Aalto University
School of Science

Influence Diagrams

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Presentation
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MS-E2191 Graduate Seminar on Operations Research
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Motivation

Modelling of decision problems

Decision trees

- Display alternatives and outcomes explicitly

⇨ Explode in complexity as the number of variables increases

Need for a **flexible** way to describe **dependencies** and **independencies** among random variables and decisions

Mathematical Basis

Probabilistic independence: $P(A, B) = P(A) \cdot P(B)$

Chain rule: $P(A, B, C) = P(A \mid B, C) \cdot P(B \mid C) \cdot P(C)$

Bayes' theorem: $P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$

Law of total probability: $P(A) = \sum_B P(A \mid B) P(B)$

Components

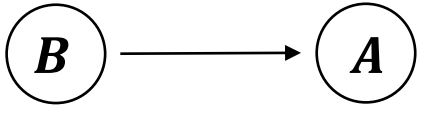
Nodes

Decision nodes □ and chance nodes ○

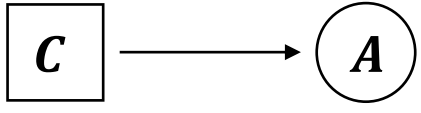
Relationships

Let $\boldsymbol{A}$, $\boldsymbol{B}$ be random variables and $\boldsymbol{C}$, $\boldsymbol{D}$ decisions:

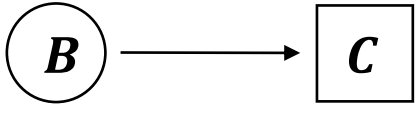
$\boldsymbol{P(A \mid B)}$, probability of $\boldsymbol{A}$ depends on outcome of $\boldsymbol{B}$



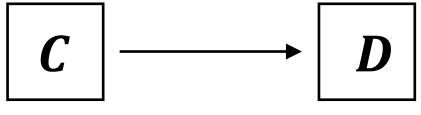
$\boldsymbol{P(A \mid C)}$, probability of $\boldsymbol{A}$ depends on decision $\boldsymbol{C}$



$\boldsymbol{\{C \mid B\}}$, decision $\boldsymbol{C}$ is based on the outcome of $\boldsymbol{B}$



$\boldsymbol{\{D \mid C\}}$, decision $\boldsymbol{D}$ is based on the decision $\boldsymbol{C}$



Graphical Manipulation

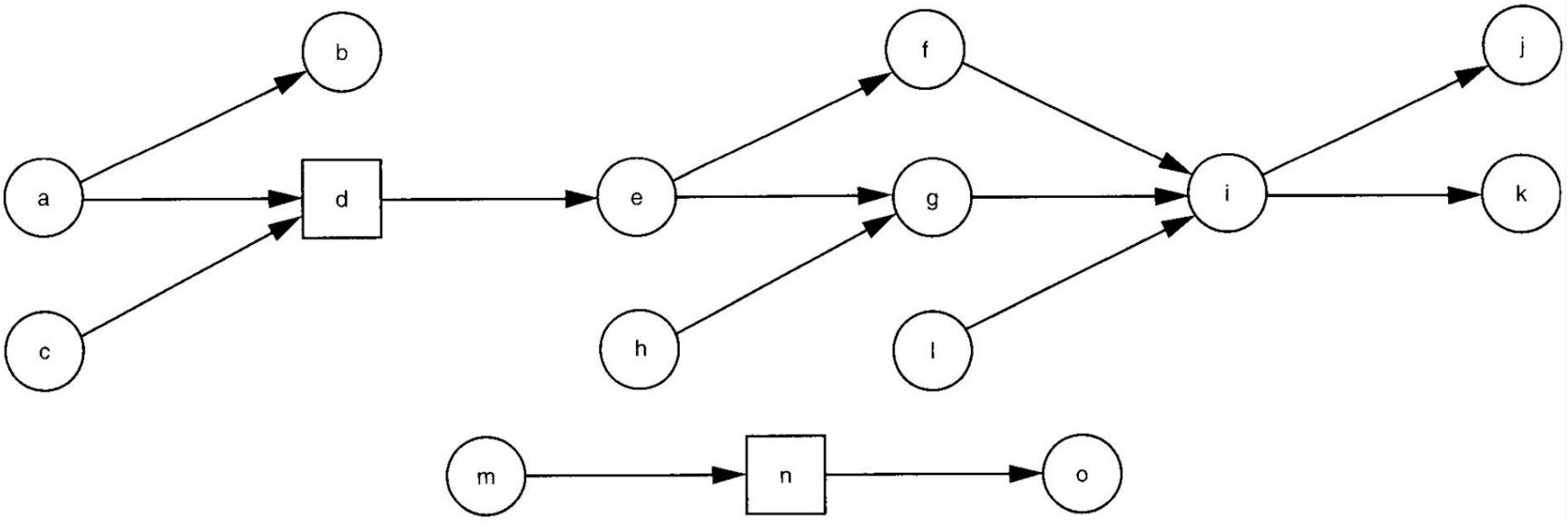
Permitted operations

1. Adding an arrow between two **chance** nodes
2. Adding an arrow **from** a **decision** node **to** a **chance** node
3. Reversing an arrow between two **chance** nodes that are based on the **same state of information**

Exception

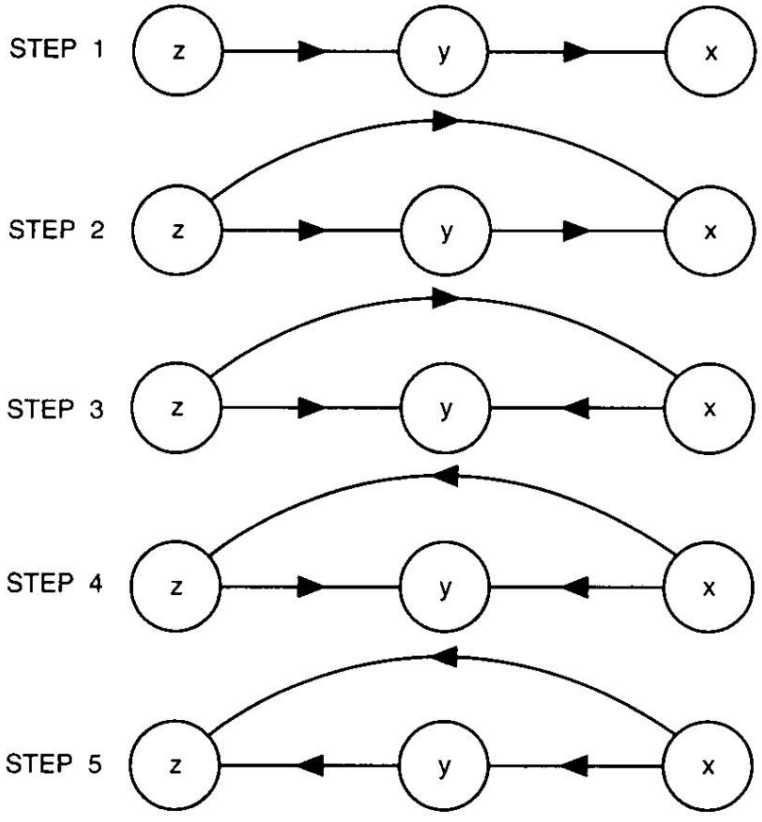
Loops are prohibited

Influence Diagram with Decision Nodes

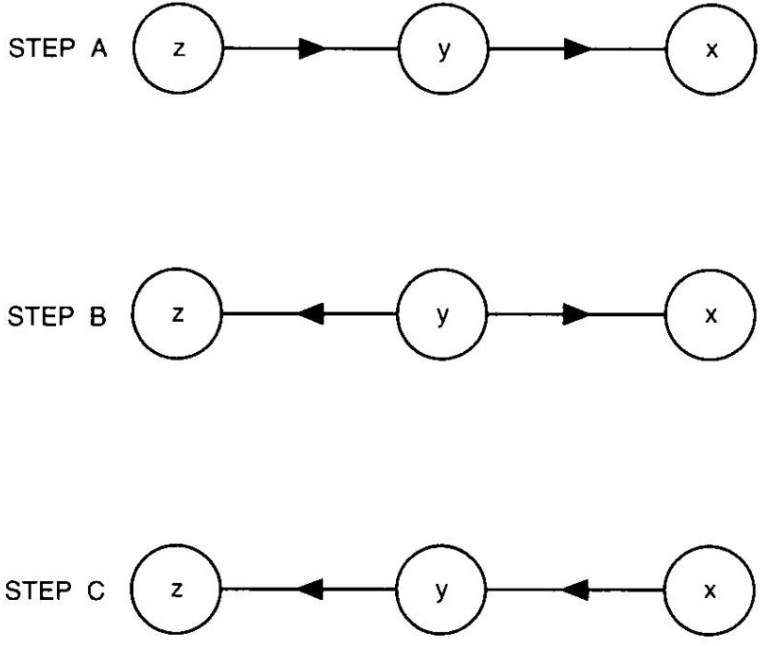


Howard, R.A. & Matheson, J.E. 2005, Influence Diagrams, Decision Analysis, Vol. 2, No. 3, pp. 127-143.

Non-Unique Results



(a) FIRST SEQUENCE OF MANIPULATIONS



(b) SECOND SEQUENCE OF MANIPULATIONS

Howard, R.A. & Matheson, J.E. 2005, Influence Diagrams, Decision Analysis, Vol. 2, No. 3, pp. 127-143.

Solving for Optimal Strategy

Influence diagram

- Specific (nonunique) order for conditional probabilities
- Information available as the basis for each decision S

Joint probability distributions $P(A, B, C \mid S)$

- Initial probability assignments $P(A \mid S)$
- Conditional probability assignments $P(B \mid A, S)$, $P(C \mid A, B, S)$
- Decision rules

Expected values for each outcome

$$\mathrm{E}(A, B, C \mid S) = \sum_{A,B,C} V(A, B, C) \cdot P(A, B, C \mid S)$$

- Value function $V(A, B, C)$

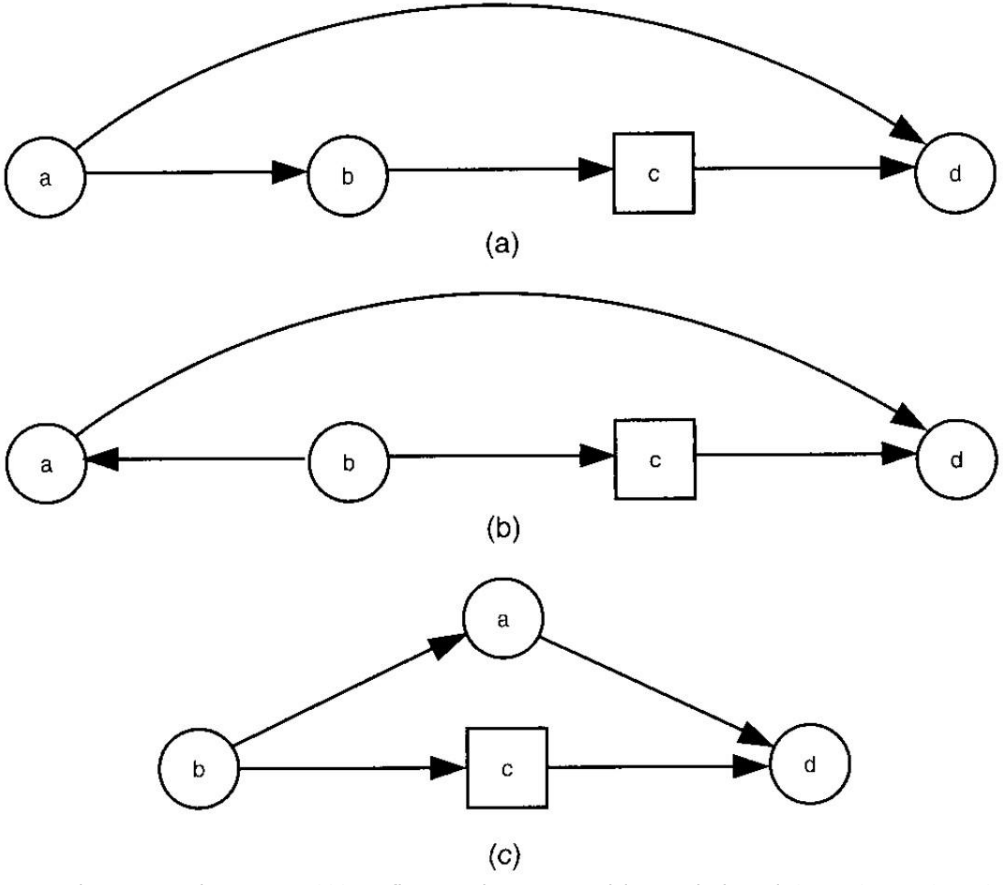
Classes of Influence Diagrams

Decision tree network

Decision network

Single DM	No forgetting	Direct correspondance to a decision tree
Total ordering among decisions	Each decision is linked to preceding decisions and their bases	A decision is linked to each node preceding it

Converting a Decision Network to a Decision-Tree Network



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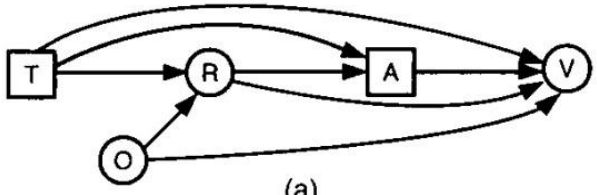
Example: The Used-Car Buyer

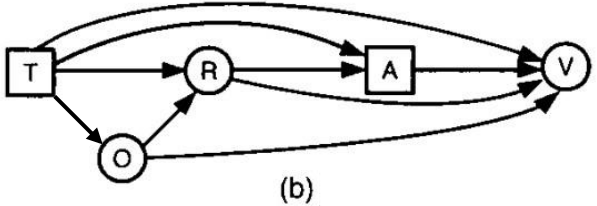
Converting an Influence Diagram to a Decision Tree

The buyer can:

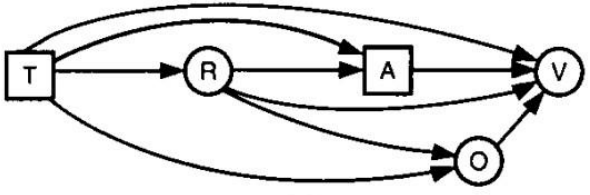
perform **test T**
with **result R,**
choose **car A**
with **condition O**
and receive **value V**



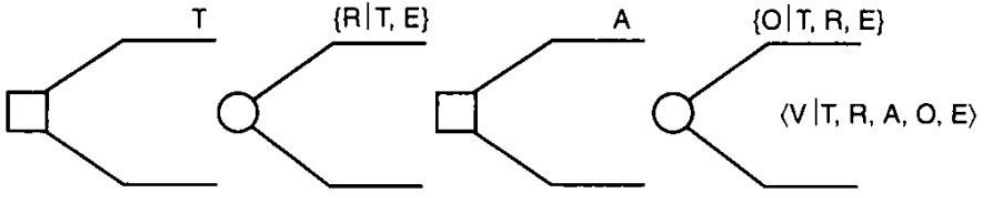
(a)



(b)



(c)



(d)

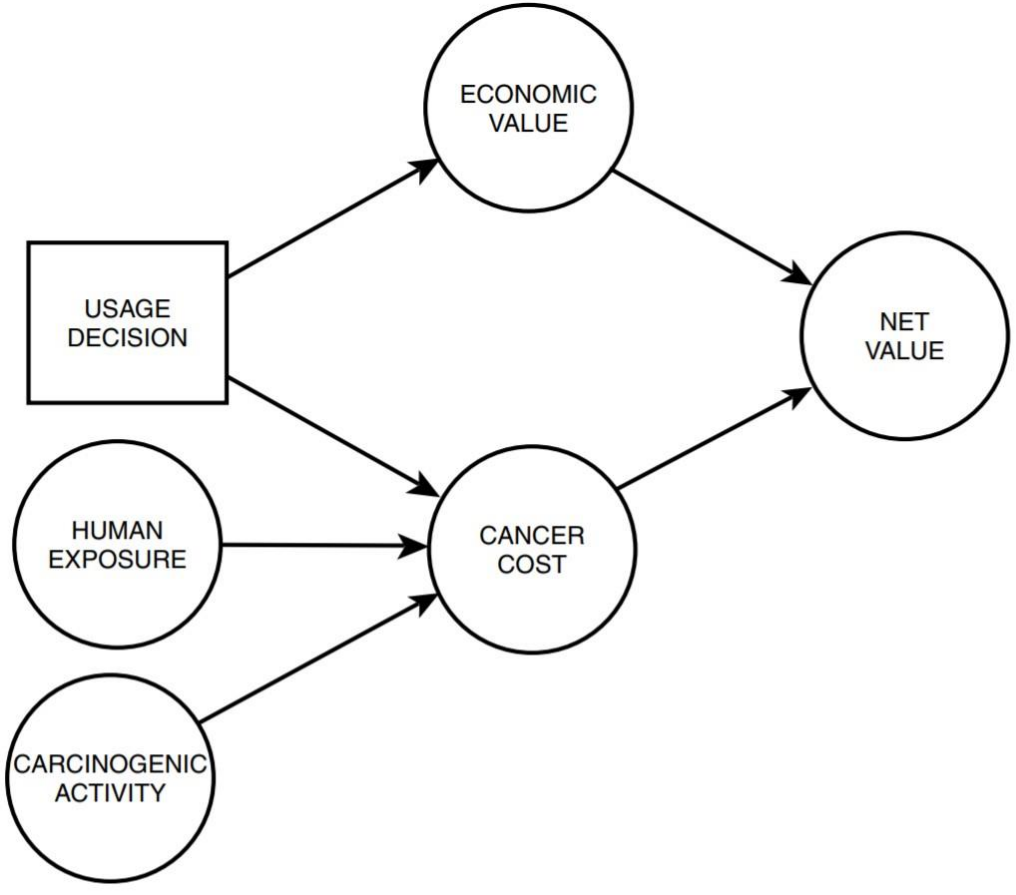
Howard, R.A. & Matheson, J.E. 2005, Influence Diagrams, Decision Analysis, Vol. 2, No. 3, pp. 127-143.

Example: Toxic-Chemical Testing

Influence Diagram for Primary Decision

Decision:

1. **Permit**
2. **Restrict**
3. **Ban**



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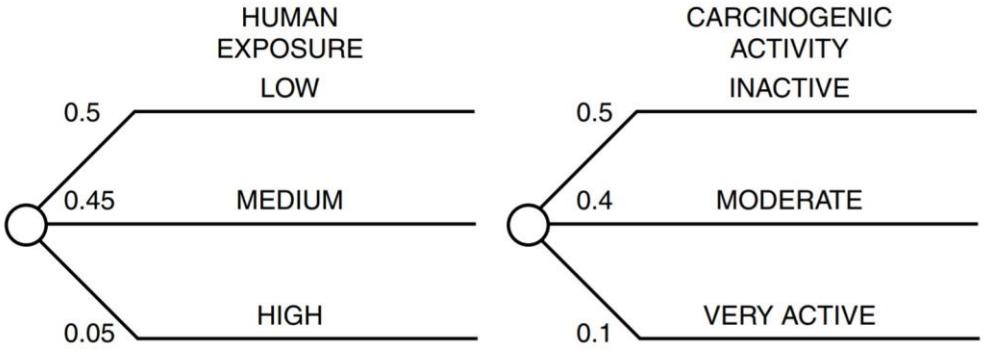
Probability Assignments and Value Functions

It costs
0M$ to **permit**,
1M$ to **restrict**
and 5M$ to **ban**
the alternative.

Initial Probability Assignments



Cancer Cost ($millions)

	PERMIT ALTERNATIVE			RESTRICT ALTERNATIVE			BAN ALTERNATIVE		
	Exposure			Exposure			Exposure		
Activity	Low	Med	High	Low	Med	High	Low	Med	High
Inactive	0	0	0	0	0	0	0	0	0
Moderate	0.5	5	50	0.05	0.5	5	0	0	0
Very active	10	100	1000	1	10	100	0	0	0

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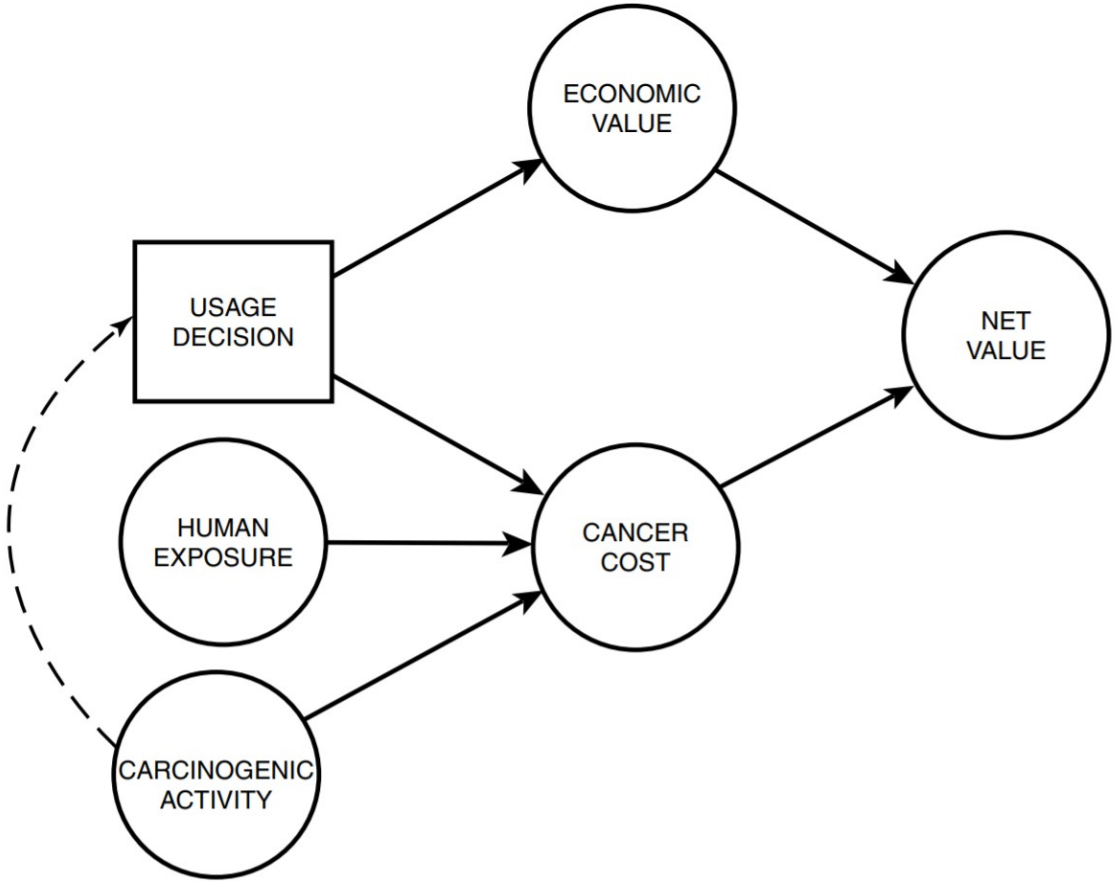
Expected Value of Perfect Information

Upper limit for the price of a variable's value in advance



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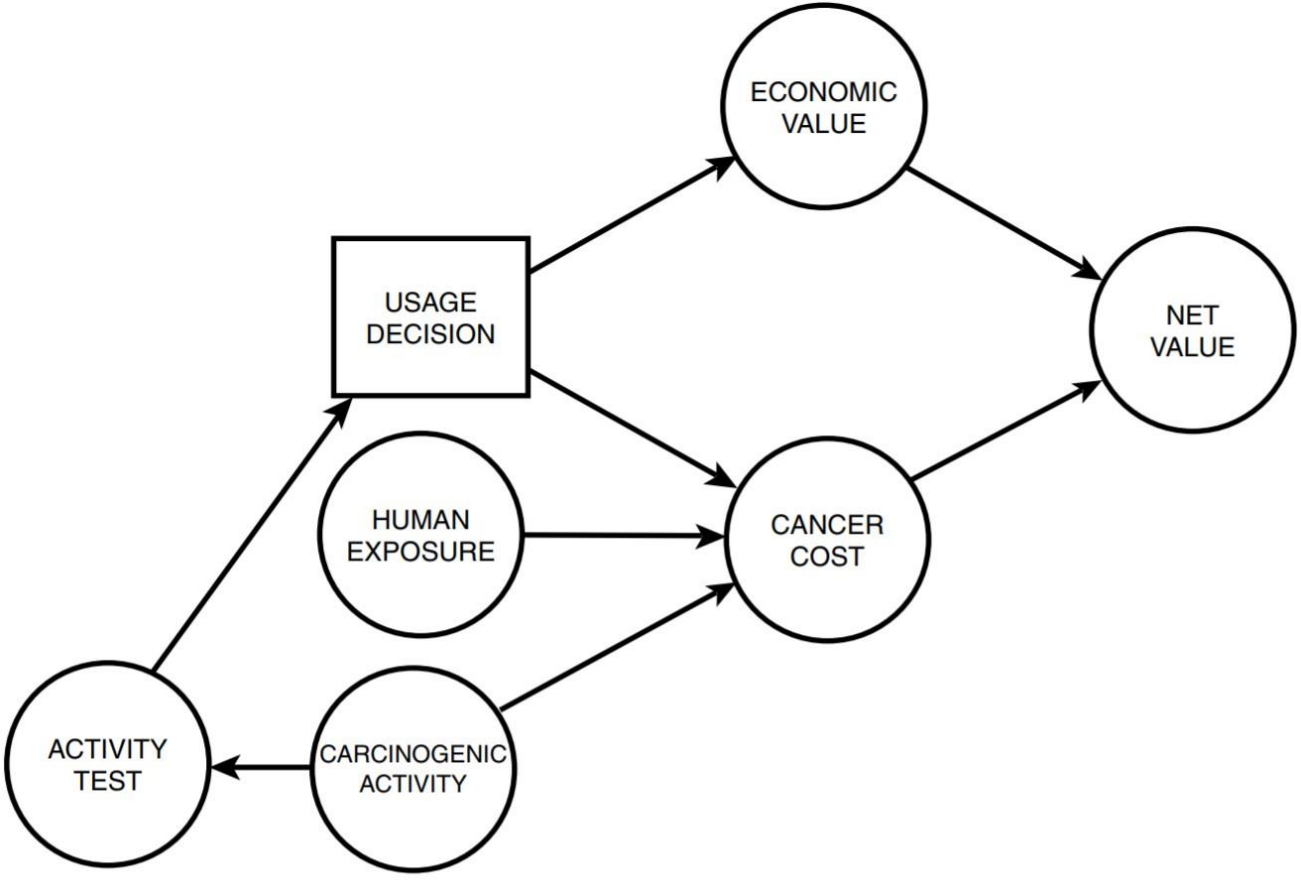
Expected Value of Sample Information

Upper limit for the price of imperfect information on variable's value



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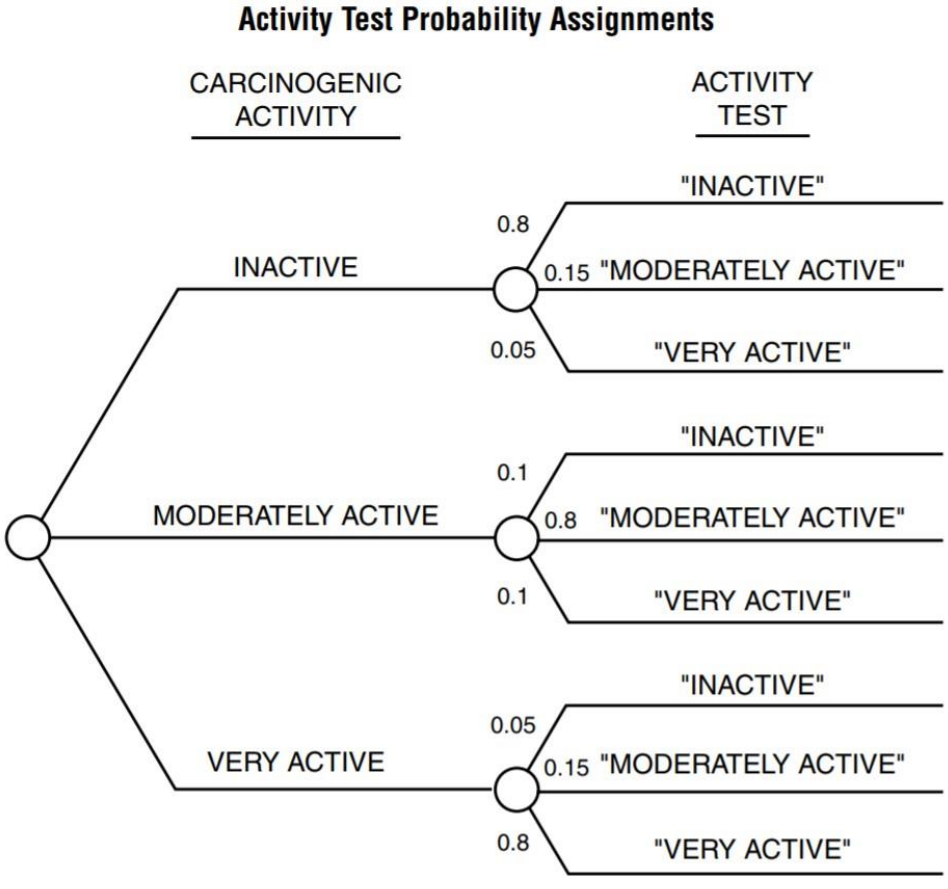
Likelihood Function of Activity Test



Optimal strategy

ACTIVITY INFORMATION
VALUE WITH = −1.96
VALUE OF = 0.25
DECISION RULE

"INACTIVE"	RESTRICT
"MODERATELY ACTIVE"	RESTRICT
"VERY ACTIVE"	BAN

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Summary

Influence diagrams

+ **Flexible** way to describe **dependencies** and **independencies** among random variables and decisions
+ Easy to model **complex systems**
- Require additional lists of decision alternatives and node states

You can

- Solve for **optimal strategy**
- Convert to a **decision tree**
- Manipulate to ask questions about **EVPI** and **EVSI**

Source

Howard, R.A. & Matheson, J.E. 2005, Influence Diagrams, Decision Analysis, Vol. 2, No. 3, pp. 127-143.

Homework

Consider the example Toxic-Chemical Testing:

Suppose that you have received information that the human exposure of the alternative is medium. Given the influence diagram for primary decision (slide 14) and the related probability assignments and value functions (slide 15), solve the optimal strategy using Excel.

Return your solution to kalle.alaluusua@aalto.fi by 25.9. 09:15.