Aalto University
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# Scenario-based portfolio selection of investment projects with incomplete probability and utility information

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# Presentation outline

- Project portfolio selection
- Modeling incomplete information
- Portfolio dominance structures
- Method for selecting non-dominated project portfolios
- Application

# Project portfolio selection

- DM’s make one-time investment decisions on multiple projects that bring value
- Goal is to find the best selection of projects to create an optimal project portfolio
  - Project outcomes depend on future scenarios
  - Resource constraints
- Contingent Portfolio Programming (CPP) method introduced last week
  - Similarly to CPP, future scenarios affect each project in the portfolio
  - Contrary to CPP, investments are one-time decisions

# Problem formulation

- DM has the possibility to do one-time investments to different projects, leading to a project portfolio

  $m$ possible investment projects $X^0 = \{x^1, \dots, x^m\}$

- Each project has some outcome in different scenarios

  $n$ disjoint scenarios $\Omega = \{s_1, \dots, s_n\}$

  value of project $x^j$ in scenario $s_i$: $x^j(s_i)$

- Investment portfolio $X$ is a subset of investment projects $X^0$

  Total portfolio value in scenario $s_i$: $X(s_i) = \sum_{x^j \in X} x^j(s_i)$

# Problem formulation

- Probability of scenario $s_i$ is $p_i$

Expected value of portfolio $X$: $\mathbb{E}[X] = \sum_{i=1}^{n} p_i X(s_i)$

- Choice of portfolio is limited by resource constraints

$$Az(X) \leq B$$
$$z(X) \in \{0,1\}^m,\ z_j = 1 \Leftrightarrow x^j \in X$$

- DM has some utility function $u(t)$

# Incomplete information

- Obtaining accurate information on probabilities of different future scenarios is often difficult or impossible
  - Expert estimates may vary a lot
  - Experts might only give ordinal estimates on scenario probabilities
- Likewise giving exact estimates of DMs utility functions can be difficult
  - DMs may have conflicting risk attitudes or opinions

  ➡ How to give recommendations on project portfolios under incomplete utility and scenario probability information?

# Agenda

- Model incomplete information
- Define dominance relations to identify non-dominated project portfolios
- Introduce a framework for ruling out portfolios that are clearly worse than others under incomplete knowledge on utilities and probabilities
  - No “one portfolio to rule them all”

# Modeling incomplete information

- Scenario probabilities cannot be accurately estimated

  Set of all possible probabilities $P^0 = \{p \in \mathbb{R} \,|\, p_i \geq 0, \sum_{i=1}^{n} p_i = 1\}$

  Set of feasible probabilities $P = \{p \in P^0 \,|\, A_p p \leq B_p\}$

- Inequality constraints set based on statements on scenario probabilities

  “scenario 2 is at least as likely than scenario 1”

  “scenario 1 occurs with at most probability 50%”

# Modeling incomplete information

- Set of feasible utility functions

$$U^0 = \{u : \mathbb{R} \to [0,1] \mid u(t) \geq u(t') \forall t \geq t'\}$$

- Obtain $U \subseteq U^0$ with constraints set based on knowledge on DM's risk preference
  - For example, assume risk-neutral (linear utility function) or risk-averse (set up requirements for concavity) DM
- **Information set** $S = P \times U$

# Portfolio dominance structure

A way to measure, which portfolios outperform others

**Definition:** Portfolio $X$ dominates portfolio $X'$ with regard to information set $S = P \times U$, denoted $X \succ_S X'$, if

$\mathbb{E}[u(X)] \geq \mathbb{E}[u(X')]$ for all $(p, u) \in S$ and

$\mathbb{E}[u(X)] > \mathbb{E}[u(X')]$ for some $(p, u) \in S$

Portfolio dominates another if its expected utility is at least as high for all feasible scenario probabilities and utility functions and there's at least one case where the utility is strictly higher

# Example with complete information

| | s1 | s2 |
|---|---|---|
| X | 5 | 3 |
| X’ | 2 | 5 |
| p | 0.5 | 0.5 |

Does portfolio $X$ dominate $X'$, knowing that the DM is risk-neutral?

$\mathbb{E}[X] = 4$ and $\mathbb{E}[X'] = 3.5 \Rightarrow X$ dominates $X'$

# Dominance relations under incomplete information

For portfolios $X$ and $X'$ and information $U \subseteq U^0$, $P \subseteq P^0$ the following apply:

*i)* $X \succ_{P\times U} X' \Leftrightarrow X \succ_{ext(P)\times U} X'$

*ii)* $X \succ_{P^0\times U} X' \Leftrightarrow X(s_i) \geq X'(s_i)\ \forall i \in \{1,\ldots,n\}$

$X$ dominates $X'$ if under information set $P \times U$ if $X$ dominates $X'$ at the extreme points of the probability constraints

If there's no probability information, $X$ dominates $X'$ if portfolio $X$ has at higher or equal utility than $X'$ in each scenario and strictly higher in at least 1 scenario

# Finding the set of non-dominated portfolios

**Goal:** Find the set of non-dominated portfolios

**Step 1:**

Rule out all dominated portfolios under

$$X \succ_{P^0 \times U} X' \Leftrightarrow X(s_i) \geq X'(s_i)\ \forall i \in \{1,\ldots,n\}$$

$\Rightarrow$ Rule out all portfolios that have equal or worse outcome than some other feasible portfolio in all scenario

# Finding the set of non-dominated portfolios

**Step 2:**

In general case $P \times U \subseteq P^0 \times U^0$: determine minimum expected utility difference between each portfolio pair $X$ and $X'$ at each extreme point $\{p^1, \ldots, p^t\} = ext(P)$

$$\min_u \mathbb{E}[u(X)] - \mathbb{E}[u(X')] = \sum_{i=1}^{n} p_i u(X(s_i)) - \sum_{i=1}^{n} p_i u(X'(s_i))$$

$X \succ_S X'$ if the minimum is non-negative for all $p^j$ and positive for some $p^j$

# Example with incomplete probability and utility information

| | s1 | s2 |
|---|---|---|
| X | 5 | 3 |
| X’ | 2 | 5 |

Does portfolio X dominate X’?

The probability of scenario 1 is estimated at
$0.4 \leq p_1 \leq 0.5$

$\Rightarrow$ Extreme values of P: $(0.5, 0.5)$ and $(0.4, 0.6)$

No assumptions on DM’s risk preference yet

# Example with incomplete probability and utility information

| | s1 | s2 |
|---|---|---|
| X | 5 | 3 |
| X’ | 2 | 5 |

Extreme value $p = (0.4, 0.6)$:

Sort unique outcome values and notate $\hat{v} = (2,3,5)$, $\hat{u} = u(\hat{v})$

$$\min_u \mathbb{E}[u(X)] - \mathbb{E}[u(X')]$$
$$= 0.4u(5) + 0.6u(3) - 0.4u(2) - 0.6u(5) = -0.4u(2) + 0.6u(3) - 0.2u(5)$$

Difference in utility is minimized with utility function $\hat{u} = (0,0,1)$

$$\min_u \mathbb{E}[u(X)] - \mathbb{E}[u(X')] = -0.2$$

$\Rightarrow X$ does not dominate $X'$

# Example with incomplete probability and utility information

Assume risk-averse DM and add constraint for utility function

$$\frac{\hat{u}_j - \hat{u}_{j-1}}{\hat{v}_j - \hat{v}_{j-1}} \geq \frac{\hat{u}_{j+1} - \hat{u}_j}{\hat{v}_{j+1} - \hat{v}_j} \Leftrightarrow \frac{u(3) - u(2)}{3 - 2} \geq \frac{u(5) - u(3)}{5 - 3} \Leftrightarrow -2u(2) + 3u(3) - u(5) \geq 0$$

$u(2) = 0$ and $u(5) = 1 \Rightarrow u(3) \geq 1/3$

Difference in utility is minimized with $\hat{u} = (0, \frac{1}{3}, 1)$

$$\min_u \mathbb{E}[u(X)] - \mathbb{E}[u(X')] = -0.4u(2) + 0.6u(3) - 0.2u(5) = 0.6/3 - 0.2 = 0$$

If p = (0.5, 0.5): $\min_u \mathbb{E}[u(X)] - \mathbb{E}[u(X')] = 0.5u(5) + 0.5u(3) - 0.5u(2) - 0.5u(5) = 0.5u(3) - 0.5u(2) = 1/6$

$\Rightarrow X$ dominates $X'$

# Application

- Application of the framework to the global forest industry
- Expert evaluations of
  - 8 scenarios
  - 24 possible actions (projects)
  - value of each action in each scenario
  - cost of each action
- Portfolio cost limited to around one third of the sum of all action costs

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**Table 1**
Expert estimates on actions' scenario values and relative costs.

| $j$ | Action title | Uncertain supply of plantation wood | Wood has no use in energy production | Uncertain/ volatile wood prices | Molecule level high-tech paper products | Nano-science has failed | All wood is certified | Wild wood markets dominate | Strong environmental regulation | Cost $a_j$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x^j(s_1)$ | $x^j(s_2)$ | $x^j(s_3)$ | $x^j(s_4)$ | $x^j(s_5)$ | $x^j(s_6)$ | $x^j(s_7)$ | $x^j(s_8)$ | |
| 1 | CO2 technology (T) | 0 | 4 | 0 | 0 | 4 | 0 | 0 | 7 | 2.5 |
| 2 | Flexible production (O) | 6 | 2 | 5 | 6 | 0 | 1 | 1 | 6 | 5 |
| 3 | Less price sensitive (O) | 0 | 4 | 0 | 4 | 4 | 4 | 0 | 0 | 1 |
| 4 | Focus on South America (O) | 1 | 0 | 0 | 0 | 5 | 1 | 0 | 1 | 5 |
| 5 | Vertical integration (O) | 7 | 2 | 6 | 5 | 5 | 6 | 0 | 1 | 2.5 |
| 6 | Deinked pulp (O) | 6 | 4 | 2 | 0 | 0 | 7 | 6 | 2 | 5 |
| 7 | Erasable paper (T) | 3 | 0 | 1 | 6 | 0 | 0 | 0 | 0 | 10 |
| 8 | Production in China (O) | 0 | 5 | 0 | 4 | 6 | 0 | 0 | 1 | 5 |
| 9 | Multi-fibre units (T) | 7 | 3 | 5 | 6 | 5 | 5 | 6 | 1 | 5 |
| 10 | Bio refineries (O) | 4 | 0 | 4 | 4 | 4 | 2 | 0 | 1 | 5 |
| 11 | Smart papers (T) | 0 | 3 | 2 | 2 | 0 | 0 | 0 | 0 | 10 |
| 12 | Hybrid media (T) | 3 | 3 | 3 | 5 | 2 | 0 | 0 | 1 | 10 |
| 13 | Broad portfolio (O) | 0 | 3 | 4 | 6 | 4 | 1 | 4 | 1 | 2.5 |
| 14 | Small mills (O) | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 5 | 5 |
| 15 | Fusions (S) | 0 | 3 | 0 | 0 | 5 | 0 | 4 | 1 | 2.5 |
| 16 | Secure raw material (S) | 6 | 3 | 7 | 3 | 3 | 7 | 7 | 1 | 5 |
| 17 | Technology company (S) | 3 | 4 | 0 | 7 | 0 | 0 | 0 | 0 | 10 |
| 18 | Traditional technology (T) | 0 | 0 | 3 | 0 | 6 | 0 | 0 | 6 | 2.5 |
| 19 | Adaptable production (O) | 5 | 0 | 4 | 0 | 0 | 1 | 5 | 1 | 5 |
| 20 | Mass-customisation (O) | 0 | 4 | 0 | 0 | 2 | 0 | 0 | 1 | 1 |
| 21 | Mini-mills (O) | 1 | 1 | 0 | 1 | 0 | 6 | 1 | 4 | 5 |
| 22 | Brand paper (O) | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 23 | Region portfolio (O) | 0 | 0 | 0 | 3 | 6 | 1 | 6 | 3 | 2.5 |
| 24 | Paper collection (S) | 0 | 2 | 3 | 1 | 3 | 7 | 6 | 1 | 5 |

**Liesiö and Salo, 2012, Scenario-based portfolio selection of investment projects with incomplete probability and utility information**

# Application

- Compare sets of non-dominated portfolios between 4 different information sets
- Probability sets
  - Rank ordering

$$P^R = \{p \in P^0 \,|\, p_3 \geq p_2 \geq p_4 \geq p_8 \geq p_5 \geq p_7 \geq p_6 \geq p_1\}$$

  - Incomplete rank ordering

$$P^{IR} = \{p \in P^0 \,|\, p_{i_a} \geq p_{i_b} \geq p_{i_c} \geq p_{i_d} \forall i_a \in \{2,3\},$$
$$i_b \in \{4,8\}, i_c \in \{5,7\}, i_3 \in \{6,1\}\}$$

- Utility functions
  - Linear $\{u_L\}$
  - Strictly increasing concave $U^A$

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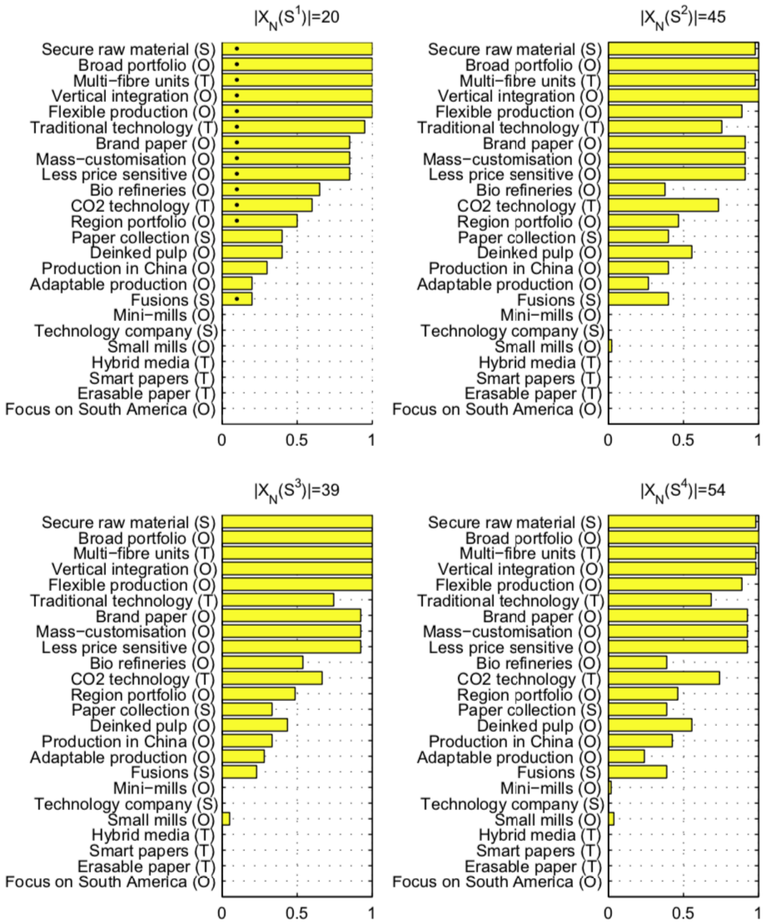


$$S^1 = P^R \times \{u^L\}$$

$$S^2 = P^{IR} \times \{u^L\}$$

$$S^3 = P^R \times U^A$$

$$S^4 = P^{IR} \times U^A$$

**Liesiö and Salo, 2012, Scenario-based portfolio selection of investment projects with incomplete probability and utility information**

# Conclusions

- Framework can be used to get a good broad overview of different investment possibilities
  - Allows interactivity: compare results between different restrictions on utility function and probability information
  - Recommendations on choosing projects can be given based on e.g. how many non-dominated portfolios they are included in, min/max portfolio utilities…
- Solving one optimal portfolio may be very sensitive to point estimates of scenario probabilities

# Reference

Liesiö and Salo, 2012, “Scenario-based portfolio selection of investment projects with incomplete probability and utility information”, *European Journal of Operational Research 217*, p. 162–172

# Thank you!

# Homework

You are a wine producer deciding on investing future development projects. You have three possible projects to invest in that give the following potential profits in three disjoint wine market scenarios:

| | S1: Alcohol legislation tightens abroad | S2: Alcohol legislation tigtens in home country | S3: Alchol legislation tightens worldwide |
|---|---|---|---|
| x1: Strengthen business in home country | 2 | 3 | 1 |
| x2: Venture into non-alcoholic wines | 6 | 3 | 5 |
| x3: Strengthen business in neighboring countries | 0 | 9 | 0 |

# Homework

You have enough resources to invest in up to two development projects.

**a)** List all feasible project portfolios. Which portfolios are not dominated, assuming you prefer more money to less and there is no information on scenario probabilities?

**b)** You get an evaluation from an expert saying that scenarios 1 and 3 are equally likely and that scenario 2 will happen with probability $0.1 \leq p_2 \leq 0.4$. List all non-dominated portfolios, assuming you are risk-neutral.

**c)** List all non-dominated assumption when addition to **b)**, we know the DM has a risk-averse utility function $U^A$ and that $u(6) = 0.8$?

**DL 16.10. 09:00**

# Finding the set of non-dominated portfolios

**Step 1:**

Rule out all dominated portfolios under

$$X \succ_{P^0 \times U} X' \Leftrightarrow X(s_i) \geq X'(s_i)\ \forall i \in \{1,\ldots,n\}$$

$\Rightarrow$ Compute all Pareto-optimal solutions to the following multiple objective binary linear programming problem:

$$\max_z \{Cz \mid Az \leq B, z \in \{0,1\}^m\}$$

where $C$ contains all scenario-specific project outcomes $[C]_{ij} = x^j(s_i)$