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Utilization of optimization models in decision trees: the Contingent portfolio programming method

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Presentation *6*

2.10.2020

MS-E2191 Graduate Seminar on Operations Research
Fall 2020

Presentation outline

- Problem description
- Problem formulation
- Solving the problem
- Conclusions and questions + homework

Problem description

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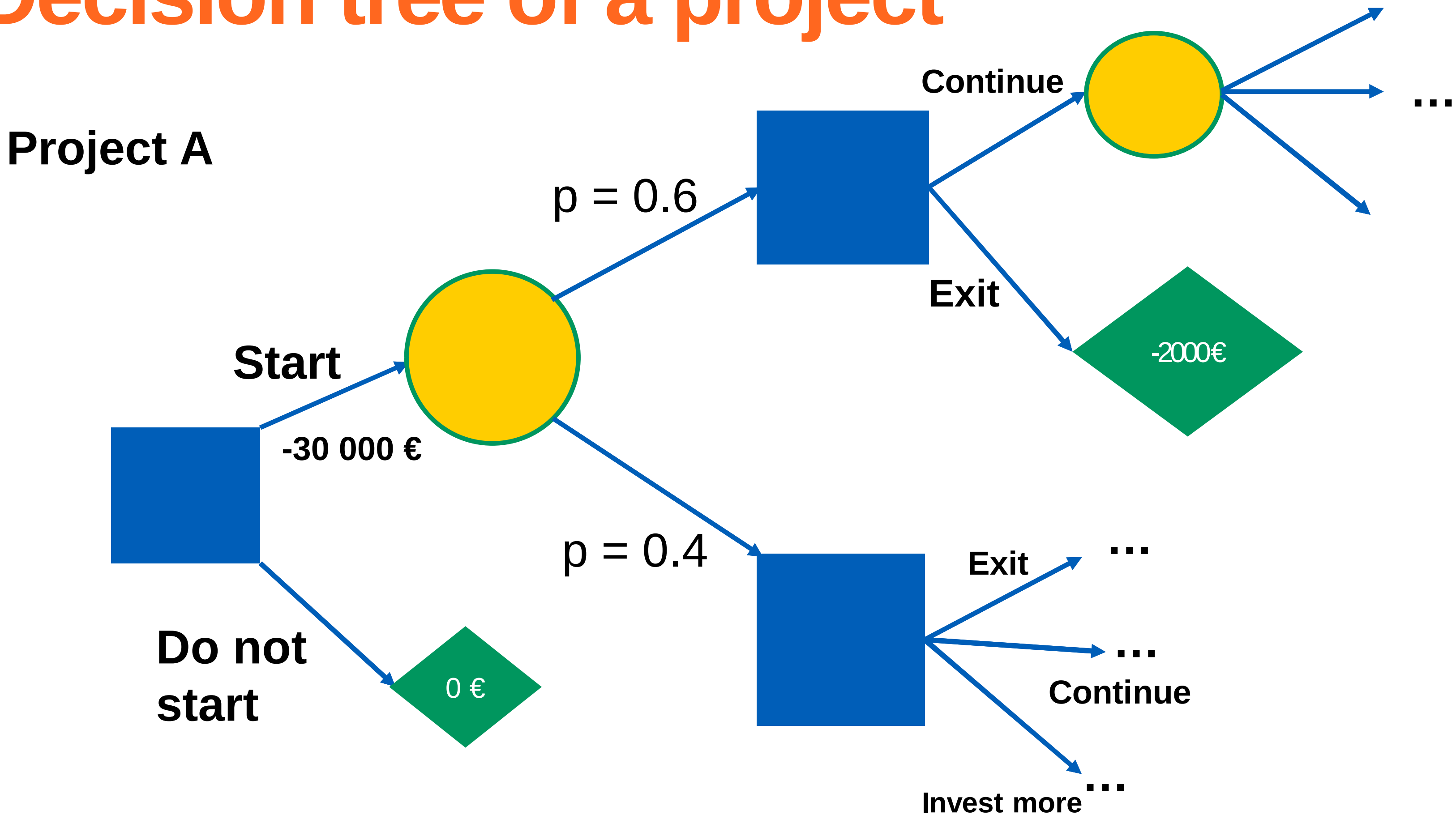
Portfolio programming

- The decision maker has the possibility to launch multiple R&D projects
- The projects have multiple stages in which the decision maker can decide how the projects are continued
- The goal is to create a plan describing which projects are started and how they are continued under different scenarios
- The decision maker has a limited amount of resources that can be allocated to the projects

Decision tree of a project



Differences between CPP and standard decision tree optimization

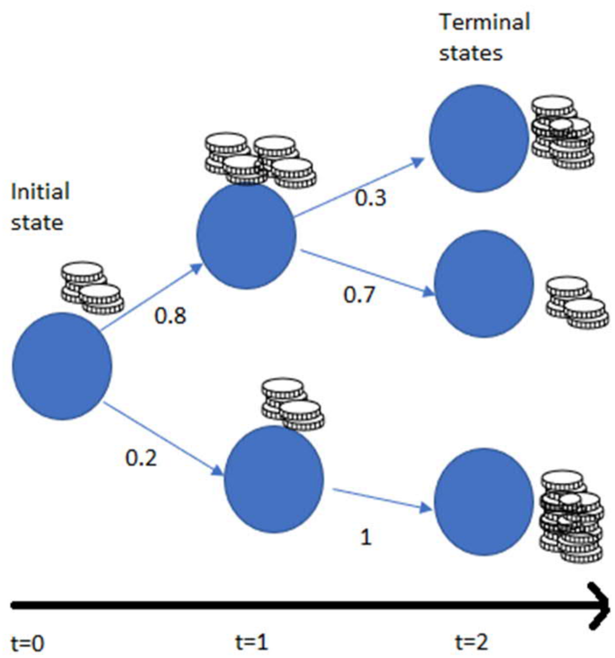
- There are multiple decision trees that cannot be solved individually
  - *Resource constraints apply to all projects*
  - *The chosen plan must be coherent with the decision maker's risk preferences*

- The chance nodes are usually shared between the project trees
  - *Government sets travelling restrictions*
  - *Pandemic ends*
  - *Other economic shocks*

  $\Rightarrow$ *In a chance node the state of the nature changes. A state tree can be drawn to visualize the probabilities of moving between these states.*

State tree

- Transitions between states are the only source of randomness
- Once a portfolio management strategy is fixed, the amount of resources at time t depends only on the state s_t
- The decision maker is interested in the probability distribution of resources at the end of the planning horizon



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Problem formulation

- *Variables*
- *Constraints*
- *Objective function*

Known information

- A state tree with probabilities
- Project trees with costs and payoffs
- Initial holdings of resources
- Other additional parameters (for example risk free interest rate)

Variables: Action variables

- For each possible choice a (binary) variable is introduced
  - Action variables X_i
- The action variables can be indexed using the project, state of the nature, and choices that are possible to make in the decision point

Variables: Surplus variables

- For each resource and state, there is a variable describing the amount of the resource
  - Surplus variables RS_s^r
- These variables are typically continuous and can be constrained to only attain nonnegative values

Variables: Deviation variables

- For each terminal state, there are two variables describing the difference of the outcome compared to a target
  - Deviation variables ΔVs_i^+ and ΔVs_i^-
- For each terminal state, either one of these variables is always zero depending on whether the outcome is better or worse than the target
- These variables are used later when the objective function is defined, and we will return to these later

Constraints: Consistency

- At the first decision points of each project exactly one alternative is chosen
  - $X_a + \cdots + X_n = 1$
- Further choices are only planned if they can follow a choice made before
  - $X_a + \cdots + X_n = X'$
- Sometimes allowing the sum to be more than one can be allowed (for example if an investment can be done multiple times)

Constraints: Resources

- The idea is to model the dependencies of resources between different state
- For example, if there is only one project which can be started for a cost of 100 euros and the initial amount of money is 500 euros, we have constraint $500 - 100X_{start} = RS_{s0}^{money}$
- If in the next state, the project can be continued for 200 euros and the risk-free interest rate is 1%, we have constraint $1.01RS_{s0}^{money} - 200X_{continue} = RS_{s1}^{money}$
- If loaning money is not allowed, $RS_{si}^{money} \geq 0$

Constraints: Deviation constraints

- For each terminal state, the deviation variables should describe the deviation from a target value t.
- For example, if money is the only interesting resource $RS_{terminal\_i}^{money} - t + \Delta_{terminal\_i}^{-} - \Delta_{terminal\_i}^{+} = 0$

Objective function

- In the end of the planning horizon, the decision maker has some amount of resources
- The amount of resources is random
- The decision maker tries to maximize the certainty equivalent of the resources
- The value in a terminal state s is additive and linear with respect to the resource standings, $V_s = \sum w_s^r RS_s^R$

Objective function

- The certainty equivalent can be approximated in many ways
- The authors suggest using lower semi-absolute deviation (LSAD) or expected downside risk (EDR) which describe the expected shortfalling from the plans expected outcome (LSAD) or from a fixed value t (EDR).
- Using EDR, the objective function is $CE = EV - \lambda * EDR$
- In this function, parameters λ and t allow to include the risk preference of the decision maker to some extent
- The expected value EV can be expressed using the surplus variables while the EDR/$LSAD$ can be written with help of the negative deviation variables ΔV_i^-

Recap: Problem formulation

- **Constraints**
  - Consistency
  - Resources
  - Deviation constraints
  - (Optional constraints)
- **Objective function**
  - Maximize expected utility
  - The authors state that reasonable risk aversion can be modeled using EDR/LSAD without turning the problem into nonlinear optimization

Solving the problem

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Solving the problem

- If the constraints and the objective function are chosen as suggested, the problem falls to mixed integer programming and can be solved using standard linear programming solvers
- Solving the problem can become complicated if there are many project-specific risks. In this case, the state tree can become too big.

Reference

This presentation is based on the following article:

Gustafsson, J., Salo, A. (2005). Contingent portfolio programming for the management of risky projects. Operations Research 53: 946-956.

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Thank you for listening!

Homework problem

In the related Excel template, a problem related to planning with two projects is presented.

1) Fill in the missing consistency constraints
2) Fill in the missing resource surplus constraints
3) Fill in the missing deviation constraints
4) Formulate the objective function. The formula for EDR is $\sum_{x:x<t}(t-x)p(x)$.
5) Solve the optimal plan
6) Remember to return your solution (alvar.kallio@aalto.fi)

- If anything is unclear, feel free to contact