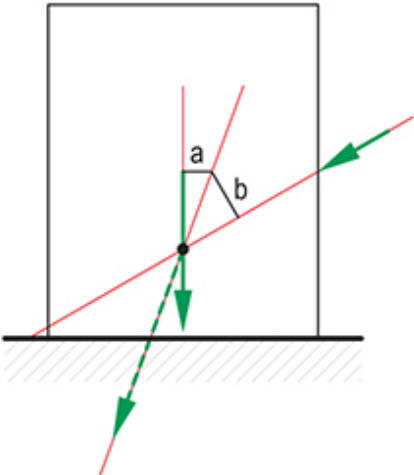
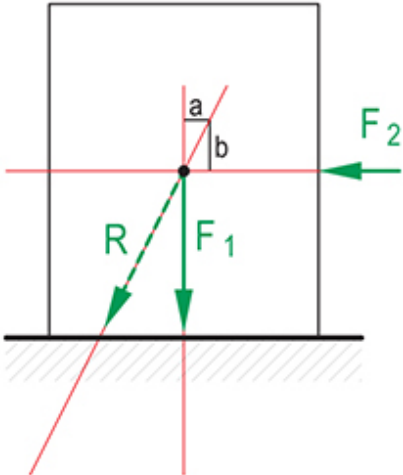


location plan



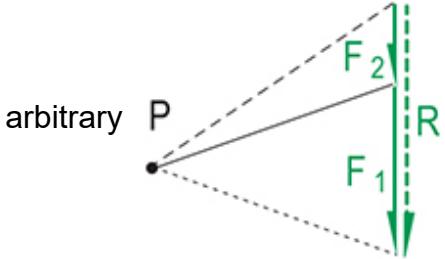
where is the resultant R located when the forces are parallel?

Lever Principle



location plan

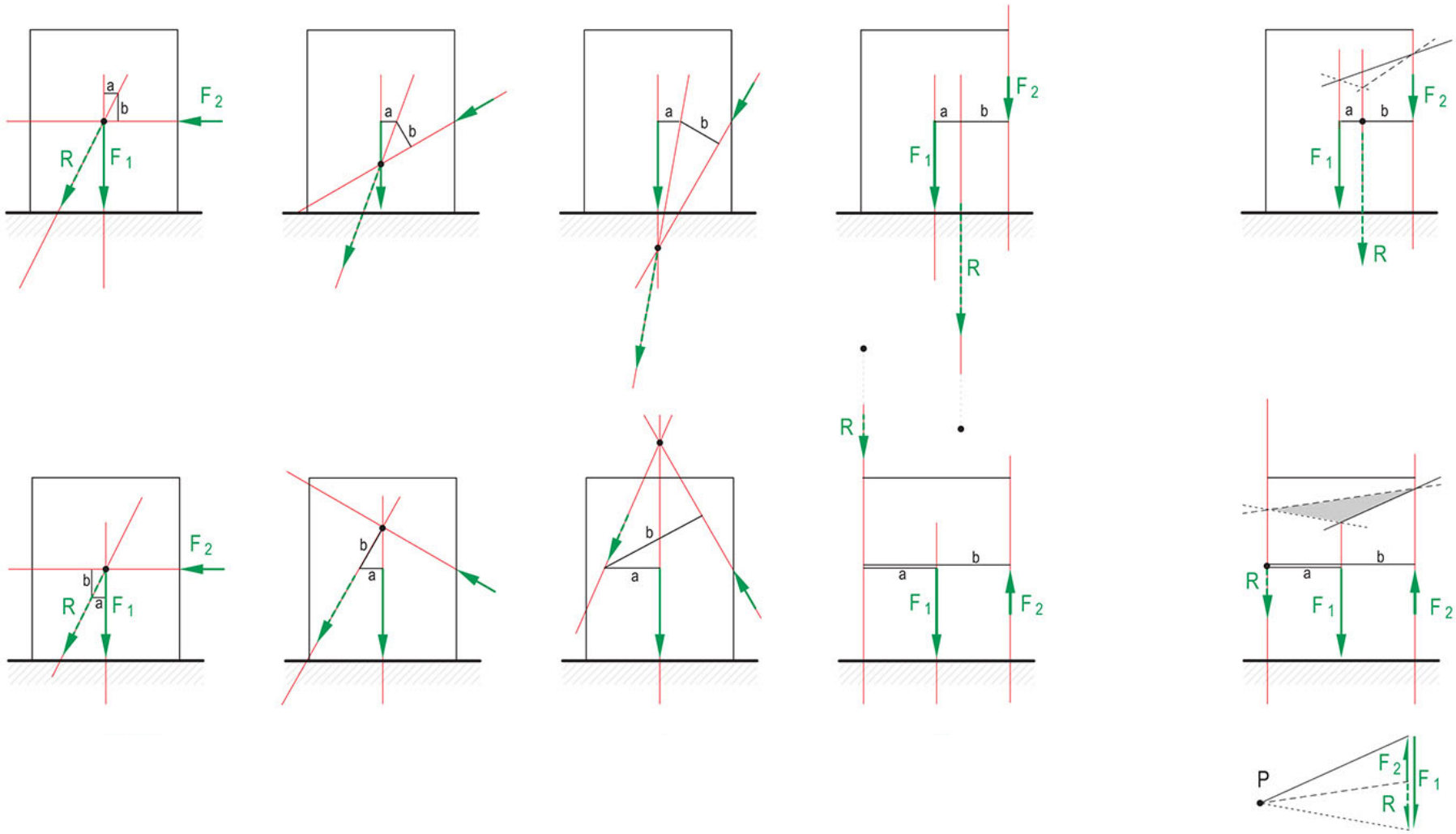
$$\frac{F_2}{F_1} = \frac{a}{b}$$



force plan

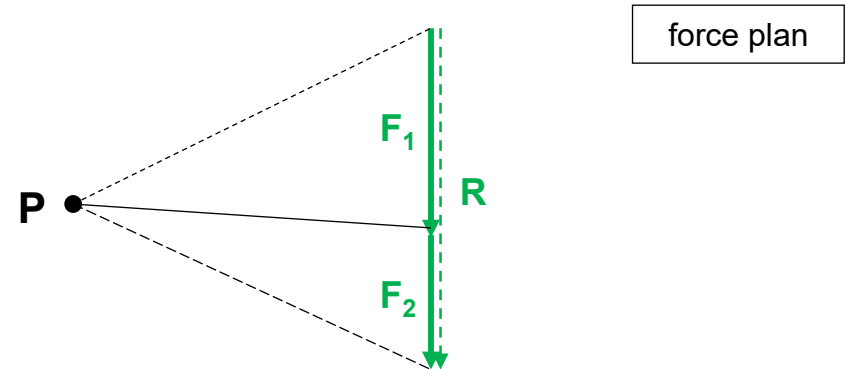
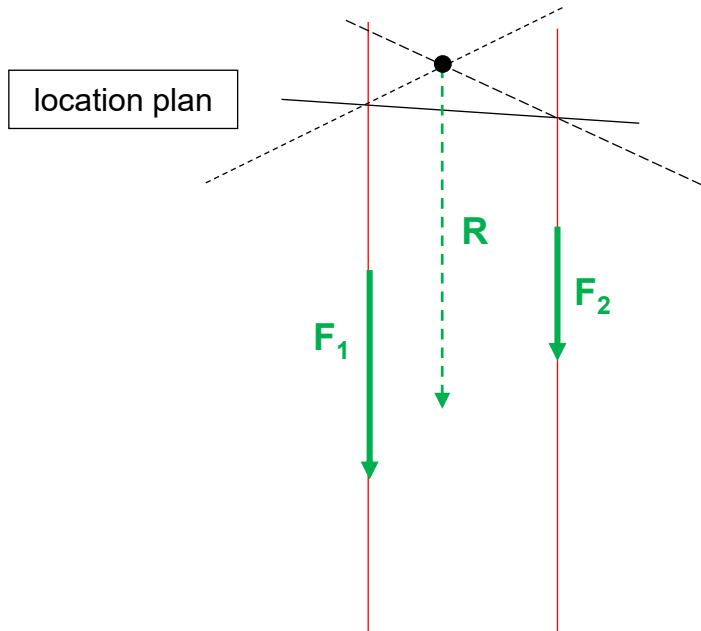
Lever Principle

$$\frac{F_2}{F_1} = \frac{a}{b}$$



Lever Principle

geometric construction of resultant



Step 1: construct R in force plan

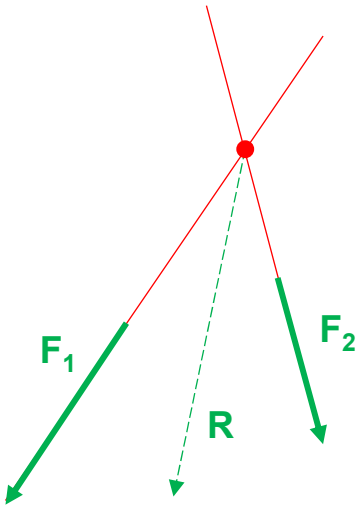
Step 2: pick P in force plan & construct rays

Step 3: move rays into location plan (parallel shift)

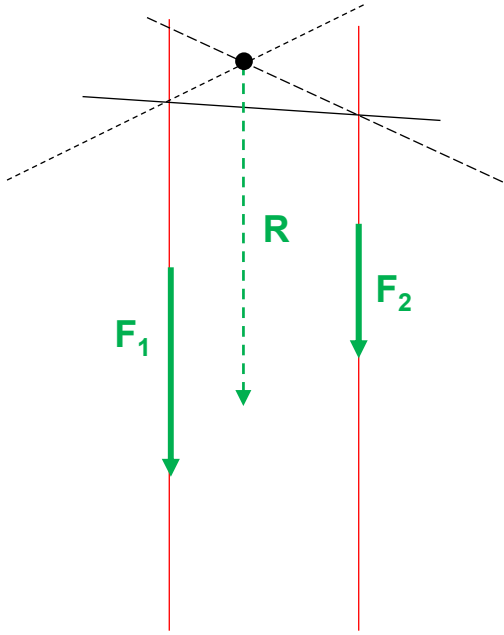
Step 4: locate R in location plan at intersection point

Basic Operation with Forces

construction of resultant

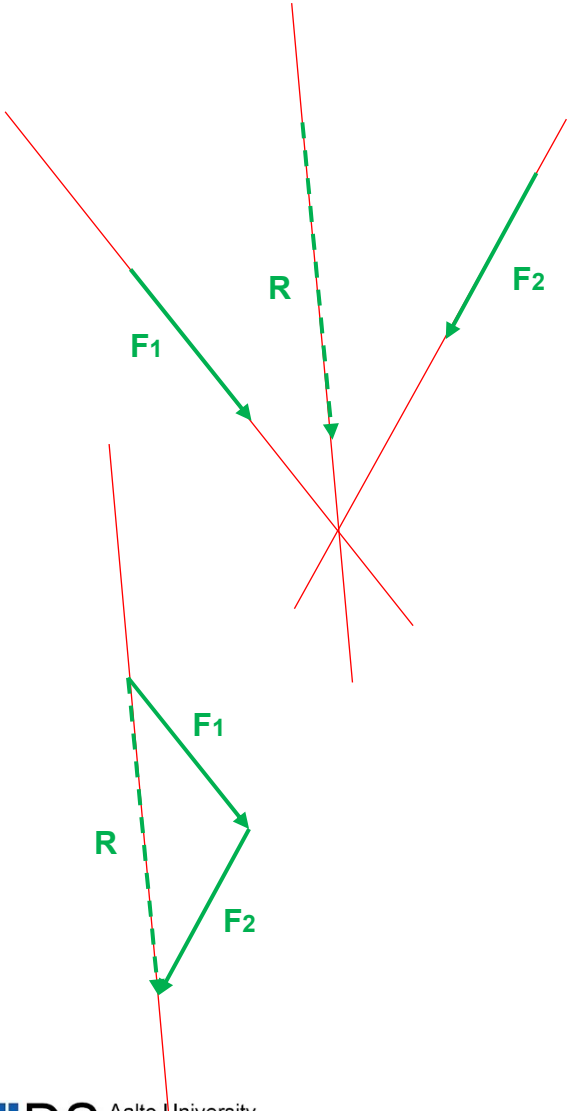


location plan

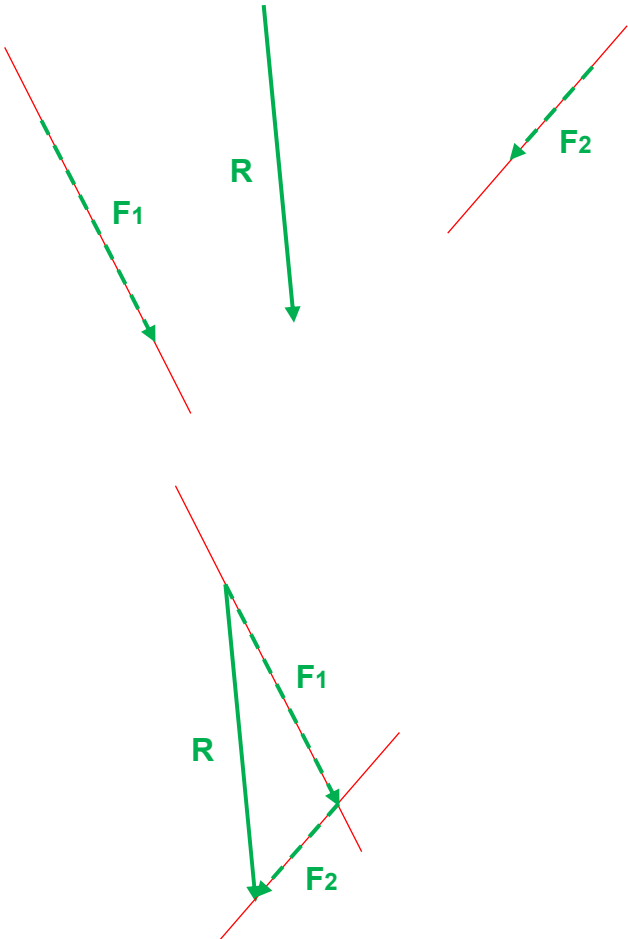


Basic Operation with Forces

two forces can be **added**
into one resulting force



one force can be **separated**
into two force



location plan

force plan



Load cases

dead load

live load

environmental load

distributed load

point load

equilibrium

location plan

force plan

resultant force

lever principle

ARK-A3001 Design of Structures_Basics Equilibrium

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Exercise 1.1

Construct the resultant of the four forces given in the location plan.

location plan



Exercise 1.2

Separate the force R into two parallel forces F_1 and F_2 in such a way that R is the resultant of these two forces and the magnitude of the forces $F_1:F_2 = 3:1$.



Exercise 1.3

The sculpture consists of three steel plates of size 160 x 160 x 7.6 cm (plate 1) respectively 160 x 213 x 5 cm (plate 2 & 3). The plates are stacked onto each other but neither screwed nor welded. Together they form a stable configuration in equilibrium. Why?

steel
 $\rho = 7850 \text{ kg/m}^3$

