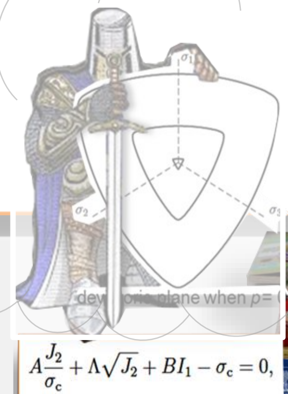
Djebar BAROUDI, PhD.
University Lecturer @ Aalto-university
Civil Engineering Department

Supporting Material in MyCourses

0. **INTRODUTION**
1. **ELASTICITY**
2. **VISCOELASTICITY** (+ basics of creep)
3. **PLASTICITY**
4. **DAMAGE** … year 2018:

damage-plasticity *ex.* Concrete Damage Plasticity, Models and Applications in Abaqus



Reading – Textbooks:

- Lemaitre and Chaboche – *Mechanics of Solid Materials*
- Ottosen & Ristinmaa – *The Mechanics of Constitutive Modeling*
- W.F. Chen, D.J. Han – *Plasticity for Structural Engineers* (only *chapters* 1-5)

Recommended elective textbooks

- ***Plasticity Theory***. Jacob Lubliner
- ***Continuum Mechanics: Elasticity, Plasticity, Viscoelasticity*** Ellis H..Dill, November 10, 2006 by CRC Press

Lemaitre & Chaboche textbook as an **e-book**:
http://proquestcombo.safaribooksonline.com.libproxy.aalto.fi/book/physics/9781107384712

Otaniemi, 14.4.2019

CONTENT

1. INTRODUTION
2. **ELASTICITY**
3. VISCOELASTICITY
4. PLASTICITY

Elasticity in Solids

Definitions

Thermodynamical framework

Elastic Solids

Isothermal Cauchy-elastic material

Green-Elastic or Hyper-elastic Materials

Examples of Non-Linear Elasticity

Hysteresis during loading and unloading

Equations of Elasticity

Minimum Master level

Material Symmetries

Degree of symmetry

Linear Elasticity – Matrix Formulation

Anisotropy

Isotropy

Limits on Elastic Parameters Values

Orthotropy

Transversal isotropy

Limits on Elastic Parameters Values

Nonlinear isotropic Hooke formulation

Generalized Hooke's Law – Examples of problems

Orthotropic case – A worked example

Good to know: layered composite (transverse orthotropy)

Transformation of Stress and Strain Components

Example exercises for training

…

Minimum L-level

Nonlinear isotropic Hooke formulation

Some general aspects

Why splitting volumetric and deviatoric (shearing)?

Thermo-elasticity

Hyperelasticity

Rubber or rubber-like Elasticity

Terminology and some definitions

Thermodynamics of rubber – enthalpic and entropic forces

Some classical models

Neo-Hookean model

Mooney-Rivlin model

Yeoh model

Ogden model

Example of Rubber Elasticity In Abaqus

W. Gilbert's experiment

On thermodynamics of elastomers

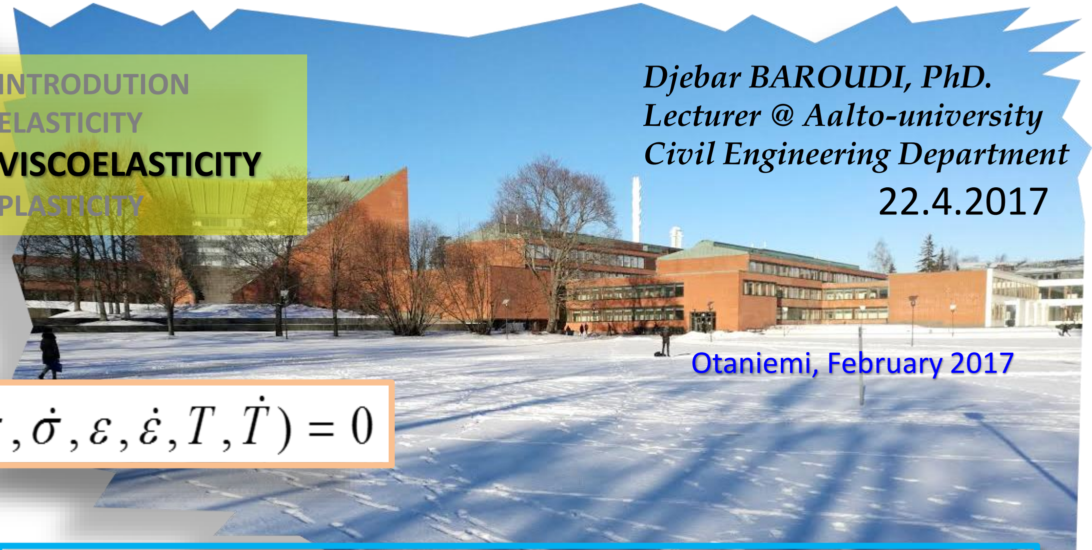
Homework

Appendix 1
Stress invariants (Recall)

Appendix 2
On Thermodynamics of Rubber
Enthalpic and Entropic forces

"The mountains **flowed** before the Lord"
(The Song of **Deborah**, Bible)
דְּבוֹרָה

Djebar BAROUDI, PhD.
Lecturer @ Aalto-university
Civil Engineering Department
22.4.2017



Otaniemi, February 2017

Viscoelasticity

$$\mathcal{F}(\sigma, \dot{\sigma}, \varepsilon, \dot{\varepsilon}, T, \dot{T}) = 0$$

Content

- **Experimental observations: evidence of viscoelastic behavior**
- **Stress relaxation at constant strain**
- **Creep at constant stress**
- **Strain-rate dependence**
- **Constitutive models in the rate form**:
  - Maxwell model
  - Kelvin-Voight model
  - Standard linear solid model
  - Burgers model
  - Generalized Maxwell model
  - Kelvin chain model

Reading: Textbooks

- Lemaitre and Chaboche – *Mechanics of Solid Materials*. **Chapter 4.3**
- Ottosen & Ristinmaa – *Introduction to time-dependent material behaviour.* **Chapter 14**

Lemaitre & Chaboche textbook as an **e-book**:
http://proquestcombo.safaribooksonline.com.libproxy.aalto.fi/book/physics/9781107384712

Primary creep:

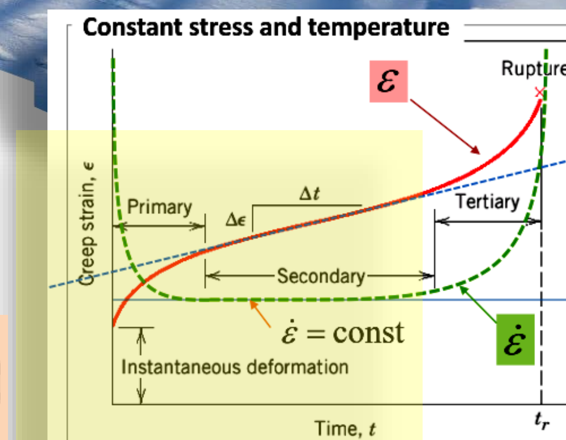
$$\varepsilon = A(\sigma)\cdot t^{1/\beta},$$

$$2 < \beta < 4$$

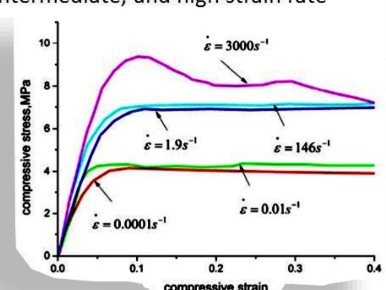
$$\varepsilon = A(\sigma)\cdot\left[t/t_{\mathrm{REF}}\right]^{1/\beta},$$

Secondary creep:

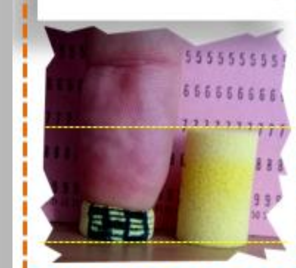
$$\dot{\varepsilon} = K_2\left[\frac{\sigma}{\sigma_{\mathrm{Ref}}}\right]^n \exp\left(-\frac{Q_c}{RT}\right)$$



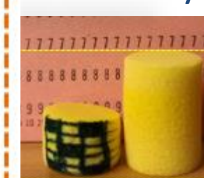
Experimental: Compressive responses of balsa wood at static, intermediate, and high strain rate



Initial loading

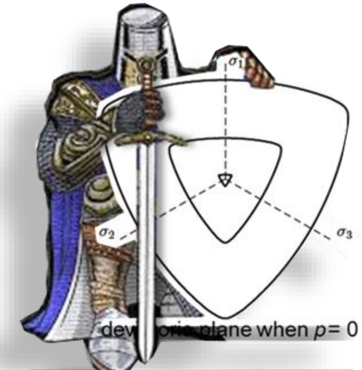


Partial recovery



Content

Engineering **Plasticity**



$$A\frac{J_2}{\sigma_c}+\Lambda\sqrt{J_2}+BI_1-\sigma_c=0,$$

Motivation

- Course of materials modelling in other universities
- Literature & textbooks
- Some historical notes on engineering plasticity
- Motivation: How engineering Plasticity is seen in Abaqus, Ansys, Lusas?
- Stress invariants
- Examples of Failure of Structures
- What is failure? Types of failures, failure envelopes and failure criteria

Plasticity

Failure hypothesis or Yield criteria

Plasticity Isotropic & Isothermal Rate-Independent

- Examples
- Some basic physics for Engineering Plasticity
- Plastic basic behavior in simple tension & compression

Modelling of uniaxial behavior in plasticity – simplified models

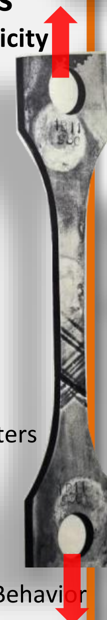
- Elastic-Perfectly Plastic Model
- Elastic-Linear Work-hardening model
- Elastic-Exponential Hardening model
- Ramberg-Osgood model
- **Tangent- and plastic modulus**
- **Hardening rules**
- **Elastic-plastic behaviour – cyclic loading**
  - ➢ **Worked uniaxial example – analytical & Abaqus**
- Loading history dependency and **strain hardening effects**
- Homework: Uniaxial Elastic-plastic behaviour : ex #1 ex #2
- Some examples of solved problems in Plasticity
  - ✓ **Plastic limit load and displacement-force relation in bending**

Engineering Plasticity

Classical theory – fundamentals

The three ingredient of the classical plasticity theory

- **Yield criteria**
- **Flow rule**
- **Hardening rule**



Yield Criteria

Pressure independent Yield criteria

- Tresca yield Criterion
- Von Mises yield Criterion

Pressure dependent Yield criteria

- Mohr-Coulomb Criterion
- Drucker-Prager Criterion
- Ottosen (1977) developed a 4-parameters failure criterion for concrete
- Hoek–Brown failure criterion
- Mohr-Coulomb yield criterion
- The Cam-Clay model

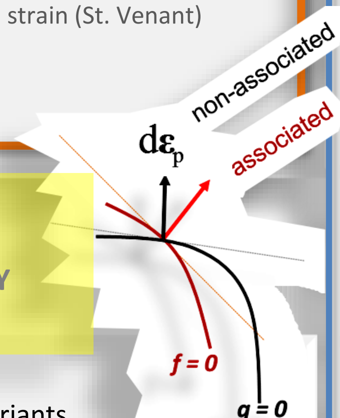
(good to know) Example of material Behavior of Clay and Silt in Otaniemi

Other types of failure criteria

- Maximum Principle Stress Criteria (Rankine)
- Maximum Principal strain (St. Venant)

Anisotropic yield criteria

...



1. INTRODUCTION
2. ELASTICITY
3. VISCOELASTICITY
4. **PLASTICITY**

Appendices

Appendix 1: Stress invariants

Appendix 2: Recommended compulsory reading

Continued

Hardening – notions

- Hardening Rules
- Examples of simple rheological models for Rate-independent plasticity
- Examples of hardening rules in Abaqus – how they looks like?

Flow rules

Flow rule & Consistency condition

Plastic strain increment

Principle of maximum plastic work

Normality rule

Consistency Condition

Associative and Non-associate Plasticity

Convexity of the criterion

Normality of the plastic flow

Some application examples of associated and non-associated plasticity

Incremental Stress-Strain Relationships

Example of a flow rule for isotropic hardening

Examples of hardening rules in Abaqus – how it looks like?

EOL

Course of **materials modelling** in other universities

TECHNISCHE UNIVERSITÄT DARMSTADT

Technische Universität Darmstadt

9.10.2016

13-02-0003-vl Werkstoffmechanik

Veranstaltungsdetails

Lehrende: Prof. Dr.-Ing. Michael Vormwald; Dipl.-Ing. Melanie Fiedler

Veranstaltungsart: Vorlesung

Orga-Einheit: FB13 Bau- und Umweltingenieurwissenschaften

Anzeige im Stundenplan: Werkstm. (V)

Fach:

Anrechenbar für:

Semesterwochenstunden: 3

Unterrichtssprache: Deutsch

Min. | Max. Teilnehmerzahl: - | -

Lehrinhalte:

- Klassifizierung der Phänomene des Deformations- und Festigkeitsverhaltens
- Lineare Elastizität
- Isotropie, Anisotropie (Orthotropie, transversale Isotropie)
- Elastoplastizität
- Idealplastizität, Isotrope und kinematische Verfestigung
- Viskoelastizität, Viskoplastizität
- Werkstoffgesetze für Stahl, Beton, Glas, Holz, Kunststoffe und Geomaterialien
- Numerische Umsetzung

Today subject

... for comparison of the courses contents at **two universities to show the relevance and importance of such course content for CIV-engineers**

Aalto-university, summer 2016

Content:

Djebar BAROUDI, PhD
Aalto-University

- **Elasticity – kimmoisuus tai elastisuus**
  (linear, hyper-elasticity, isotropy, anisotropy, orthotropy)
- **Viscoelasticity - viskoelastisuus**
- **Viscoplasticity** or creep – **viskoplastisuus** ... viruminen
- **Failure hypotheses - lujuushypoteesit**
- **Plasticity - plastisuus**
  associative, non-associative
- **Damage - vauriotuminen**
  damage-plasticity *ex.* Concrete Damage Plasticity, Model in Abaqus

If time...

Ref: thanks go to an exchange student for providing the course content-list above

Massachusetts Institute of Technology

Lecture 12: Fundamental Concepts in Structural Plasticity - MIT ...

[PDF] Unit 4 Equations of Elasticity

[PDF] engineering viscoelasticity - MIT

[PDF] Constitutive Equations - ...

[PDF] Lecture XI: Introduction ...

[PDF] Development of Constit...

[PDF] mechanical properties o...

[PDF] engineering viscoelastic...

Lecture 15: Elastic Constitut...

[PDF] Unit 4 Equations of Elas...

16.20 Structural Mechanics, ...

[PDF] Lecture 15 Study Guide - Use of Elastic Constitutive Relation...
https://ocw.mit.edu/resources/res-2-002...for.../MITRES2_002S10_lec15.pdf
Use of Elastic. Constitutive. Relations in Total. Lagrangian. Formulation. • Basic cons...
modeling **material** response. • Linear and nonlinear elasticity.

[PDF] engineering viscoelasticity - MIT
web.mit.edu/course/3/3.11/www/modules/visco.pdf
by D Roylance - 2001 - Cited by 298 - Related articles
Oct 24, 2001 - of Nonlinear **Viscoelastic Materials**, Dover Publications, New York, 1989 ..... This expression is a "**constitutive**" equation for our fictitious Maxwell ...

[PDF] Module 3 Constitutive Equations - MIT
web.mit.edu/16.20/.../3_Constitutive/Constitutive_files/module_3_no_solutions.pdf
particular the fourth-order **elasticity** or stiffness tensor describing Hooke's **Law**. ... **relations** for a linear elastic **material** exploiting these symmetries as follows:.

[PDF] mechanical properties of materials - MIT
web.mit.edu/course/3/3.225/book.pdf
elastic but nonlinear), and not all linear **materials** are elastic (**viscoelastic materials** ... The stress-strain, or "**constitutive**," **law** of the **material** must be extended to ...

3.11 Mechanics of Materials F01 - MIT
web.mit.edu/course/3/3.11/www/lectures.html
Hooke's law and **constitutive equation** for elastic behaviour (*Readings: ... Roylance's Introduction to Composite **Materials**) ... **VISCOELASTICITY** (2 weeks):

[PDF] Structural Analysis of Viscoelastic Materials
https://stuff.mit.edu/afs/athena/course/3/3.91/www/slides/williams_aiaa.pdf
by ML WILLIAMS - Cited by 238 - Related articles
Although **viscoelastic materials** have been incorporated as .... first is the manner in which the **viscoelastic** stress-strain law ...... upon the **constitutive equation**.

MIT OpenCourseWare | Materials Science and Engineering | 3.91J ...
https://dspace.mit.edu/bitstream/handle/1721.1/41943/.../Materials.../index.htm
Stress, Transformations of Stress and Strain, and Hookean **Elasticity** ... Matrix form of **constitutive equations** (PDF) ... Composite **Materials** and Rule of Mixtures.

[PDF] A Fractional K-BKZ Constitutive Formulation for Describing the ...
web.mit.edu/nnf/publications/GHM221.pdf
K-BKZ framework and suitably modify it for power-**law materials** exhibiting Mittag- ... **constitutive** model and damping function results in a nonlinear **viscoelastic** ...

[PDF] Part 1: Continuum mechanics and ...

[PDF] Formulations of viscoelastic constit...

[PDF] Lecture 12: Fundamental Concepts in Structural Plasticity - MIT ...
https://ocw.mit.edu/courses/mechanical.../2-080j.../MIT2_080JF13_Lecture12.pdf
Plastic properties of the **material** were already introduced briefly earlier in the ..... The elastic **constitutive equation** can also be written in an alternative form, ...

Constitutive equations and failure criteria for amorphous polymeric ...
https://dspace.mit.edu/handle/1721.1/17543
by BP Gearing - 2002 - Cited by 10 - Related articles
Constitutive equations and failure criteria for amorphous polymeric solids. Research and Teaching Output of the **MIT** Community ... 1988; Arruda & Boyce, 1993) on modeling the **plastic** deformation of amorphous polymers. ... an amorphous **material** based on the notion that the **constitutive relations** for such **materials** should ...

[PDF] mechanical properties of materials - MIT
web.mit.edu/course/3/3.225/book.pdf
The stress-strain, or "**constitutive**," **law** of the **material** must be extended to include these .... and growth, while shear stresses underlie yield and **plastic** slip.

[PDF] Lallit Anand - MIT MechE - Massachusetts Institute of Technology
meche.mit.edu/sites/default/files/cv/anand_cv_2017_0.pdf
Special issue of the International Journal of **Plasticity** in Honor of Lallit Anand, Volume ... **Constitutive equations** for the rate-dependent deformation of **metals** at.

[PDF] Continuum Mechanics - MIT
web.mit.edu/abeyaratne/Volumes/RCA_Vol_II.pdf
by R Abeyaratne - Related articles
May 11, 2012 - 2.073: Solid Mechanics: **Plasticity** and Inelastic Deformation,. 2.075: Advanced ..... 8.5.2 Imposing Symmetry Requirements on **Constitutive** Response Functions.218 ... 9 Elastic **Materials**: Micromechanical **Models**. 253.

Modules | Mechanics of Materials | Materials Science and Engineering ...

[PDF] Module 3 Constitutive Equations - MIT
web.mit.edu/16.20/.../3_Constitutive/Constitutive.../module_3_with_solutions.pdf
MODULE 3. **CONSTITUTIVE EQUATIONS** ρ σ. 1. E. ˆ ψ = 1. 2. Eq2. Figure 3.1: Stress-strain curve for a linear **elastic material** subject to uni-axial stress σ (Note.

16.20 Structural Mechanics, Spring 2013 | 3. Constitutive Equations
web.mit.edu/16.20/homepage/3_Constitutive/Constitutive.html
Understand basic stress-strain response of engineering **materials**. Quantify the linear **elastic** stress-strain response in terms of tensorial quantities and in ...

[PDF] Block 3 -Materials and Elasticity Lecture M17: Engineering Elastic ...
https://ocw.mit.edu/courses/aeronautics-and.../16.../materials.../zm17_20.pdf
continuum version of a **constitutive law** - at least for linear **elastic materials** spq = E ?emn. **Elasticity**. Where does it come from? 2. Increasingly, **materials** are ...

[PDF] Elasticity (and other useful things to know) - MIT OpenCourseWare
https://ocw.mit.edu/courses/electrical-engineering.../6.../07lecture06split.pdf
Cite as: Carol Livermore, course **materials** for 6.777J / 2.372J Design and Fabrication of Microelectromechanical ... **Constitutive equations** of linear **elasticity**.

[PDF] mechanical properties of materials - MIT
web.mit.edu/course/3/3.225/book.pdf
Elasticity is a form of **materials** response that refers to immediate and .... The stress-strain, or "**constitutive**," **law** of the **material** must be extended to include these ...

[PDF] Unit 4 Equations of Elasticity
https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-20...fall.../unit4.pdf
MIT - 16.20. Fall, 2002. Unit 4 ... Stress - Strain Relations (**Constitutive Relations**) ..... The "type" of **material** (with regard to **elastic** behavior) dictates the number.

[PDF] Constitutive Equations - Massachusetts Institute of Technology
web.mit.edu/course/3/3.11/www/modules/const.pdf
by D Roylance - 2000 - Cited by 10 - Related articles
Oct 4, 2000 - With these **constitutive relations**, the vital role of the **material** is reasserted: ... **material**. Isotropic **elastic materials**. In the general case of a linear ...

[PDF] Lecture 15 Study Guide - Use of Elastic Constitutive Relations in Tot...
https://ocw.mit.edu/resources/res-2-002...for.../MITRES2_002S10_lec15.pdf
Use of **Elastic**. **Constitutive**. **Relations** in Total. Lagrangian. Formulation. • Basic considerations in modeling **material** response. • Linear and nonlinear **elasticity**.

[PDF] E - Massachusetts Institute of Technology
web.mit.edu/16.unified/www/FALL/materials/Lectures/M3.2-Unified08.pdf
MIT - 16.001/16.002. Fall, 2008 ... ...explain the meaning of the **elasticity** and compliance tensors and ... ...employ a continuum version of the **constitutive law of elasticity** ... Different types of **material** exhibit different stress-strain behavior.

Linear elasticity in anisotropic materials

- General case:
  - Stress is a second rank tensor
  - Strain is a second rank tensor
  - Elastic constants form a fourth rank tensor
- There is lots of symmetry in all the tensors
- Can represent stress as a 1 x 6 array and strain as a 1 x 6 array
- The elastic constants form a 6 x 6 array, also with symmetry

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{pmatrix}$$

Cite as: Carol Livermore, course materials for 6.777J / 2.372J Design and Fabrication of Microelectro... OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD...

Example of a lecture slide on **Elasticity @ MIT…yes**, student has to **begin from the basics to become a master** and then he can innovate, otherwise *innovointi ja luovuus insinöörisovellutuksissa on yhtä tyhjän kanssa …vaan kansan viihdettä (DBA, 2017)*